Credit Card Competition and Naive Hyperbolic Consumers*

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Abstract

We consider the credit card market with hyperbolic consumers. We show that naive hyperbolic borrowers might be unresponsive to the interest rates and credit limits of credit card offers when these offers contain a grace period. Consequently, we demonstrate that there might be no competition with regard to the interest rate and credit limit even if the consumer accepts only one card. We determine whether credit card companies can exploit time-inconsistent consumers and gain positive expected profits. We show that, in fact, there are circumstances in which both zero and positive expected profits are possible.

The credit card industry has evoked much interest in the last few years due to the conjunction of persistently high interest rates and what appears to be vigorous competition among credit card providers (Ausubel, 1991). The two questions that have been most puzzling are why interest rates remain high despite the competition and, why consumers continue to borrow at these high rates. Moreover, there is no agreement in the literature as to whether banks/companies earn competitive profits in this market (Evans and Schmalensee, 2005). Motivated by these questions, we present a theoretical model of credit card competition to determine the possibility for positive expected profits in the market equilibrium. The main features of the model are time-inconsistent consumers and a grace period offer in credit card contracts. We therefore also contribute to the debate as to whether time-inconsistent consumers

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are “money pumps” or whether competition effectively eliminates the disadvantage arising from this inconsistency.[1]

We show that there are positive expected profit equilibria for some parameter values and zero profit equilibria for other parameter values. We also find that there are multiple equilibria with zero and positive expected profits for still other parameter values.

As discussed in Ausubel (1991), we believe that there are three groups of consumers in the credit card market. The first group contains convenience users who never pay interest on their cards and are termed "deadbeats" in the credit card industry jargon. The second group comprises borrowers who revolve and pay interest. This group searches for the lowest interest rate available. Finally, the last group encompasses those who believe that they are not going to borrow on their cards, but end up borrowing due to commitment problems. This group is not searching for the lowest interest rate, and therefore is the most profitable group for the companies. This is because the first group does not generate interest revenue and the second group consists of high-risk consumers. In this paper, we analyze the contracts offered to the last group.

In our model, there is an initial period of contracting, followed by three consumption periods.[2] In the initial period, two credit card companies simultaneously offer contracts that are defined by the interest rate and the credit limit. The contracts include a grace period, which we specify as a period in which the consumer may pay his debt without interest.

The consumer is time-inconsistent and has a constant income at each period.[3] We model time inconsistency using the quasi-hyperbolic discount structure from Phelps and Pollak (1968) and Laibson (1997). The consumer chooses one (or both) of the contracts during the contracting period according to his plan for future consumption. If there is at least one contract offering enough credit (more than the amount the consumer wants to borrow during each period), the consumer chooses only one contract because there is an infinitesimal cost of accepting a contract. Otherwise, he chooses both contracts. At each period, he decides how much to borrow and how much to pay back depending on his plan for future consumption. The consumer does not pay interest if the previous period’s borrowing is less than what he can pay back in the current period from his income (grace period feature). For convenience, we consider the consumer making decisions at different periods as different selves of the consumer. Each period self underestimates future consumptions. We analyze only

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[1] For a detailed discussion, see Laibson and Yariv (2004)

[2] We will use the terms "contracting period", "initial period", or "period zero" interchangeably throughout the paper.

[3] Strotz (1956) claims that people would not obey their optimal plan of the present moment if they were allowed to reconsider their plans in future periods. Because people are impatient, they give more weight to the earlier time as it approaches, and this causes time inconsistent behavior. There is a significant amount of evidence for the existence of time-inconsistent preferences. See, for instance, Thaler (1981), Frederick, and Loewenstein and O’Donoghue (2002).

the consumer with contracting period self believing to borrow less than income at each consumption period and with period one self borrowing more than income.

If the consumer is not allowed to default (he must pay his borrowings back by the last period), companies can safely offer high enough credit limits and the consumer chooses only one contract. The contracting period self does not take the interest rate into account when choosing a contract because he believes he will not pay interest. However, the first period self ends up borrowing more than his income, and thus pays interest. Therefore, the contracting period self's indifference to interest rates eliminates the interest rate competition among the companies, while the first period self's high borrowing activity creates a positive profit for the chosen contract.

We also analyze the problem by allowing the consumer to default. There is an exogenously given cost of default to the consumer. We interpret this cost as the cost of bankruptcy proceedings and of receiving unfavorable terms in any contract in the future after declaring bankruptcy. If this cost is high enough (the consumer's risk of default is low enough), we obtain positive profit equilibria, as in the no default case. If the cost is small enough, we obtain zero profit equilibria. This is because none of the companies can offer a large enough credit limit without inducing default, and the consumer chooses both contracts at the contracting period. Although the contracting period self is indifferent to interest rates, the future period selves are not, since they realize they will pay interest. The second period self pays back the higher interest rate company within the grace period in order to minimize the interest payment. Therefore, the higher interest rate company does not earn a positive profit. Foreseeing this, credit card companies compete with regard to interest rates, resulting in zero profits.

In the literature, there are experimental and empirical studies that analyze consumer behavior in the credit card market.[4] There are also a few theoretical papers that provide alternative explanations for the sticky and high interest rates. Parlour and Rajan (2001) construct a model in which the competing firms cannot sustain zero-profit equilibria if the incentive to default is high enough and if there is multiple contracting. Parlour and Rajan's (2001) results depend on a moral hazard problem because the consumer decides whether or not to default during the same stage in which he decides which offers to accept. In our model, as opposed to that of Parlour and Rajan (2001), neither bankruptcy nor multiple contracting results in positive profits. On the contrary, bankruptcy and multicontracting are factors that work against positive profits. If we do not include bankruptcy in our model, then we would observe only positive profit equilibria. Moreover, the predictions of our model and that of Parlour and Rajan (2001) are completely opposite in regards to the effects of the bankruptcy law changes of 2005. The new law increased the cost of default for the consumers. According to Parlour and Rajan's (2001) model, this change should decrease the positive profits closer to a competitive level. According to our model,

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[4] See Calem and Mester (1995), Ausubel (1999), Laibson, Repetto and Tobacman (2001) and Ausubel and Shui (2004)

however, this change should increase the positive profits.

DellaVigna and Malmendier (2004) carefully analyze a firm's profit-maximizing contract design when the consumers are partially naive hyperbolic discounters. They determine that a firm prices "investment goods" less than the marginal cost and "leisure goods"—e.g. credit card financed consumptions—higher than the marginal cost if the firm proposes a two-part tariff . They also show that this result does not depend on the monopoly assumption, although the profits realized by the firms are zero under perfect competition. They provide a number of evidence for the predictions of their model from a variety of markets (e.g. health clubs, credit cards). Eliaz and Spiegler (2006) characterize the menu of contracts a monopolist would offer when facing dynamically inconsistent consumers. They also provide examples for a variety of markets, including credit cards, for the application of their results. These two papers analyze monopoly contracts when the consumers are dynamically inconsistent. Our interest, however, concerns the competitive contracts and the possibility of positive profits under these contracts when consumers are dynamically inconsistent. Moreover, our model is specific to the credit card market, mainly due to inclusion of the default risk. It is crucial to include the default risk in a model specific to the credit card market in order to contribute to the literature on the possibility of noncompetitive profits. Ausubel (1991) finds an evidence for noncompetitive profits, but Evans and Schmalensee (2005) criticize this evidence on the basis of not adjusting for the differences in risk factor. Moreover, credit card companies argue that the interest rates are high because of the high default risks but not because of the noncompetitive prices, as opposed to Ausubel (1991). Our results work against the claims of the credit card companies, as we show that the higher risk of default should result in lower credit limits rather than higher interest rates on offered contracts. This result can be observed only if one includes credit limits as well as interest rates in contract specifications, as in our model.

In another paper, Brito and Hartley (1995) show that rational individuals may choose to pay interest on a credit card rather than pay the transaction costs on regular bank loans. We are interested in the competition between credit card companies rather than the competition between credit card financing and other forms of financing. Investigating the competition between credit card companies shows that we should observe lower credit limit offers accompanied by lower interest rates in the credit card market if the risk of default is high, although Brito and Hartley (1995) argue that the high interest rates are due to higher risk.

In our model, we include the grace period feature of the credit card market. To the best of our knowledge, this feature was not included in the previous credit card competition models. If there was no grace period in the contracts, then there would not be any convenience users and these contracts would be —correctly— treated as loans. The grace period feature enables credit card companies to earn a profit from consumers who would not otherwise borrow at these high rates.

In summary, we show that some naive hyperbolic consumers are unresponsive to interest rates because of the grace period feature of contracts. This behavior of the naive consumers may result in positive expected profits for the companies. We demonstrate that there are in fact circumstances in which both zero and positive expected profits can be possible.

1 The Model

There are three periods of consumption preceded by an initial period, which is designated for contracting only.[5] One good is present at each consumption period. The model contains three agents: one consumer and two companies who compete against each other for the credit card business of the consumer. Companies know the consumer's type, so that this is a model of competition for each type of consumer.

1.1 *The Consumer*

Following Phelps and Pollak (1968) and Laibson (1997), we use quasi-hyperbolic discounting to model the time-inconsistent consumer. In the literature, two kinds of consumers are discussed. The first kind is called naive, as he is not aware of his time inconsistency. Specifically, he knows that his future discounting today is $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ..\}$, and believes that from tomorrow on it will be $\{1, \delta, \delta^2, \delta^3, ..\}$, although in reality it will be $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ..\}$. The second kind, called sophisticated, is aware of his time inconsistency. He knows that his future discounting today is $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ..\}$, and he correctly anticipates that it will be $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ..\}$ from tomorrow onward. O'Donoghue and Rabin (2001) introduce a model to represent a partially naive hyperbolic consumer who is aware of his time inconsistency but who underestimates its severity. The partially naive hyperbolic consumer knows that his future discounting today is $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ..\}$, and he incorrectly believes that it will be $\{1, \beta'\delta, \beta'\delta^2, \beta'\delta^3, ..\}$ from tomorrow onward with $\beta < \beta'$.

We analyze only the naive hyperbolic consumer with positive time discounting—$\delta \in [0, 1]$ and $\beta \in [0, 1]$.[6] The consumer chooses contracts at the initial period and consumes the consumption good in the consumption periods. There is an arbitrarily small cost for accepting a contract. The consumer's total utility is also affected by whether he defaults at the last period. In each period $t = 0, 1, 2$, the consumer aims to maximize U_t, where[7]

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[5] We need at least three periods to observe an interest-bearing debt.

[6] We can get the same results with partially naive hyperbolic consumers as well, although the calculations will be much more demanding. We provide an example of the case with the sophisticated hyperbolic consumer in Section 3.1.

[7] We do not need to write U_3 because period three is the terminal period, which implies that the period two plan is implemented.

$$\begin{aligned} U_0 &= \beta\delta\left[u(c_1^0)+\delta u(c_2^0)+\delta^2 u(c_3^0)+\delta^2 v(d^0)\right], \\ U_1 &= u(c_1^1)+\beta\delta u(c_2^1)+\beta\delta^2 u(c_3^1)+\beta\delta^2 v(d^1), \\ U_2 &= u(c_2^2)+\beta\delta u(c_3^2)+\beta\delta v(d^2). \end{aligned} \tag{1}$$

At this point, it is convenient to consider the consumer that makes decisions at different periods as different selves of the consumer. Here $c_\tau^t \in \mathbb{R}_+$ is the consumption at period τ according to period t self, and $d^t \in \{-1, 0\}$ is the period t self's plan concerning defaulting: $d^t = -1$ denotes default and $d^t = 0$ denotes no default. The function u is strictly increasing, concave, and satisfies the standard Inada conditions. The function v satisfies $v(-1) = -C$ and $v(0) = 0$, where $C > 0$ is the exogenous cost of default. As the consumer becomes more of a risky prospect, this cost of bankruptcy is lower for him.

The consumer chooses a trade $y_j^t = (s_j, n_{j1}^t, p_{j2}^t, n_{j2}^t, p_{j3}^t, d^t)$ for each company's card $j = 1, 2$ during each period $t = 0, 1, 2$. Here $s_j \in \{0, 1\}$, where $s_j = 1$ denotes that the contract is signed with company j and $s_j = 0$ denotes that it is rejected; $n_{j\tau}^t \in \mathbb{R}_+$ is the new borrowing from company j at time τ according to the period t self; $p_{j\tau}^t \in \mathbb{R}_+$ is the repayment to company j at time τ according to the period t self. At each consumption period, the consumer receives an income of m.

These trades of the consumer determine the consumption plan of each period t self:

$$\begin{aligned} c_1^t &= m + \sum_{j=1}^{2} s_j n_{j1}^t;\ t = 0, 1. \\ c_2^t &= m + \sum_{j=1}^{2} s_j \left(n_{j2}^t - p_{j2}^t\right);\ t = 0, 1, 2. \\ c_3^t &= m - \left(1 + d^t\right) \sum_{j=1}^{2} s_j p_{j3}^t;\ t = 0, 1, 2, 3. \end{aligned} \tag{2}$$

The consumer chooses the trades sequentially. The consumption and default plan of self t does not affect the plans of the subsequent selves' except for the trades completed at time t —those for which $t = \tau$.

1.2 *The Companies and the Class of Contracts*

For simplicity, we assume that the only source of revenue is from the interest payments. Company j's profit is $\Pi_j = (1 + d^2)\,(n_{j1}^1 - p_{j2}^2) r_j + d^2\left(n_{j1}^1 + n_{j2}^2 - p_{j2}^2\right)$. Each credit card company j charges an interest rate of r_j for loans of more than one period, although it is not permitted to charge interest for only one-period loans (the grace period). A credit card company loses everything lent if the consumer defaults.

The company's strategy set consists of contracts specified by a credit limit l and an interest rate $r \in [0,1]$.[8] The consumer's total debt cannot be greater than his credit limit. Moreover, his repayment cannot be higher than his income or his total debt at any period.

1.3 *Strategic Interaction*

The two companies make simultaneous contract offers and the consumer decides which one to choose at the initial period; subsequently the consumer makes two sequential decisions as described in Section 1.1. Therefore, the only strategic game between the companies and the consumer takes place at period zero.

We focus on the pure strategy subgame perfect equilibria of this game.[9] From this point on we will examine only the subgame perfect equilibria with three tie-breaking conventions; (i) if the consumer wants to choose one contract and is indifferent among any two contracts, then either choice is equally likely; (ii) if a company is indifferent among two contracts to offer and if the credit limits iof these contracts are higher than m, then the company offers the one with the lower credit limit; (iii) if a company is indifferent among two contracts to offer and if the credit limits of these contracts are lower than m, then the company offers the one with the higher credit limit.[10]

2 Analysis

We first determine how the consumer chooses among the contracts given that he is not allowed to default. Our approach is as if there is only one interest rate affecting the consumer's problem. We later confirm that only one interest rate is relevant for the consumer in equilibrium, even if he accepts two different contracts. Second, we analyze the consumer's decision regarding default. Third, we explain the companies' objective functions and determine the best responses for two different cases; when only one contract is selected (designates the "first case") or when both contracts are selected (designates the "second case"). Fourth, we show the existence of the equilibrium; we assume that a particular case holds and calculate the best responses, and then we check whether the strategies are consistent with that assumed case. If they are not consistent, we carry out the same exercise for the other case.

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[8] Our results would not change as long as the upper bound for the interest rate is finite.

[9] As a difference from the standard Subgame Perfect Equilibrium, a naive hyperbolic consumer has wrong beliefs about his future decisions. This does not create any problems for the definition of the equilibrium in our case, since there is no strategic game after the contracting period, but just a decision-making problem.

[10] See the discussion of the first convention in Section 4. The last two conventions are just to restrict the set of equilibria and our results are not affected by these conventions.

2.1 *The Consumer's Behavior with no Default Option*

The consumer needs to pay all of his debt back by the last period. Therefore, he cannot borrow more than m at the second period. This means that new borrowing at the second period will never create interest revenue. On the other hand, borrowing at the first period may result in interest revenue, as there are two periods for the consumer to pay back his debt following the first period. We analyze only the first-period new borrowing as the interest revenue results from the first-period borrowing only.[11]

2.1.1 *Exponential consumer* ($\beta = 1$)

We first analyze the exponential consumer as our benchmark case ($0 < \delta < 1$ and $\beta = 1$). The exponential consumer is time consistent, and therefore the first period borrowing according to the period zero self (believed amount of borrowing) and according to the period one self (actual amount of borrowing) is the same ($n_1^0 = n_1^1$).[12] From this point on we will use n_1^0 for the first period borrowing, keeping in mind that $n_1^0 = n_1^1$.

Proposition 1 *For an exponential consumer, there is a* δ^*, *such that* $n_1^0 < m$ *for* $\delta^* < \delta$. *In other words, the consumer does not borrow more than his income if his* δ *is high enough. This cutoff* δ^* *is irrespective of the interest rate offered. There is also a* $\delta^{**}_{r=1}$, *such that* $n_1^0 > m$ *for* $\delta < \delta^{**}_{r=1}$. *In other words, the consumer borrows more than his income (and consequently pays interest) if his* δ *is low enough, even with the highest interest rate* ($r = 1$). *The consumer borrows exactly* m *if* $\delta^{**}_{r=1} \leq \delta \leq \delta^*$ *with* $r = 1$.

Proof. See the Appendix. ■

Let us explain whether an exponential consumer with δ discount factor borrows or not with the aid of the following diagram.

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[11] The interest revenue will come only from the first period borrowing even if we allow the consumer to default. If the consumer borrows more than m at the second period, he cannot pay it back at the third period, declares bankruptcy and does not pay interest. If he borrows less than m at the second period, then he can pay it back at the third period and does not pay interest.

[12] In general, $n_\tau^t = n_\tau^\tau$ and $p_\tau^t = p_\tau^\tau$ for all $0 \leq t \leq \tau$.

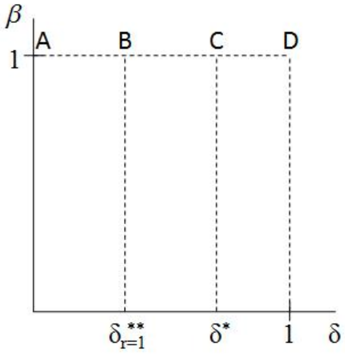

Fig. 1. δ Cutoffs for an Exponential Consumer's Borrowing

The x-axis shows the δ discount factor in $[0, 1]$. The y-axis shows the β hyperbolic discount factor in $[0, 1]$. Note that $\beta = 1$ for an exponential consumer. In this diagram, if the consumer is between point C and point D, the consumer borrows less than his income, irrespective of the interest rate. If $r = 1$ and the consumer is between point B and point C, he borrows the exact amount m and does not pay interest. If $r = 1$ and the consumer is between point A and point B, he borrows more than his income, cannot pay his debt back within the grace period, and pays interest.[13]

Proposition 2 *If the consumer is an exponential discounter, only zero profit equilibrium exist.*

Proof. If $\delta > \delta^*$, the contracting period self knows that he will not consume more than m at any consumption period and he will not pay interest. Therefore, he is unresponsive to interest rates. Nevertheless, being unresponsive to interest rates does not hurt him, since he will not pay interest anyway in the future as $n_1^0 = n_1^1$. The companies earn zero profit even without competition on interest rates.

If $\delta < \delta^*$, no equilibrium contract will have a positive interest rate. Suppose that company i offers a contract with interest rate $r_i > 0$ and $\delta < \delta_{r_i}$ (the consumer correctly anticipates paying interest in the future). If company j offers $0 < r_j < r_i$, the consumer will choose the contract offered by company j to minimize his interest pay-

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[13] Note that the cutoff at $\delta^{**}_{r=1}$ would shift to the right if the interest rate was smaller, and people with higher δ than $\delta^{**}_{r=1}$ would pay interest. In the proof of the previous proposition, we show that $\delta^{**}_{r=1} \leq \delta^{**}_{r} \leq \delta^{**}_{r=0} = \delta^*$. Since r will be endogenously determined, we want δ cutoffs to be free of r. Therefore, we determine the cutoffs for $r = 0$ and $r = 1$. Later, we show that our analysis holds for all $r \in [0, 1]$.

ment. This will create interest rate competition, resulting in zero profit equilibrium.[14]
■

In the previous proof, the first type of consumer mentioned is a convenience user. He uses his credit card just for convenience or to utilize the grace period, and he does not pay interest. We would not observe this type of a consumer in the credit card market if there was no grace period feature in the contracts. The second type of consumer mentioned is a revolver who searches for the lowest rate.

2.1.2 *Naive hyperbolic consumer* ($\beta < 1$)

A naive hyperbolic consumer is time inconsistent ($\beta < 1$) and is not aware of it. Therefore, he underestimates his future borrowings, in contrast to the exponential consumer; the initial-period self's believed amount of borrowing for the first period, n_1^0, is always less than the actual amount of borrowing, n_1^1.

In this section, we show that there exists a naive hyperbolic consumer (specified by δ and β) with an initial-period self who does not plan to borrow more than his income at any consumption period, but with a period one self who ends up borrowing more than his income and paying interest. First, we show the existence of an initial-period self who does not plan to borrow more than his income even at a zero interest rate (namely $\delta > \delta^*$), and second, we show the existence of a period one self who ends up borrowing more than his income even at the highest interest rate, $r = 1$.

Proposition 3 *For a naive hyperbolic consumer, there is a* δ^**, such that* $n_1^0 < m$ *for all* $\delta > \delta^*$*. There is also a* $\beta^{**}(\delta^*)$*, such that* $n_1^0 < m < n_1^1$ *for all* (δ, β) *where* $\delta > \delta^*$*,* $\beta < \beta^{**}(\delta^*)$*, and* $r = 1$.

Proof. See the Appendix. ■

In the following diagram, we demonstrate how a naive hyperbolic consumer's period one self may end up borrowing more than his income even though his initial-period self plans not to.

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[14] The companies are free to choose any credit limit without inducing default, as the consumer never defaults. Therefore, the credit limits will not affect the consumer's choice, as each company can offer a high enough credit limit.

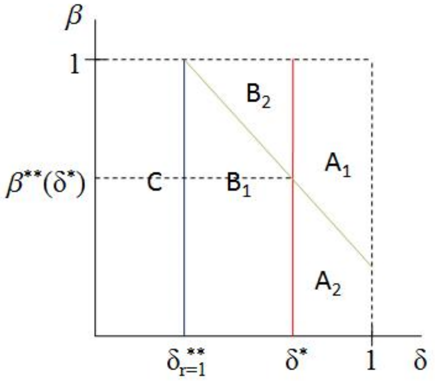

Fig. 2. δ and β Cutoffs for a Naive Hyperbolic Consumer's Borrowing

The initial period self believes that the β discount factor does not affect his future consumption plans, and therefore he will be acting just like an exponential consumer. If the consumer is in region A_1 or A_2, he believes that he will borrow less than his income, even with a zero interest rate. If he is in region C, he believes that he will borrow more than his income and pay interest even at the highest interest rate. If he is in region B_1 or B_2, he may believe that he will borrow more than his income, depending on the interest rate.

On the other hand, the period one self takes β into account when deciding how much to borrow. For the period one self, the interest rate (r) and the exponential discount factor (δ) are not the only determinants of borrowing anymore, but also the hyperbolic discount factor (β). Therefore, the vertical line at $\delta^{**}_{r=1}$ separating the interest payers from convenience users when $r = 1$, transforms into the downward sloped line in the diagram.[15] If the consumer is in region A_1, he does not borrow more than his income. If he is in region A_2, B_1, or C, he borrows more than his income and pays interest, even at the highest interest rate. Note that there is a conflict between what the initial period self believes and what the period one self ends up doing if the consumer is in region A_2, irrespective of the interest rate. Proposition 3 shows the existence of consumers in this region.

From this point on, we will analyze the region A_2 consumers only. The initial period self for this type of consumer is unresponsive to interest rates. However, the period one self ends up borrowing more than his income and paying interest, as opposed to the initial-period self's belief.

Since there is an arbitrarily small cost for accepting a contract, the consumer will choose only one contract if there is at least one company offering a large enough credit

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[15] This line does not have to be a straight one.

limit. Suppose the offered credit limits are l_1 and l_2,

- if $\max\{l_1, l_2\} \geq \max\{n_1^0, n_2^0\} \equiv n^0 > \min\{l_1, l_2\}$, then the consumer accepts the contract with the higher credit limit only;
- if $\min\{l_1, l_2\} \geq n^0$, then the consumer accepts one contract randomly;
- if $\max\{l_1, l_2\} < n^0$, then the consumer accepts both contracts.

Description of the equilibrium Recall that we consider a consumer who is in region A_2 and who never defaults. First of all, his initial period self is unresponsive to interest rates, and therefore the companies do not have to compete on interest rates. Second, he never defaults, and therefore the companies do not need to restrict the credit limits to prevent him from defaulting. Third, his period one self ends up borrowing more than his income and paying interest. All of these features create a perfect environment for noncompetitive profits, as each company chooses the profit maximizing interest rate and credit limit, just like a monopolist.

Proposition 4 *If the consumer is a naive hyperbolic discounter with* $\delta > \delta^*$, $\beta < \beta^{**}(\delta^*)$ *and if he never defaults, only positive profit equilibrium exists.*

Proof. Consumer's unresponsiveness to interest rates together with no possibility of defaulting enable companies to offer monopoly contracts. The contracting period self of the consumer chooses one contract randomly, and the company with the chosen contract earns a positive profit, as the period one self borrows more than his income. ■

2.2 *The Consumer's Plan for Defaulting*

The consumer's plan for defaulting at each period may differ from his previous plans due to time inconsistency. At $t = 0$, $t = 1$, and $t = 2$, the consumer compares his total utility from defaulting and not defaulting, which provides three different cutoffs, namely C_0, C_1, and C_2. If the consumer's exogenously given cost of default (C) is higher than these cutoffs, he chooses not to default. The relevant cutoff, the highest one of these three, depends on the number of contracts selected.

Lemma 1 *Suppose* l_1 *and* l_2 *are in the appropriate ranges, such that the consumer chooses only one contract and the first case occurs (*$\max\{l_1, l_2\} > n^0$*). The unselected contract's interest rate does not affect the relevant cutoff. The relevant cutoff is* $C' = \max\{C_0(l_1, l_2), C_1(l_1, r), C_2(l_1, r)\}$, *such that* (l_1, r) *is the selected contract.*[16]

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[16] We can prove that $C_t < C_{t+1}$ for $t = 0, 1$, if the consumer at period t is planning to default. The same comparison cannot be made easily if he is not planning to default at period t, since he may start planning to default in the subsequent period.

Proof. See the Appendix. ■

Lemma 2 *Suppose* l_1 *and* l_2 *are in the appropriate ranges, such that the consumer chooses both contracts and the second case occurs* ($\max\{l_1, l_2\} < n^0$)*. The higher interest rate does not affect the relevant cutoff. The relevant cutoff is*

$C' = \max\{C_1(l_1, l_2, r), C_2(l_1, l_2, r)\}$*, such that* (l_1, r) *is the lower interest rate contract.*

Proof. See the Appendix. ■

2.3 *Credit Card Companies' Behavior and Best Responses*

In this section, we show the specific shape of the best response functions (in terms of credit limits) for each case. The best responses are symmetric. In the first case, the best response credit limit of a company is at first constant, then decreases at the same rate as the other company's credit limit increases. In the second case, the best response credit limit of a company always decreases at the same rate as the other company's credit limit increases. These specific shapes of the best response functions guarantee the existence of an equilibrium.

2.3.1 *The first case (Only one contract is selected)*

There should be at least one contract that offers a credit limit more than the consumer's believed amount of borrowing for the first case to occur. Let us assume that the first case holds. Let us also assume that the selected contract is i, and the other one is j. The consumer's interest-bearing debt is $l_i - m$ for the selected contract if $l_i > m$, and is zero otherwise. We can write the chosen company's objective function as follows:

$$\max_{l_i, r} \ (l_i - m)r \tag{3}$$

$$s.t.$$

$$m \leq l_i \leq n_1^1 \tag{4}$$

$$0 \leq r \leq 1 \tag{5}$$

$$\max\{C_0(l_i, l_j), C_1(l_i, r), C_2(l_i, r)\} \leq C \tag{6}$$

Note that l_j affects the chosen company's optimal choice through the default constraint (6) and only if $C_0(l_i, l_j)$ is the relevant cutoff. However, it is not clear when $C_0(l_i, l_j)$ will be the relevant cutoff. Two different entities affecting the comparison between $C_0(l_i, l_j)$ and $\max\{C_1(l_i, r), C_2(l_i, r)\}$:

1. The potential credit limit to default on tends to make $C_0(l_i, l_j)$ higher than $\max\{C_1(l_i, r), C_2(l_i, r)\}$: The higher the potential credit limit to default on, the

higher will be the utility from defaulting. The potential credit limit to default on if he starts planning default at period zero $(l_i + l_j)$ is higher than if he starts planning default at period one (l_i).

2. The expenditure level on the credit card tends to make $\max\{C_1(l_i, r), C_2(l_i, r)\}$ higher than $C_0(l_i, l_j)$: The higher expenditure level on credit card causes the utility from defaulting to be higher. The consumer spends more at the consumption periods than he had planned in period zero.

As a result, these two opposite forces make it complicated to compare $C_0(l_i, l_j)$ with $\max\{C_1(l_i, r), C_2(l_i, r)\}$. Nevertheless, we can easily show that $C_0(l_i, l_j = 0)$ is less than $\max\{C_1(l_i, r), C_2(l_i, r)\}$, as in the proof of Lemma 2. We can then use this information to calculate the best responses in the following Lemma. Let us define $l_i^*(l_j)$ as the chosen company's best response credit limit to the other company's credit limit offer l_j.

Lemma 3 *Given that only one contract is selected, there is a cutoff* l_j''*, such that the best response credit limit of the first company:*

- *is constant at* $l_i^*(0)$ *for* $l_j < l_j''$*,*
- *decreases at the same rate with* l_j *for* $l_j'' \leq l_j < l_j'' + l_i^*(0)$*,*
- *is zero for* $l_j'' + l_i^*(0) \leq l_j$*.*

Proof. See the Appendix. ■

Lemma 4 *Given that only one contract is selected, the best response interest rate determined together with the best response credit limit is*

- *positive if* $l_i^*(0) > m$*,*
- *zero if* $l_i^*(0) \leq m$*.*

Proof. See the Appendix. ■

2.3.2 *The second case (Both contracts are selected)*

The second case occurs if no company offers a contract with a credit limit more than the consumer's believed amount of borrowing. The reason for offering such low credit limits is the binding default constraint, specifically

$\max\{C_1(l_i, l_j, r), C_2(l_i, l_j, r)\} = C$.

Lemma 5 *Given that both contracts are selected, each company offers a zero interest rate.*

Proof. See the Appendix. ■

Lemma 6 *Given that both contracts are selected, the best response credit limit of each company decreases at the same rate as the increase in the other company's credit limit until it becomes zero.*

Proof. See the Appendix. ■

2.3.3 *Zero and positive profit equilibrium regions*

Let us partition the Cartesian plane with companies' credit limit offers on the axes into three different regions, as shown in the figure below. If the consumer is going to choose one contract in equilibrium, the best responses should lie in region two and/or region three. Otherwise the best responses should lie in region one. Sometimes, some parts of the best responses calculated according to the first case may lie in region one. Since the best responses lying in region one are not consistent with the assumed case, there will be a discontinuity in the best responses in region one. We provide two examples of this situation in the appendix and show that it does not create a problem for the existence of an equilibrium.

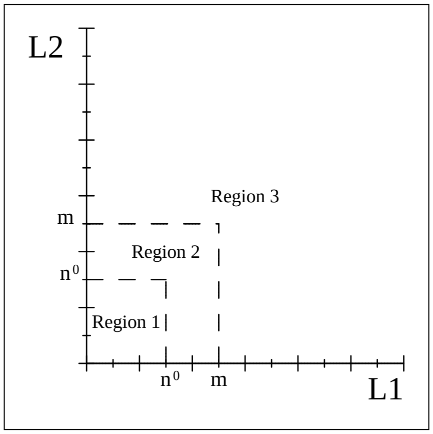

Fig. 3. Zero and Positive Profit Equilibrium Regions

All equilibria, which occur in region three, provide positive profit because the consumer chooses only one contract and there is no competition concerning interest rates. Region one and region two equilibria provide zero profit. In region two, the consumer chooses only one contract, but the credit limit on that contract prevents the consumer from borrowing more than his income. In region one, the consumer chooses both contracts, and zero interest rates in these contracts result in zero profit equilibria.

2.4 *Description of the Equilibria*

The following two propositions show the existence of positive and zero profit equilibria.

Proposition 5 *If $l_i^*(0) > m$, there exists a positive profit equilibrium.*

Proof. Depending on the value of l_j'', there are two possibilities:

1. If $l_j'' > l_i^*(0)$, then the best responses cross each other at $(l_j^*(0), l_i^*(0))$, which is a positive profit equilibrium (see Figure 4).

2. If $l_j'' \leq l_i^*(0)$, then there always exists a positive profit equilibrium at $(l_j'', l_i^*(0))$, since the best responses are symmetric and each of them is a decreasing line with -1 slope for $l_j'' \leq l_j < l_j'' + l_i^*(0)$ (see Figure 5). Note that $(l_j'', l_i^*(0))$ is an equilibrium, even if some other parts of the best response functions calculated according to the first case lies in region one (see Example 3 in the Appendix).

■

Proposition 6 *If $l_i^*(0) \leq m$, there exists a zero profit equilibrium.*

Proof. Depending on the values of $l_i^*(0)$ and l_j'', there are three possibilities:

1. If $n^0 \leq l_i^*(0)$ and $l_i^*(0) < l_j''$, then the best responses cross each other at $(l_j^*(0), l_i^*(0))$, which is a zero profit equilibrium (see Figure 6).

2. If $n^0 \leq l_i^*(0)$ and $l_j'' \leq l_i^*(0)$, then there exists a zero profit equilibrium at $(l_j'', l_i^*(0))$, since the best responses are symmetric and each of them is a decreasing line with -1 slope for $l_j'' \leq l_j < l_j'' + l_i^*(0)$ (see Figure 7). Note that $(l_j'', l_i^*(0))$ is an equilibrium, even if some other parts of the best response functions calculated according to the first case lies in region one (see Example 4 in the Appendix).

3. If $l_i^*(0) < n^0$, then two contracts are chosen in equilibrium. The best responses overlap in region $(0, l_j^*(0)) \times (l_i^*(0), 0)$ and provide zero profit equilibria (see Figure 8).

■

Remark 1 *The last two propositions cover all possible cases, depending on the parameter values, thereby proving the existence of an equilibrium.*

Under some parameter values, both zero and positive profit equilibria may be possible, as shown in Example 3 in the Appendix.

3 Extensions

In this section, we first provide a discussion of the case with a sophisticated hyperbolic discounter. We then determine whether the consumer would have been better off had we restricted the credit limits to income. Finally, we assess whether the equilibria in the original model would survive if the grace period interest rate were endogenous.

3.1 *Sophisticated Hyperbolic Consumer*

If the consumer is sophisticated, we do not obtain the same results. Although we could not solve the problem in general due to analytical intractability, we have an interesting example. Assume that the offered credit limits are the same. If the consumer is sophisticated, we expect him to choose the lower interest rate contract knowing that he is going to pay interest. This is true for some parameter values. On the other hand, surprisingly, for other parameter values, the consumer chooses the higher interest rate contract. The zero period self of the sophisticated consumer wants to limit his future selves' spending on credit. If he cannot do this by choosing a lower credit limit, he can choose a higher interest rate and consequently decrease his future selves' credit card spending, as if using the higher interest rate contract as a commitment device.[17]

3.2 *Consumer Welfare*

We calculate the consumer welfare according to the initial-period self. Whether he is better off with the restriction on the offered credit limits depends on the utility function and discount factors δ and β. We conclude that the welfare effects of credit limit regulation would be ambiguous. We provide an example for each case.

Example 1 *Let* $u(x) = x^{1/2}$, $\delta = 0.61$ *and* $\beta = 0.8$, *if the consumer's cost of default is high enough and if we do not restrict the credit limits, we find the total utility as* $U_0' = 2.1177m^{1/2}$. *If we restrict the credit limits to income, then the total utility is* $U_0'' = 2.1273m^{1/2}$. *Since* $U_0' < U_0''$, *the consumer would have been better off had we restricted the credit limits to income.*

Example 2 *Let* $u(x) = x^{1/2}$, $\delta = 0.61$, *and* $\beta = 0.9$. *If the consumer's cost of default is high enough and if we do not restrict the credit limits, we find the total utility as* $U_0' = 2.1262m^{1/2}$. *If we restrict credit limits to income, then the total utility is* $U_0'' = 2.1258m^{1/2}$. *Since* $U_0' > U_0''$, *the consumer would not have been better off had we restricted the credit limits to income.*

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[17] We have a numerical example for each case. It is available upon request.

3.3 *Endogenous Grace Period*

Our base model treats the introductory interest rate as an exogenous value set as zero, which corresponds to the grace period. Incorporating the competition on the introductory interest rate is left for future research. In this section, we determine which of the equilibria would survive if the introductory interest rate were also endogenous. We specify each contract j with a credit limit l_j, an introductory interest rate r_{j1}—for one-period loans— and a regular interest rate r_{j2}—for two-period loans—. Note that interest revenue from a one-period loan is possible in this case. Moreover, the introductory interest rate may also affect the number of contracts the consumer accepts. We divide the discussion into the following two cases. Let $((l_1, 0, r_{12}), (l_2, 0, r_{22}))$ denote an equilibrium.

Claim 1 *Some of the positive profit equilibria survive when we make the introductory interest rate endogenous.*

This claim follows from the subsequent arguments:

1. Let $l_1^*, l_2^* > m$. If a company deviates to offer a contract with a positive introductory interest rate, then the consumer will never choose that contract. Therefore, there is no profitable deviation from the current state.

2. Let $l_1^* > m$ and $n^0 > l_2^*$. If the first company deviates to offer a positive introductory interest rate, then the consumer chooses both contracts, since the second contract does not provide a high enough credit limit. By deviating, the first company increases his revenue from one-period loans, but decreases his revenue from two-period loans. Therefore, it is not clear whether this deviation is profitable.

Claim 2 *Some of the zero profit equilibria survive when we make the introductory interest rate endogenous.*

This claim follows from the subsequent arguments:

1. Let $m > l_1^*, l_2^* > n^0$. If a company deviates to offer a contract with a positive introductory interest rate, then the consumer will never choose that contract. As a result, there is no profitable deviation from the current equilibrium.

2. Let $m > l_1^* > n^0 > l_2^*$. If the first company deviates to offer a positive introductory interest rate, then the consumer chooses both contracts, since the second contract does not provide a high enough credit limit. By deviating, the first company increases his revenue from zero to a positive number and the current equilibrium does not survive.

3. Let $n^0 > l_1^*, l_2^*$. If a company deviates to offer a positive introductory interest rate, that company can make a positive revenue (because the consumer accepts both contracts) and the current equilibrium does not survive.

4 Discussion and Conclusion

In our model, we focus on two aspects of a credit card contract: the interest rate and the credit limit. Cash back and reward points (which can be modeled as transfers to the contracting period self of the consumer) are other aspects of credit card contracts that were not common ten years ago. If we include these aspects as well, then positive expected profits are not possible, even if there was no interest rate competition.

Consumer's time inconsistency and naivete, and credit card companies' grace period offers result in a positive expected profit equilibrium. If there is no grace period, then there would not be any convenience users in the credit card market, which is not consistent with what we observe in this market.

The small cost of applying for a credit card makes the consumer choose only one card when the offered credit limits are higher than the consumer's believed amount of borrowing. Once the consumer decides to get only one contract and if he is indifferent among the offered contracts, then he chooses one contract randomly. If the consumer believes that he might make a mistake and borrow more than his income, then he chooses the contract with the lower interest rate and our result for noncompetitive interest rates and a positive profit equilibrium would not hold. One might describe this as the "trembling-hand perfectness" problem with our equilibria. This problem, however, does not arise if there is an infinitesimal cost of investigating a contract. Consider the environment in which the consumer receives the offers in his mailbox in envelopes. He opens one envelope randomly. If the contract in the envelope has a high enough credit limit, more than the believed amount of borrowing, he would not open the second envelope and would accept the first contract. This is because it is costly to investigate each contract.

In our model, the consumer's cost of bankruptcy is exogenously given. With the recent changes in the bankruptcy law, one would expect this cost to increase for the consumer.[18] If this cost increases, the consumer is less likely to default and the credit card companies are more likely to obtain positive expected profits.

Since our results depend on the consumer's incorrect belief of his future consumption, we can easily obtain the same results for more than three periods of consumption, with appropriate cutoffs for δ and β. Moreover, with the exact same reasoning, we can produce similar results with partially naive hyperbolic consumers as well.

In our model, we do not permit contracting after the initial period. If we did, our results would still hold in this three periods of consumption model. This is because applying for a lower interest rate card does not decrease the interest payment on the first-period debt, which is charged on the initially chosen card. As a result, the only consumer who applies for a new card in the first period is the one planning to default. Therefore, there would be no credit card offers to this consumer after the

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[18] The new bankruptcy law, which took effect on October 2005, introduced stricter bankruptcy proceedings, such as more stringent conditions to file under chapter 7 and a mandatory financial management education program.

initial period. On the other hand, the case of more than three periods of consumption is more complicated. At any period, applying for a lower interest rate card decreases the interest payments on future debts. If the consumer cannot foresee his future interest-bearing debts because of the severe naive time inconsistency, then he does not apply for a lower interest rate card and our results still hold. However, there may be consumers who are able to anticipate future interest payments. In that case, we may approach the problem in two different ways. First, we may look at the entry deterrent strategies of the initially selected company, which may be to increase the consumer's risk of default by offering a higher credit limit, and consequently to prevent other companies from offering a contract to the consumer. Second, we may examine the case in which the initially selected company is allowed to match interest rates on future contracts offered by entering companies. In any of these cases, there is first-mover advantage, and the companies do not need to compete with regard to the interest rate in order to be the first-mover. Therefore, positive profit equilibria may still exist.

In summary, in this paper we show that positive expected profits might be possible in the credit card market. This is because there are some naive hyperbolic consumers who are unresponsive to interest rates on contracts with a grace period.

5 References

1. **Ausubel, L. M. (**1991). 'The failure of competition in the credit card market', *American Economic Review*, vol. 81(1), pp. 50-81.

2. **Ausubel, L. M.** (1999). 'Adverse selection in the credit card market', Working Paper, University of Maryland.

3. **Ausubel, L. M. and Shui, H.** (2004). 'Time inconsistency in the credit card market', Working Paper, University of Maryland.

4. **Brito, D. L. and Hartley, P.R.** (1995). 'Consumer rationality and credit cards', *The Journal of Political Economy,* vol. 103(2), pp. 400-33.

5. **Calem, P. S. and Mester, L. J.** (1995). 'Consumer behavior and the stickiness of credit-card interest rates', *The American Economic Review*, vol. 85(5), pp. 1327-36.

6. **Chakravorti, S.** (2003). 'Theory of credit card networks: a survey of the literature', *Review of Network Economics, vol.* 2(2), pp. 50-68.

7. **DellaVigna, S. and Malmendier, U.** (2004). 'Contract design and self control', *Quarterly Journal of Economics*, vol. 119(2), pp. 353-402.

8. **DellaVigna, S. and Malmendier, U.** (2006). 'Paying not to go to the gym', *American Economic Review*, vol. 96(3), pp. 694-719.

9. **Eliaz, K. and Spiegler, R.** (2006). 'Contracting with diversely naive agents', *Review of Economics Studies*, vol. 73(3), pp. 689-714.

10. **Evans, D. S. and Schmalensee, R.** (2005). *Paying with Plastic: The Digital Revolution in Buying and Borrowing,* Massachusets: MIT press.

11. **Frederick, S., Loewenstein, G. and O'Donoghue, T.** (2002). 'Time Discounting and Time Preference: A Critical Review', *Journal of Economic Literature*, vol. 40(2), pp. 351-401.

12. **Laibson, D.** (1997). 'Golden Eggs and Hyperbolic Discounting', *The Quarterly Journal of Economics*, vol. 112(2), pp. 443-77.

13. **Laibson, D., Repetto, A. and Tobacman, J.** (2003). 'A debt puzzle', in (P. Aghion *et. al.*, eds), *Knowledge, Information, and Expectations in Modern Economics: In Honor of Edmund S. Phelps,* pp. 228-66, Princeton: Princeton University Press.

14. **Laibson, D. and Yariv, L.** (2004). 'Safety in Markets: An Impossibility Theorem for Dutch Books', Society for Economic Dynamics Meeting Paper 867.

15. **Parlour, C. and Rajan, U.** (2001). 'Competition in Loan Contracts', *American Economic Review*, vol. 91(5), pp. 1311-28.

16. **Phelps, E. S. and Pollak, R.A.** (1968). 'On Second-Best National Saving and Game-Equilibrium Growth', *Review of Economic Studies,* vol. 35(2), pp. 185-99.

17. **O'Donoghue, T. and Rabin, M.** (2001). 'Choice and Procrastination', *Quarterly Journal of Economics*, vol. 116(1), pp. 121-160.

18. **Strotz, R. H.** (1956). 'Myopia and Inconsistency in Dynamic Utility Maximization', *Review of Economic Studies,* vol. 23(3), pp. 165-80.

19. **Thaler, R. H.** (1981). 'Some Empirical Evidence on Dynamic Inconsistency', *Economics Letters*, vol. 8(3), vol. 201-7.

20. **The Library of Congress.** 2005. Thomas.

    http://thomas.loc.gov/cgi-bin/bdquery/z?d109:SN00256:@@@L&summ2

    =m&|TOM:/bss/d109query.html. (accessed October 24, 2005).

21. **Building a Better Credit Report.** 2005. Federal Trade Commission. http://www.ftc.gov/bcp/conline/pubs/credit/bbcr.htm. (accessed October 10, 2005)

22. **Your Credit Scores.** 2005. Consumer Federation of America and FairIsaac. http://pueblo.gsa.gov/cic_text/money/creditscores/your.htm. (accessed October 10, 2005)

6 Appendix

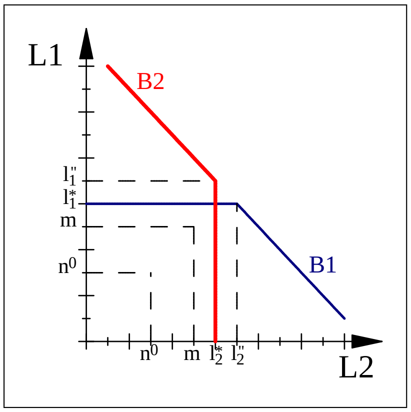

Fig. 4. Positive Profit Equilibrium

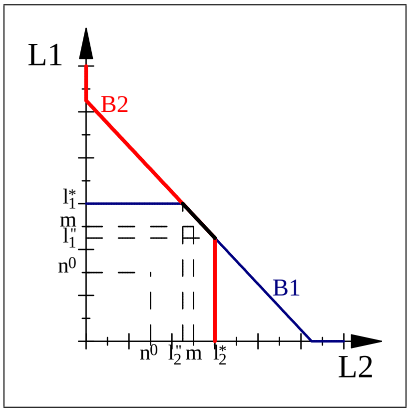

Fig. 5. Positive Profit Equilibria

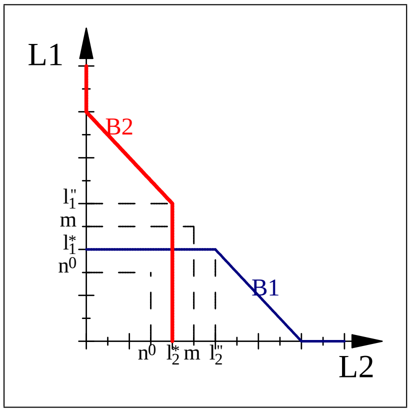

Fig. 6. Zero Profit Equilibrium

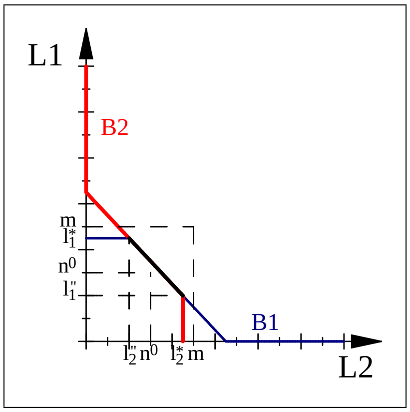

Fig. 7. Zero Profit Equilibria

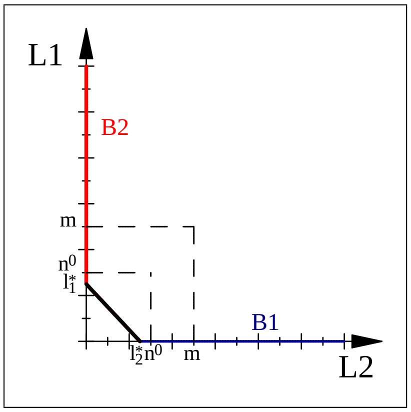

Fig. 8. Zero Profit Equilibria

Proof of Proposition 1. It is a dominant strategy for the consumer to pay as much of his borrowing as possible within the grace period. Therefore, if $n_1^t \leq m$, then $p_2^t = n_1^t$ and $p_3^t = n_2^t$; if $n_1^t > m$, then $p_2^t = m$ and $p_3^t = (n_1^t - m)(1+r) + n_2^t$.Now we can write the consumer's total utility function in terms of new borrowings (n_τ^t), income (m), and interest rate (r) only. The total utility function may take two different forms, depending on the value of the first period borrowing:

- If $n_1^t \leq m$, then $U_{n_1^t \leq m} = \delta[u(m+n_1^t) + \delta u(m - n_1^t + n_2^t) + \delta^2 u(m - n_2^t)]$.
- If $n_1^t \geq m$, then $U_{n_1^t \geq m} = \delta[u(m+n_1^t) + \delta u(n_2^t) + \delta^2 u(m - (n_1^t - m)(1+r) - n_2^t)]$.

Therefore, we will solve these two utility functions separately, and show that:

- $\max U_{n_1^0 \leq m}$ has an internal solution ($n_1^0 < m$) for $\delta > \delta^*$, and a boundary solution ($n_1^0 = m$) for $\delta \leq \delta^*$, irrespective of the interest rate.
- $\max U_{n_1^0 \geq m}$ has a boundary solution ($n_1^0 = m$) for $\delta \geq \delta^{**}_{r=1}$, and an internal solution ($n_1^0 > m$) for $\delta < \delta^{**}_{r=1}$ when $r = 1$.

We will complete the proof by demonstrating that $\delta^{**}_{r=1} \leq \delta^*$.

1. For $U_{n_1^0 \leq m}$, we can write the objective function of the consumer at time zero as follows:

$$\max_{n_1^0,\, n_2^0} \delta\left[u(m+n_1^0) + \delta u(m - n_1^0 + n_2^0) + \delta^2 u(m - n_2^0)\right] \tag{7}$$
$$s.t.\ n_1^0 \leq m, \ \ -n_1^0 \leq 0, \ and \ -n_2^0 \leq 0.$$

FOCs if the constraints are not binding:

$$u'(m - n_1^0 + n_2^0) = \delta u'(m - n_2^0) \tag{8}$$
$$u'(m + n_1^0) = \delta u'(m - n_1^0 + n_2^0) \tag{9}$$

We know that $n_1^0 = n_2^0 = 0$, when $\delta = 1$. Moreover, it is possible to show that $\frac{\partial n_1^0}{\partial \delta} < 0$ and $\frac{\partial n_2^0}{\partial \delta} < 0$. Let us take the derivative of (8) and (9) with respect to δ :

$$u''(m - n_1^0 + n_2^0)(-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) = u'(m - n_2^0) + \delta u''(m - n_2^0)(-\frac{\partial n_2^0}{\partial \delta}) \tag{10}$$
$$u''(m + n_1^0)\frac{\partial n_1^0}{\partial \delta} = u'(m - n_1^0 + n_2^0) + \delta u''(m - n_1^0 + n_2^0)(-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) \tag{11}$$

- Suppose $\frac{\partial n_1^0}{\partial \delta} \geq 0$, then $(-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) > 0$ and $\frac{\partial n_2^0}{\partial \delta} > 0$ by (11), which are in contradiction with (10). Therefore, $\frac{\partial n_1^0}{\partial \delta} < 0$.

- Now we know $\frac{\partial n_1^0}{\partial \delta} < 0$. Suppose $\frac{\partial n_2^0}{\partial \delta} \geq 0$, then $(-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) > 0$, which is in contradiction with (10). Therefore, $\frac{\partial n_2^0}{\partial \delta} < 0$.

  As n_1^0 decreases with δ, we can find δ^*, such that $n_1^0 \leq m$ for $\delta > \delta^*$ and $n_1^0 > m$ for $\delta \leq \delta^*$ from the following equalities.

  $$\begin{aligned} u'(n_2^0) &= \delta^* u'(m - n_2^0) \\ u'(2m) &= \delta^* u'(n_2^0) \end{aligned} \tag{12}$$

  As a result, the first constraint ($n_1^0 < m$) is not binding for $\delta > \delta^*$, and by the definition of the maximum:

  $$\begin{aligned} & \max_{n_1^0 < m,\ n_2^0} \delta \left[u(m + n_1^0) + \delta u(m - n_1^0 + n_2^0) + \delta^2 u(m - n_2^0) \right] \\ \geq \quad & \max_{n_1^0 = m,\ n_2^0} \delta \left[u(m + n_1^0) + \delta u(m - n_1^0 + n_2^0) + \delta^2 u(m - n_2^0) \right] \end{aligned} \tag{13}$$

  On the other hand, it is binding ($n_1^0 = m$) for $\delta \leq \delta^*$.

2. For $U_{n_1^0 \geq m}$, we can write the objective function of the consumer at time zero as follows:

   $$\begin{aligned} & \max_{n_1^0,\ n_2^0} \delta \left[\begin{array}{c} u(m + n_1^0) + \delta u(n_2^0) \\ +\delta^2 u(m - (n_1^0 - m)(1 + r) - n_2^0) \end{array} \right] \\ & s.t. \ -n_1^0 \leq -m, \ \ -n_1^0 \leq 0, \ and \ -n_2^0 \leq 0. \end{aligned} \tag{14}$$

   FOCs if the constraints are not binding:

   $$u'(n_2^0) = \delta u'(m(2 + r) - n_1^0(1 + r) - n_2^0) \tag{15}$$

   $$u'(m + n_1^0) = \delta(1 + r) u'(n_2^0) \tag{16}$$

   As we showed previously, n_1^0 decreases with δ, and we can find a $\delta_{r=1}^{**}$, such that $n_1^0 \leq m$ for $\delta \geq \delta_{r=1}^{**}$ and $n_1^0 > m$ for $\delta < \delta_{r=1}^{**}$ from the following equalities.

   $$\begin{aligned} u'(n_2^0) &= \delta_r^{**} u'(m - n_2^0) \\ u'(2m) &= \delta_r^{**} (1 + r) u'(n_2^0) \end{aligned} \tag{17}$$

   This means that the first constraint is binding ($n_1^0 = m$) for $\delta \geq \delta_{r=1}^{**}$, and we can write the objective function of the consumer as follows:

   $$\max_{n_2^0} \delta \left[u(2m) + \delta u(n_2^0) + \delta^2 u(m - n_2^0) \right] \tag{18}$$

   On the other hand, it is not binding ($n_1^0 > m$) for $\delta < \delta_{r=1}^{**}$, and by the definition of the maximum:

$$
\begin{aligned}
& \max_{n_1^0 > m,\ n_2^0} \delta[u(m+n_1^0) + \delta u(n_2^0) + \delta^2 u(m - \left(n_1^0 - m\right)(1+r) - n_2^0)] \quad (19) \\
\geq \ & \max_{n_1^0 = m,\ n_2^0} \delta\left[u(m+n_1^0) + \delta u(n_2^0) + \delta^2 u(m - \left(n_1^0 - m\right)(1+r) - n_2^0)\right]
\end{aligned}
$$

It is clear from (12) and (17) that $\delta^{**}_{r=1} \leq \delta^*$ and $\delta^{**}_{r=1} \leq \delta^{**}_r \leq \delta^{**}_{r=0} = \delta^*$.

For $\delta > \delta^*$, (13) becomes a strict inequality with $n_1^0 < m$. For $\delta^* \geq \delta \geq \delta^{**}_{r=1}$, (13) becomes an equality, which is also equal to (18) with $n_1^0 = m$. For $\delta < \delta^{**}_{r=1}$, (13) becomes an equality and (19) becomes a strict inequality with $n_1^0 > m$. This means that the zero period self of the consumer believes that he will not pay interest on his credit card debt if $\delta \geq \delta^{**}_{r=1}$.

■

Proof of Proposition 3.

We showed that an exponential consumer with $\delta > \delta^*$ correctly believes that he will borrow less than his income. A naive hyperbolic consumer with an exponential discount factor $\delta > \delta^*$ possesses the exact same belief ($n_1^0 < m$), but his belief may not be correct, depending on his hyperbolic discount factor, in contrast to the exponential consumer. Let us analyze the naive hyperbolic consumer with $\delta > \delta^*$. As before, the total utility function at the first period may take one of the following two different forms, depending on the first period borrowing:

- If $n_1^t \leq m$, then $U_{n_1^t \leq m} = u(m+n_1^t) + \beta\delta u(m - n_1^t + n_2^t) + \beta\delta^2 u(m - n_2^t)$.
- If $n_1^t \geq m$, then $U_{n_1^t \geq m} = u(m+n_1^t) + \beta\delta u(n_2^t) + \beta\delta^2 u(m - (n_1^t - m)(1+r) - n_2^t)]$.

We follow the similar steps as before and solve these two utility functions separately, and showing that:

- $\max U_{n_1^1 \leq m}$ has a boundary solution ($n_1^0 = m$) for all (δ, β), where $\delta > \delta^*$ and $\beta < \beta^*(\delta^*)$,
- $\max U_{n_1^1 \geq m}$ has an internal solution ($n_1^0 > m$) for all (δ, β) where $\delta > \delta^*$ and $\beta < \beta^{**}(\delta^*)$.

We will complete the proof by demonstrating that max $U_{n_1^1 \leq m} \leq \max U_{n_1^1 \geq m}$ for all (δ, β), where $\delta > \delta^*$ and $\beta < \beta^{**}(\delta^*)$.

1. For $U_{n_1^1 \leq m}$, we can write the objective function of the consumer at time one as follows:

$$
\begin{aligned}
& \max_{n_1^1,\ n_2^1} u(m+n_1^1) + \beta\delta u(m - n_1^1 + n_2^1) + \beta\delta^2 u(m - n_2^1) \quad (20) \\
& s.t.\ n_1^1 \leq m, \ -n_1^1 \leq 0, \ and \ -n_2^1 \leq 0.
\end{aligned}
$$

FOCs if the constraints are not binding:

$$u'(m - n_1^1 + n_2^1) = \delta u'(m - n_2^1) \tag{21}$$

$$u'(m + n_1^1) = \beta\delta u'(m - n_1^1 + n_2^1) \tag{22}$$

For $\delta > \delta^*$ and $\beta = 1$, the first constraint ($n_1^1 \leq m$) is not binding. Moreover, it is possible to show that $\frac{\partial n_1^1}{\partial \beta} < 0$ and $\frac{\partial n_2^1}{\partial \beta} < 0$. Let us take the derivative of (21) and (22) with respect to β:

$$u''(m - n_1^1 + n_2^1)(-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) = \delta u''(m - n_2)(-\frac{\partial n_2^1}{\partial \beta}) \tag{23}$$

$$u''(m + n_1^1)\frac{\partial n_1^1}{\partial \beta} = \delta u'(m - n_1^1 + n_2^1) + \beta\delta u''(m - n_1^1 + n_2^1)(-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) \tag{24}$$

- Suppose $\frac{\partial n_1^1}{\partial \beta} \geq 0 \Rightarrow (-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) > 0$ and $\frac{\partial n_2^1}{\partial \beta} > 0$ by (24). However, these two inequalities are in contradiction with (23). Therefore, $\frac{\partial n_1^1}{\partial \beta} < 0$.
- We know that $\frac{\partial n_1^1}{\partial \beta} < 0$. Suppose $\frac{\partial n_2^1}{\partial \beta} \geq 0$, then $(-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) < 0$ by (23), which is a contradiction. Therefore, $\frac{\partial n_2^1}{\partial \beta} < 0$.

From (21), let $\varepsilon_1 = (m - n_1^1 + n_2^1) - (m - n_2^1)$ and rearrange it to get $n_1^1 = 2n_2^1 - \varepsilon_1$. For any β as $\delta \to 1$, $\varepsilon_1 \to 0$ and $n_1^1 \to 2n_2^1$. Rewrite (22) as $u'(m + 2n_2^1) = \beta u'(m - n_2^1)$. As $\beta \to 0$, $\frac{u'(m+2n_2^1)}{u'(m-n_2^1)} \to 0$, then $n_2^1 \to m$, since the denominator will be infinity and the numerator will be a finite number. Consequently, $n_1^1 \to 2m$. As a result, there is a $\beta^*(\delta^*)$ such that the first constraint ($n_1^1 \leq m$) is binding for $\delta > \delta^*$ and $\beta < \beta^*(\delta^*)$ and the objective function is:

$$\max_{n_1^1 = m,\ n_2^1} u(m + n_1^1) + \beta\delta u(m - n_1^1 + n_2^1) + \beta\delta^2 u(m - n_2^1) \tag{25}$$

2. For $U_{n_1^1 \geq m}$, we can write the objective function of the consumer at time one as follows:

$$\max_{n_1^1,\ n_2^1} \begin{bmatrix} u(m + n_1^1) + \beta\delta u(n_2^1) \\ +\beta\delta^2 u(m - (n_1^1 - m)(1 + r) - n_2^1) \end{bmatrix} \tag{26}$$
$$s.t. \ -n_1^1 \leq -m, \ -n_1^1 \leq 0, \ and \ -n_2^1 \leq 0.$$

FOCs if the constraints are not binding:

$$u'(n_2^1) = \delta u'(m(2 + r) - n_1^1(1 + r) - n_2^1) \tag{27}$$

$$u'(m + n_1^1) = \beta\delta(1 + r)u'(n_2^1) \tag{28}$$

For $\delta > \delta^*$, in order to determine the change in n_1^1 and n_2^1 with β, we take the derivative of (27) and (28) with respect to β:

$$u''(n_2^1)(\frac{\partial n_2^1}{\partial \beta}) = \delta u''(m(2 + r) - n_1^1(1 + r) - n_2^1)(-\frac{\partial n_1^1}{\partial \beta}(1 + r) - \frac{\partial n_2^1}{\partial \beta}) \tag{29}$$

$$u''(m + n_1^1)(\frac{\partial n_1^1}{\partial \beta}) = \delta(1 + r)u'(n_2^1) + \beta\delta(1 + r)u''(n_2^1)\frac{\partial n_2^1}{\partial \beta} \tag{30}$$

- Suppose $\frac{\partial n_1^1}{\partial \beta} \geq 0$, then $\frac{\partial n_2^1}{\partial \beta} > 0$ by (30). However, these inequalities provide a contradiction in (29). Therefore, $\frac{\partial n_1^1}{\partial \beta} < 0$.
- Given that $\frac{\partial n_1^1}{\partial \beta} < 0$, suppose $\frac{\partial n_2^1}{\partial \beta} \leq 0$, then (29) gives a contradiction. Therefore, $\frac{\partial n_2^1}{\partial \beta} > 0$.

From (27), let $\gamma_1 = n_2^1 - (m(2+r) - n_1^1(1+r) - n_2^1)$ and rearrange it to obtain $n_1^1 = \frac{\gamma_1 - 2n_2^1 + m(2+r)}{1+r}$. For any β as $\delta \to 1$, then $\gamma_1 \to 0$, $n_1^1 \to \frac{-2n_2^1 + m(2+r)}{1+r}$. Rewrite (28) as $u'(m + \frac{-2n_2^1 + m(2+r)}{1+r}) = \beta(1 + r)u'(n_2^1)$. As $\beta \to 0$, $\frac{u'\left(\frac{-2n_2^1 + m(2+r)}{1+r}\right)}{u'(n_2^1)} \to 0$, then $n_2^1 \to 0$, since the denominator will be infinity and the numerator will be a finite number. Consequently, $n_1^1 \to m\frac{2+r}{1+r} > m$. As a result, there is a $\beta^{**}(\delta^*)$, such that the first constraint $(-n_1^1 \leq -m)$ is not binding for $\delta > \delta^*$ and $\beta < \beta^{**}(\delta^*)$, and by the definition of the maximum:

$$\begin{aligned} & \max_{n_1^1 > m,\ n_2^1} \left[\begin{array}{c} u(m + n_1^1) + \beta\delta u(n_2^1) \\ +\beta\delta^2 u(m - (n_1^1 - m)(1 + r) - n_2^1) \end{array} \right] \\ > & \max_{n_1^1 = m,\ n_2^1} \left[\begin{array}{c} u(m + n_1^1) + \beta\delta u(n_2^1) \\ +\beta\delta^2 u(m - (n_1^1 - m)(1 + r) - n_2^1) \end{array} \right] \end{aligned} \tag{31}$$

It is easy to show that $\beta^{**}(\delta^*) < \beta^*$ from (21), (22), (27), and (28). Finally, for all (δ, β) where $\delta > \delta^*$ and $\beta < \beta^{**}(\delta^*)$, by (25) and (31):

$$\begin{aligned} & \max_{n_1^1 > m,\ n_2^1} u(m + n_1^1) + \beta\delta u(n_2^1) + \beta\delta^2 u(m - \big(n_1^1 - m\big)(1 + r) - n_2^1) \\ > & \max_{n_1^1 = m,\ n_2^1} u(m + n_1^1) + \beta\delta u(n_2^1) + \beta\delta^2 u(m - \big(n_1^1 - m\big)(1 + r) - n_2^1) \\ = & \max_{n_1^1 = m,\ n_2^1} u(m + n_1^1) + \beta\delta u(m - n_1^1 + n_2^1) + \beta\delta^2 u(m - n_2^1) \end{aligned} \tag{32}$$

This means that the consumer ends up paying interest on his credit card debt, in opposition to his zero period belief that he would not.

■

Proof of Lemma 1.

1. At the initial period, the consumer's total utility is:

   $$\max_{n_1^0} \beta\delta \left[u(m+n_1^0) + \delta u(m+l_1+l_2-n_1^0) + \delta^2 u(m)\right]$$

   if he plans to default, and

   $$\max_{n_1^0 \leq m,\ n_2^0} \beta\delta \left[u(m+n_1^0) + \delta u(m-n_1^0+n_2^0) + \delta^2 u(m-n_2^0)\right]$$

   if he does not plan to default.

   Therefore, the cutoff cost of default in order not to plan to default at the initial period is:

   $$C_0 = \frac{1}{\delta^2}\left\{\begin{array}{c} \max\limits_{n_1^0} \left[u(m+n_1^0) + \delta u(m+l_1+l_2-n_1^0) + \delta^2 u(m)\right] \\ -\max\limits_{n_1^0 \leq m,\ n_2^0} \left[u(m+n_1^0) + \delta u(m-n_1^0+n_2^0) + \delta^2 u(m-n_2^0)\right] \end{array}\right\} \tag{33}$$

2. When the consumer reaches the first period with the chosen contract (which is (l_1, r)), he realizes that his actual debt is more than his income. Therefore, the consumer's total utility is:

   $$\max_{n_1^1 \leq l_1} \left[u(m+n_1^1) + \beta\delta u(m+l_1-n_1^1) + \beta\delta^2 u(m)\right]$$

   if he plans to default, and

   $$\max_{n_1^1 \leq l_1,\ n_2^1} \left[u(m+n_1^1) + \beta\delta u(n_2^1) + \beta\delta^2 u(m-(n_1^1-m)(1+r)-n_2^1)\right]$$

   if he does not plan to default.

   Therefore, the cutoff cost of default in order not to plan to default at the first period is:[19]

   $$C_1 = \frac{1}{\beta\delta^2}\left\{\begin{array}{c} \max\limits_{n_1^1 \leq l_1} \left[u(m+n_1^1) + \beta\delta u(m+l_1-n_1^1) + \beta\delta^2 u(m)\right] \\ -\max\limits_{n_1^1 \leq l_1,\ n_2^1} \left[\begin{array}{c} u(m+n_1^1) + \beta\delta u(n_2^1) \\ +\beta\delta^2 u(m-(n_1^1-m)(1+r)-n_2^1) \end{array}\right] \end{array}\right\} \tag{34}$$

3. When the consumer reaches to the second period, the consumer's total utility is:

   $$\left[u(m+l_1-n_1^{1*}) + \beta\delta u(m)\right]$$

   if he plans to default, and

   $$\max_{n_2^2} \left[u(n_2^2) + \beta\delta u(m-(n_1^{1*}-m)(1+r)-n_2^2)\right]$$

   if he does not plan to default.

---

[19] If the credit limit of the accepted contract is less than m, then C_1 and C_2 would be different. However, the analysis of the subsequent parts would still hold true.

Therefore, the cutoff cost of default in order not to plan to default at the second period is:

$$C_2 = \frac{1}{\beta\delta}\left\{ \begin{array}{c} [u(m+l_1-n_1^{1*})+\beta\delta u(m)] \\ -\max\limits_{n_2^2}[u(n_2^2)+\beta\delta u(m-(n_1^{1*}-m)(1+r)-n_2^2)] \end{array} \right\} \tag{35}$$

As a result, the relevant cutoff is:

$$C' = \max\{C_0(l_1,l_2), C_1(l_1,r), C_2(l_1,r)\}. \tag{36}$$

■

Proof of Lemma 2. If the total credit limit offered is more than m, the consumer pays his debt on the higher interest rate card in full within the grace period, and consequently, the higher interest rate company does not earn interest revenue. For convenience, we take the first company as the one offering the lower interest rate r. The cutoff cost of default for each period is as follows:[20]

$$\begin{aligned} C_0 &= \frac{1}{\delta^2}\left\{ \begin{array}{c} \max\limits_{n_1^0 \le l_1+l_2}\left[u(m+n_1^0)+\delta u(m+l_1+l_2-n_1^0)+\delta^2 u(m)\right] \\ -\max\limits_{n_1^0 \le m, n_2^0}\left[u(m+n_1^0)+\delta u(m-n_1^0+n_2^0)+\delta^2 u(m-n_2^0)\right] \end{array} \right\} \\ C_1 &= \frac{1}{\beta\delta^2}\left\{ \begin{array}{c} \max\limits_{n_1^1 \le l_1+l_2}\left[u(m+n_1^1)+\beta\delta u(m+l_1+l_2-n_1^1)+\beta\delta^2 u(m)\right] \\ -\max\limits_{n_1^1 \le l_1+l_2, n_2^1}\left[\begin{array}{c} u(m+n_1^1)+\beta\delta u(n_2^1) \\ +\beta\delta^2 u(m-(n_1^1-m)(1+r)-n_2^1) \end{array} \right] \end{array} \right\} \\ C_2 &= \frac{1}{\beta\delta}\left\{ \begin{array}{c} [u(m+l_1+l_2-n_1^{1*})+\beta\delta u(m)] \\ -\max\limits_{n_2^2}[u(n_2^2)+\beta\delta u(m-(n_1^{1*}-m)(1+r)-n_2^2)] \end{array} \right\} \end{aligned} \tag{37}$$

These expressions for cutoffs are found in the same way as in the previous proof, except that the credit limits at periods one and two are l_1+l_2 instead of l_1.

In this case, the credit limit on which to default is the same at each period, and therefore we can at least show that C_0 is less than $\max\{C_1, C_2\}$.

If $\beta = 1$, then C_1 would be written as follows:

$$C_1 = \frac{1}{\delta^2}\left\{ \begin{array}{c} \max\limits_{n_1^1 \le l_1+l_2}\left[u(m+n_1^1)+\delta u(m+l_1+l_2-n_1^1)+\delta^2 u(m)\right] - \\ \max\limits_{m \le n_1^1 \le l_1+l_2, n_2^1}\left[\begin{array}{c} u(m+n_1^1)+\delta u(n_2^1) \\ +\delta^2 u(m-(n_1^1-m)(1+r)-n_2^1) \end{array} \right] \end{array} \right\} \tag{38}$$

To compare C_0 and C_1, let's rewrite the expression for C_0 below:

$$C_0 = \frac{1}{\delta^2}\left\{ \begin{array}{c} \max\limits_{n_1^0 \le l_1+l_2}\left[u(m+n_1^0)+\delta u(m+l_1+l_2-n_1^0)+\delta^2 u(m)\right] \\ -\max\limits_{n_1^0 \le m, n_2^0}\left[u(m+n_1^0)+\delta u(m-n_1^0+n_2^0)+\delta^2 u(m-n_2^0)\right] \end{array} \right\} \tag{39}$$

---

[20] If the total credit limit is less than m, then the expressions for C_1 and C_2 would be different. However, the Lemma 2 would still hold true.

When we analyze (38) and (39), we can see that the first maximization expressions are the same for both. Therefore, any difference between C_0 and C_1 is due to the second expressions. From Proposition 1, we know that the profit maximizing n_1 is less than m for $\delta > \delta^*$. Therefore, the second expression in the bracket of (39) is greater than the second expression in the bracket of (38) for $\delta > \delta^*$. Consequently $C_0 < C_1$. Now, if we show that C_1 decreases with β, then we can compare C_0 and C_1.

$$\frac{\partial C_1}{\partial \beta} = -\frac{\left[u(m + n_1^{1*}) - u(m + n_1^{1**})\right]\delta^2}{\left(\beta\delta^2\right)^2} < 0 \tag{40}$$

such that n_1^{1*} and n_1^{1**} represent the profit maximizing n_1 in the case of planning and not planning default, respectively. Accordingly, $C_1 > C_0$ for $\beta \leq 1$. Therefore, $C' = \max\{C_1(l_1, l_2, r), C_2(l_1, l_2, r)\}$. ■

Proof of Lemma 3. At each period, the credit limit on which to default is the same when $l_j = 0$. Therefore, we can show that $C_0(l_i, 0) < \max\{C_1(l_i, r), C_2(l_i, r)\}$ as in the proof for Lemma 2. Note that only C_0 changes with l_j but not C_1 or C_2 :

$$\frac{\partial C_0}{\partial l_j} = \frac{u'(m + l_i + l_j - n_1^{0*})}{\delta} > 0. \tag{41}$$

Therefore, there is a cutoff $l'_j \in [0, \infty)$, such that:

$$\begin{aligned} C_0(l_i, l_j) &< \max\{C_1(l_i, r), C_2(l_i, r)\} \text{ for } l_j < l'_j \\ &\quad\ \text{and the relevant cutoff is } \max\{C_1, C_2\}, \\ \max\{C_1(l_i, r), C_2(l_i, r)\} &\leq C_0(l_i, l_j) \text{ for } l'_j \leq l_j \\ &\quad\ \text{and the relevant cutoff is } C_0. \end{aligned} \tag{42}$$

For $l_j < l'_j$, the best response is a constant at $l_i^*(0)$, since the relevant cutoff, namely $\max\{C_1, C_2\}$, does not change with l_j.[21] Once the relevant cutoff becomes C_0, it increases with l_j and starts to bind the consumer's cost of default at some $l''_j \in [l'_j, \infty)$. The best response remains as a constant at $l_i^*(0)$ for $l'_j \leq l_j < l''_j$, since the default constraint (6) is not yet binding.

Note that C_0 does not change with the interest rate but does change with the credit limits, specifically $0 < \frac{\partial C_0}{\partial l_i} = \frac{\partial C_0}{\partial l_j}$. Therefore, the best response function $l_i^*(l_j)$ decreases at the same rate with l_j for $l''_j \leq l_j < l''_j + l_i^*(0)$ to satisfy the binding default constraint ($C_0(l_i, l_j) = C$). Once the best response function becomes zero at $l''_j + l_i^*(0)$, it stays at zero for $l''_j + l_i^*(0) \leq l_j$. ■

Proof of Lemma 4. Suppose that ($l_i^*(0) > m, r = 0$) is the best response credit limit and interest rate when $l_j = 0$, and therefore the company makes zero profit. Since the consumer is unresponsive to interest rates, only a binding default constraint

---

[21] Note that there can be more than one $l_i^*(0)$ and we assume that the company offers the lowest one from our second tie breaking convention to restrict the set of equilibria.

$(\max\{C_1(l_i(0), r = 0), C_2(l_i(0), r = 0)\} = C)$ can explain the zero interest rate. However, the company can charge a positive interest rate by decreasing the credit limit just a little bit and still satisfy the binding default constraint $(\max\{C_1(l_i(0) - \epsilon, r > 0), C_2(l_i(0) - \epsilon, r > 0)\} = C)$, consequently realizing a positive profit. Therefore, $l_i^*(0) > m$ will always be accompanied by a positive interest rate. As l_j increases from zero to l_j'', the best response credit limit $l_i^*(l_j)$ remins constant at $l_i^*(0)$ with a positive interest rate. As l_j increases from l_j'' (when the default constraint starts to bind), $l_i^*(l_j)$ decreases but the interest rate remains positive, since decreasing the interest rate does not affect the default constraint $(C_0(l_i, l_j) = C)$. As a result, the best response interest rate determined together with the best response credit limit is positive if $l_i^*(0) > m$.

Suppose that $(l_i^*(0) = m, r > 0)$ is the best response credit limit and interest rate when $l_j = 0$, and therefore the company makes zero profit. Only a binding default constraint $(\max\{C_1(l_i(0) = m, r), C_2(l_i(0) = m, r)\} = C)$ can explain the credit limit to be m. However, the company can provide a higher credit limit by decreasing the interest rate just a little bit and still satisfy the binding default constraint $(\max\{C_1(l_i(0) > m, r - \epsilon > 0), C_2(l_i(0) > m, r - \epsilon > 0)\} = C)$, and consequently make a positive profit. Therefore, $l_i^*(0) = m$ is always accompanied by a zero interest rate.

If $l_i^*(0) < m$, the profit for the selected company is zero, since this much credit limit prevents the consumer from spending more than his income and therefore he avoids paying interest. There are many best response contracts, which provide a zero profit for the company (e.g. $(l_i^*(0) = 0, r > 0)$). In order to restrict the set of equilibria, we use our third tie-breaking convention, which we mentioned in Section 1.3, and analyze the contract with the highest credit limit, which is the one with zero interest rate. ■

Proof of Lemma 5. This is because of the following reasons:

- if $l_i + l_j > m$, the consumer realizes that he will pay interest when he reaches the first period, therefore he pays back the higher interest rate company first within the grace period to minimize his interest payment. As a result, the higher interest rate company does not earn profit from interest. Foreseeing this, the credit card companies compete with regard to interest rates, which derives the interest rates to zero.
- if $l_i + l_j \leq m$, neither of the companies earns a positive profit, since the total credit limit does not allow the consumer to borrow more than his income. It is possible to offer several different contracts resulting in zero profit (e.g. $(l = 0, r > 0)$). Among them, from our third tie breaking convention we only analyze the contract with the highest credit limit, which is the one with zero interest rate.[22]

---

[22] If an interest rate has a positive value, then the corresponding credit limit can be increased by decreasing the interest rate without inducing default.

■

Proof of Lemma 6. As one company's credit limit increases, the other company should decrease either the credit limit or the interest rate to keep satisfying the binding default constraint ($\max\{C_1(l_i, l_j, r = 0), C_2(l_i, l_j, r = 0)\} = C$). Since the interest rate is already zero, the only way to do this is to decrease the credit limit. From the proof of Lemma 2, $0 < \frac{\partial C_1}{\partial l_i} = \frac{\partial C_1}{\partial l_j}$, and $0 < \frac{\partial C_2}{\partial l_i} = \frac{\partial C_2}{\partial l_j}$, an increase in one company's credit limit should be accompanied by the same amount of decrease in the other company's credit limit. Therefore, a company's best response credit limit decreases at the same rate as the increase in the other company's credit limit until it becomes zero. ■

Example 3 *Let the best response functions be as in the following graph under the first case:*[23]

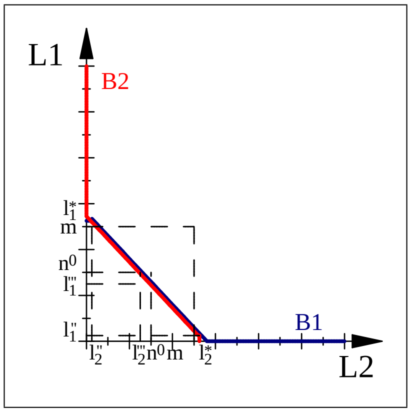

Fig. 9. Best Responses Under the First Case

Note that $l_i < n^0$ for $l_j''' < l_j < n^0$. This means that the consumer should have accepted both contracts for $l_j''' < l_j < n^0$, and consequently B_i cannot be the best response in that region. We need to draw another best response, B_i', in this region under the second case.

A decreasing B_i in $(l_j'', l_j'' + l_i^*(0))$ tells us that the consumer's cost of default is binding:

$$C = C_0(l_1, l_2). \tag{43}$$

We also know that the competition among the firms drives the interest rates down to zero in region one.

By the proof of Lemma 2,

$$C_0(l_1, l_2) < \max\{C_1(l_1, l_2, 0), C_2(l_1, l_2, 0)\}. \tag{44}$$

---

[23] B_1 and B_2 are distinguished by the dark line and the light line, respectively.

By (43) and (44),

$$C < \max\{C_1(l_1, l_2, 0), C_2(l_1, l_2, 0)\} = C'. \tag{45}$$

Therefore, firm i should offer a smaller l_i^* for each l_j in (l_j''', n^0) to prevent the consumer from defaulting. As a result, B_i' lies under B_i in (l_j''', n^0):

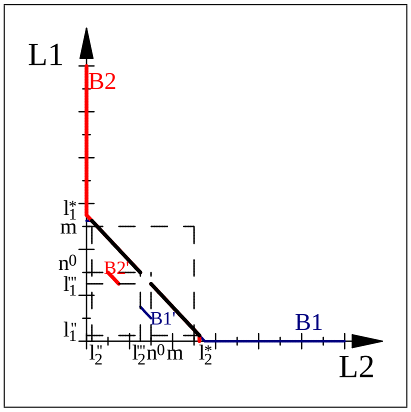

Fig. 10. Zero and Positive Profit Equilibria are Possible Under the Same Parameter Values

In this case, both zero and positive profit equilibria are possible, and they are indicated by the dark and the thick line in Figure 10. In this example, note that B_i' does not lie in $(l_i''', n^0) \times (l_j''', n^0)$. If it was in that region, there would be additional zero profit equilibria.

Example 4 *Let the best response functions be given as in Figure 11 under the first case. Based on the same reasoning we provided in the previous example, B_1 and B_2 cannot be the best responses in $(l_j'', n^0) \times (l_i'', n^0)$. We need to draw the best responses B_1' and B_2' in that area under the second case. Therefore, the best responses are as shown in Figure 12 or in Figure 13.*

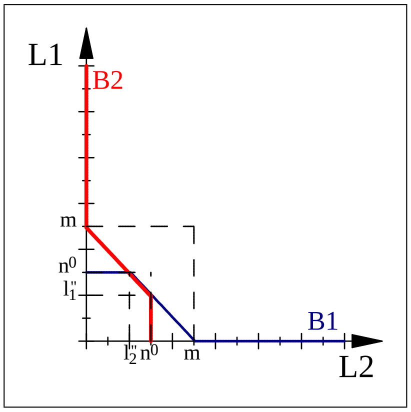

Fig. 11. Best Responses Under the First Case

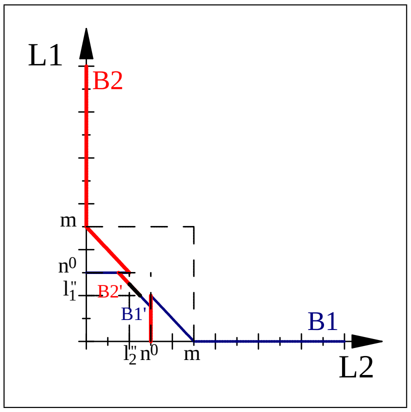

Fig. 12. Zero Profit Equilibria

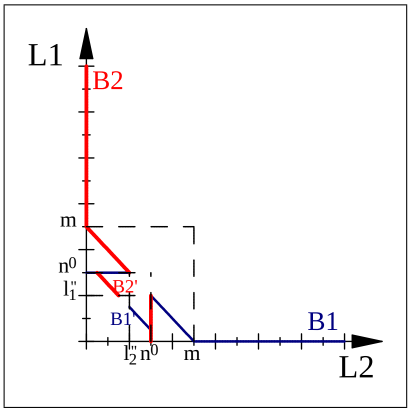

Fig. 13. Zero Profit Equilibria

Note that (l_j'', n^0) *and* (n^0, l_i'') *provide zero profit equilibria in both graphs. In Figure 12, there are additional zero profit equilibria in* $(l_2'', n^0) \times (l_1'', n^0)$.