Unparticles and Inflation

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The Niels Bohr International Academy

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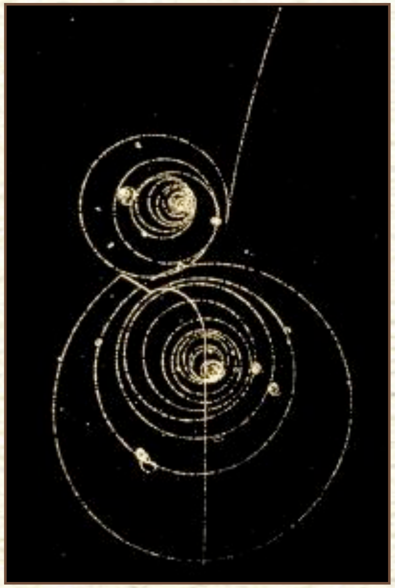
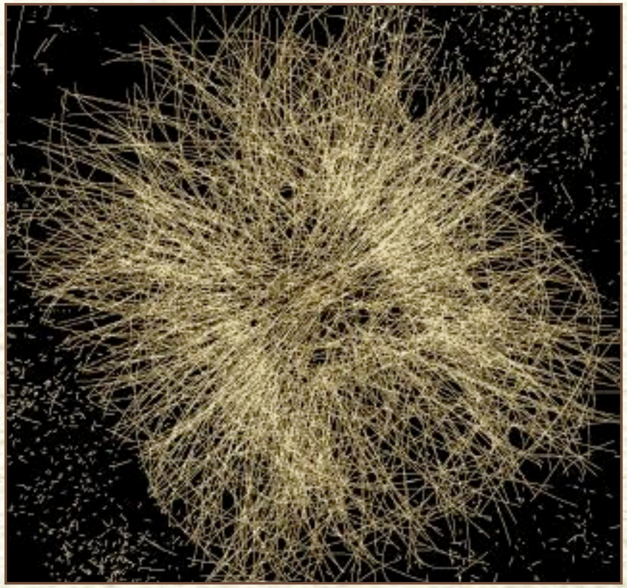
Prelude: Unparticles as a warning

As particle experiments become more complex,
how well prepared are we for the unexpected?



Bubble chamber decay of a K^+
© 1973, CERN



ALICE (LHC): Simulated Pb-Pb collision
© 2003, CERN

There is a danger that we might be
blinded by our expectations

Overview:

- ☐ introducing unparticles
- ☐ unparticles in flat space
- ☐ the inflaton as an unparticle
- ☐ unparticle loops & the inflaton

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Introducing unparticles

What is an unparticle?

It is a scale-invariant (or conformal) field

with scaling dimension d

Georgi, 2007

a few conventions

σ = an unparticle in an expanding universe

χ = an unparticle in flat space

φ = an ordinary scalar particle

Conformal invariance provides powerful constraints

- it fixes the propagator
- unparticles in a conformally flat backgrounds are closely related to those in flat backgrounds

Particles in a conformal background

Before worrying about unparticles, let us look at a conformal *particle* in an expanding universe

a few preliminaries

the metric: $ds^2 = a^2(\eta)\,[\,d\eta^2 - d\vec{x}\cdot d\vec{x}\,]$

$a(\eta)$ is the *scale factor*

$H = a' / a^2$ is the *Hubble scale*

In flat space, it enough to set $m = 0$

In curved space, the curvature itself can break the conformal symmetry

$\therefore$ we ought to consider a massless, *conformally coupled* scalar field

Particles in a conformal background

A massless, conformally coupled field, $\varphi(\eta,x)$

$$\boxed{S[\varphi] = \int d^4x\ \sqrt{-g}\,[\,\frac{1}{2} g^{\mu\nu}\,\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{12}R\varphi^2\,]}$$

Expand the field in eigenmodes

$$\varphi(\eta,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\left[\varphi_k(\eta)\, e^{i\vec{k}\cdot\vec{x}}\, a_{\vec{k}} + \varphi_k^*(\eta)\, e^{-i\vec{k}\cdot\vec{x}}\, a_{\vec{k}}^{\dagger}\right]$$

To obtain the following equation of motion,

$$\boxed{\varphi_k'' + 2\,\frac{a'}{a}\,\varphi_k' + \frac{a''}{a}\,\varphi_k + k^2\varphi_k = 0}$$

Rescale by $a(\eta)$ raised to the scaling dimension ($d = 1$)

$$\varphi_k(\eta) = \frac{\chi_k(\eta)}{a(\eta)} \qquad \Longrightarrow \qquad \chi_k'' + k^2\chi_k = 0$$

Unparticles in a conformal background

We no longer have an explicit action

$$S[\sigma(x)] = \ ?$$

But we can still take advantage of
the conformality of the field

to relate unparticles in curved space (σ)
to those in flat space (χ)

$$\sigma(\eta,\vec{x}) = \frac{\chi(\eta,\vec{x})}{a^d(\eta)}$$

So, we first need to review the behavior of
unparticles in flat space

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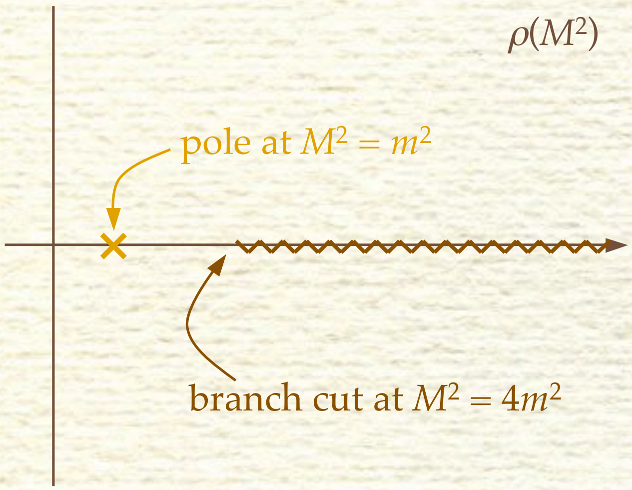
A short review of particles

In a Lorentz-invariant theory, the propagator can be written in the Lehmann-Källen spectral form

$$\langle 0 | \, T[\chi(x)\chi(y)] \, |0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \int_0^\infty dM^2 \, \frac{i\rho(M^2)}{k_0{}^2 - k^2 - M^2 + i\varepsilon}$$

What are the properties of $\rho(M^2)$ for a particle?



Renormalization conditions:

1) pole at the physical mass

2) with residue i

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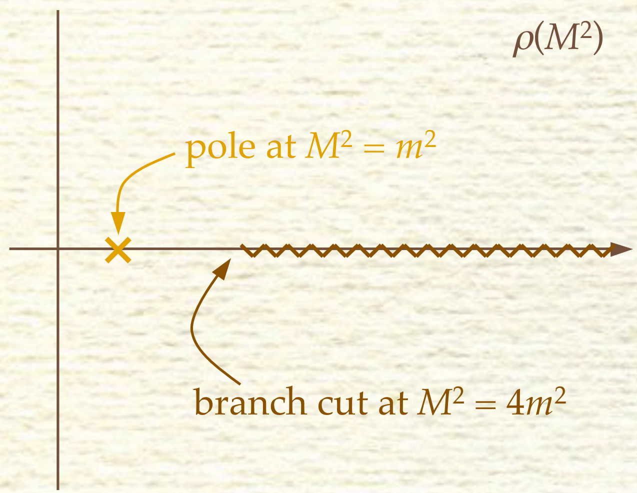

Renormalization conditions:

1) pole at the physical mass

2) with residue i

unparticles have no mass poles

nor a normalization

A short review of unparticles

In a Lorentz-invariant theory, the propagator can still be written in the Lehmann-Källen spectral form

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What are the properties of $\rho(M^2)$ for an unparticle?

1) The field is scale invariant
$\Rightarrow$ fixes $\rho(M^2)$ up to a normalization

2) No canonical normalization
$\Rightarrow$ match with the phase space of d massless particles

Georgi, 2007

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$$2d \quad = \quad 4 \quad + \quad 2 \quad + \quad [\rho] \quad - \quad 2$$

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$$A_d = \frac{d-1}{(16\pi^2)^{d-1}} \frac{2\pi}{(\Gamma(d))^2}$$

Georgi, 2007

Limits on d

Is any possible scaling dimension d
allowed for an unparticle?

Unitarity places a lower bound on d (> 1)

Grinstein, Intriligator & Rothstein, 2008

But if we simply integrate over the spectral index M^2,

we discover $1 \leq d < 2$

$$\langle 0 | T[\chi(x)\chi(y)] | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{A_d}{2} \frac{(-1)^{d-2}}{\sin \pi d} \frac{i}{(k_0{}^2 - k^2 + i\varepsilon)^{2-d}}$$

Note that a massless scalar particle is an unparticle too

$$d \to 1: \quad \frac{d-1}{(16\pi^2)^{1-1}} \frac{\pi}{(\Gamma(1))^2} \frac{(-1)^{1-2}}{\sin \pi d} \frac{i}{(k_0{}^2 - k^2 + i\varepsilon)^{2-1}} \quad \to \quad \frac{i}{k_0{}^2 - k^2 + i\varepsilon}$$

Wightman functions

In a time-dependent background,
it is often more useful to write the propagator as

$$\langle 0|\, T[\chi(x)\chi(y)]\, |0\rangle$$
$$= \int \frac{d^3\vec{k}}{(2\pi)^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \left[\Theta(t-t')\, \tilde{\Gamma}_k^{>}(t,t') + \Theta(t'-t)\, \tilde{\Gamma}_k^{<}(t,t') \right]$$

$\tilde{\Gamma}_k^{>}(t,t') = \tilde{\Gamma}_k^{<}(t',t)$ is the unparticle Wightman function

$$\tilde{\Gamma}_k^{>}(t,t') = \frac{A_d}{2}\,\frac{(-1)^{d-2}}{\sin \pi d} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi}\, \frac{i e^{-ik_0\cdot(t-t')}}{(k_0^2 - k^2 + i\varepsilon)^{2-d}}$$

Integrating over k_0, we find

$$\tilde{\Gamma}_k^{>}(t,t') = \frac{-i}{(8\pi^2)^{d-1}}\,\frac{\sqrt{\pi}}{2\sqrt{2}}\,\frac{1}{\Gamma(d)} \left(\frac{t-t'}{k}\right)^{\frac{3}{2}-d} H^{(2)}_{d-\frac{3}{2}}\,[k(t-t')]$$

Hankel function

Wightman functions in cosmology

In a conformally flat universe, it is easy to write the cosmological propagator in terms of the flat one,

$$\boxed{\langle 0 | T[\sigma(\eta,\vec{x})\sigma(\eta',\vec{y})] \, |0\rangle = \frac{\langle 0 | T[\chi(\eta,\vec{x})\chi(\eta',\vec{y})] \, |0\rangle}{a^d(\eta) \, a^d(\eta')}}$$

So, the cosmological Wightman function is

$$\Gamma_k^{>}(\eta,\eta') = a^{-d}(\eta) \, a^{-d}(\eta') \, \tilde{\Gamma}_k^{>}(\eta,\eta')$$

or,

$$\Gamma_k^{>}(\eta,\eta') = \frac{-i}{(8\pi^2)^{d-1}} \frac{\sqrt{\pi}}{2\sqrt{2}} \frac{1}{\Gamma(d)} \left(\frac{\eta-\eta'}{k}\right)^{\frac{3}{2}-d} \times a^{-d}(\eta) \, a^{-d}(\eta') \, H^{(2)}_{d-\frac{3}{2}}[k(\eta-\eta')]$$

We are now ready to calculate the effects of unparticles on the primordial perturbations

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Unparticles as inflatons

Perhaps some of the unnatural features of the inflaton could be explained if it were an unparticle

The conformal properties could protect small parameters of the potential

Unfortunately, in the slowly rolling regime, this hope is not realized —

The power spectrum is not flat

Some simplifying assumptions:

1) de Sitter space

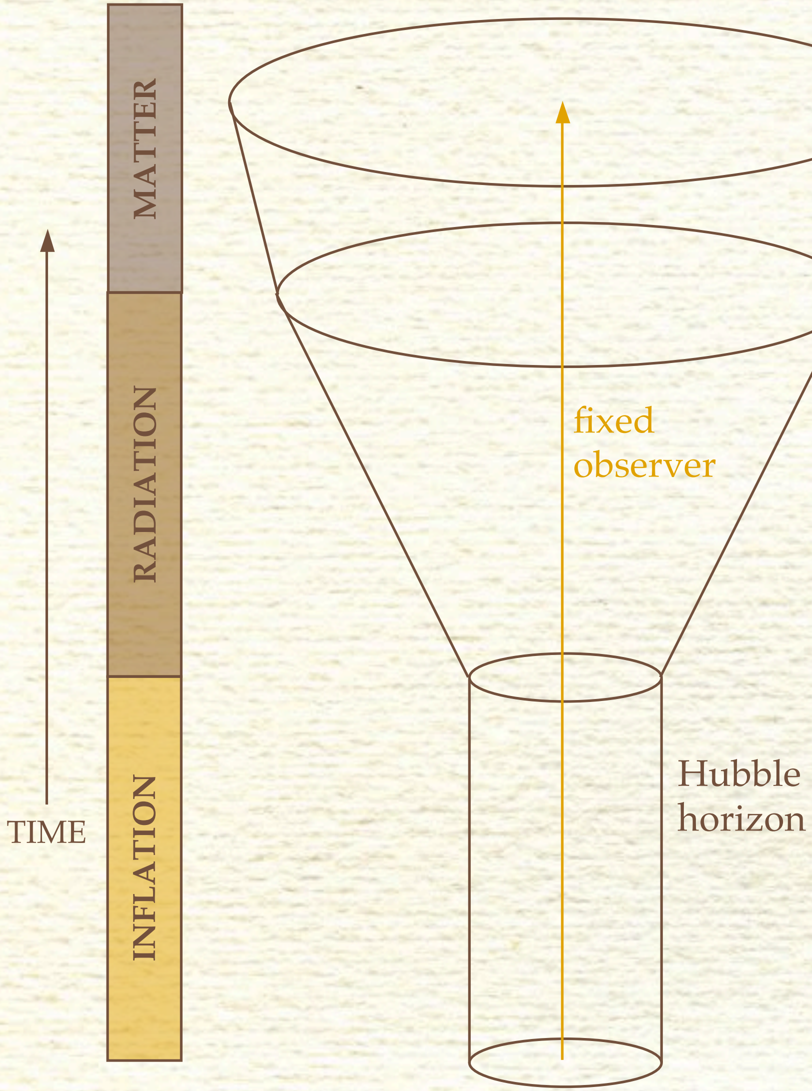
2) neglect mixing with gravity

$$a(\eta) = -\frac{1}{H\eta}$$

Let us see how the calculation proceeds

How inflation makes structure

(a *very* brief review)



"cosmological coordinates"

$$ds^2 = dt^2 - a^2(t)\, d\vec{x} \cdot d\vec{x}$$

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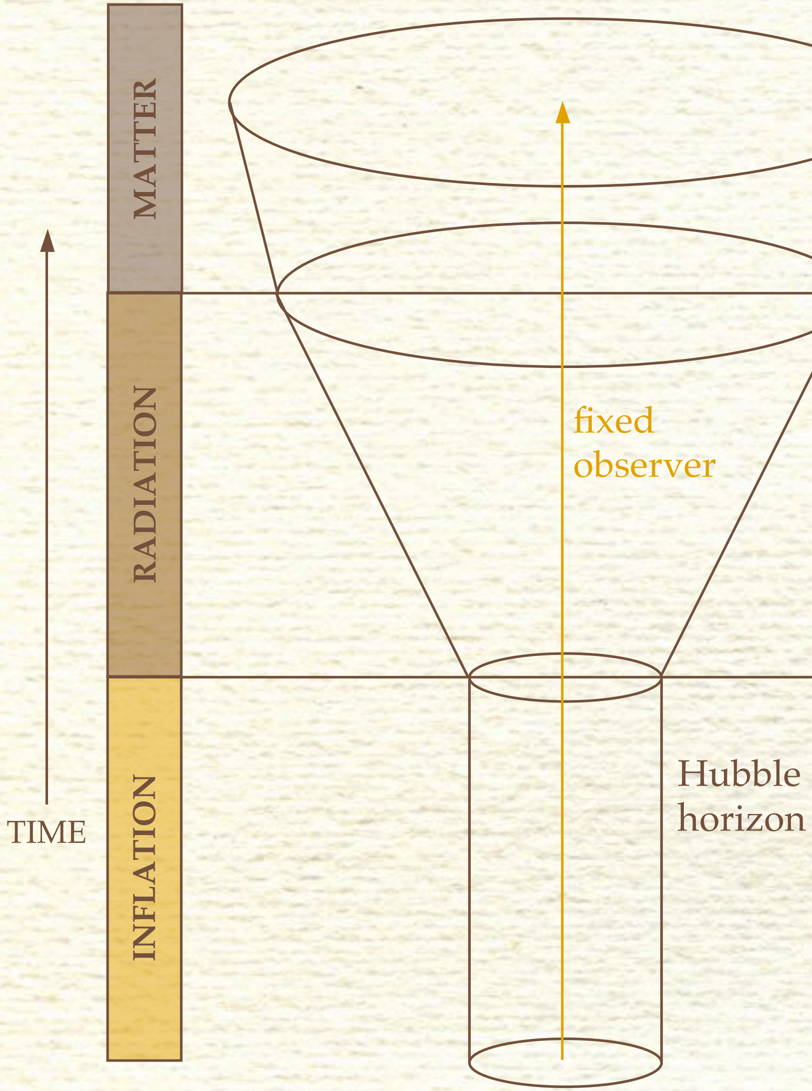

Hubble horizon	physical sizes $L_{\text{phys}} \propto a(t)$
$\frac{1}{H} = \frac{3}{2}t$	$L \propto t^{2/3}$
$\frac{1}{H} = 2t$	$L \propto t^{1/2}$
$\frac{1}{H} \approx \text{const}$	$L \propto e^{Ht}$

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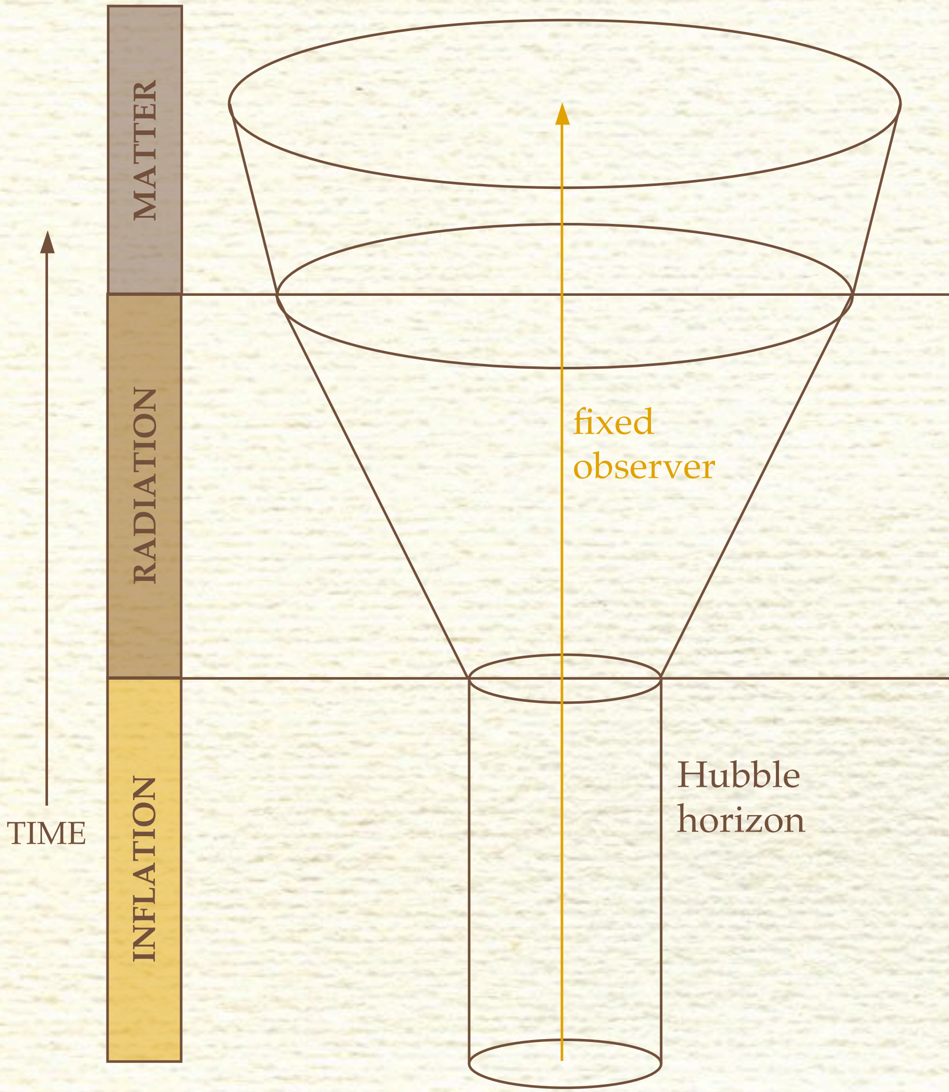

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$\frac{1}{H} = \frac{3}{2}t$ **FASTER**	$L \propto t^{2/3}$
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$\frac{1}{H} \approx \text{const}$	$L \propto e^{Ht}$ **MUCH FASTER**

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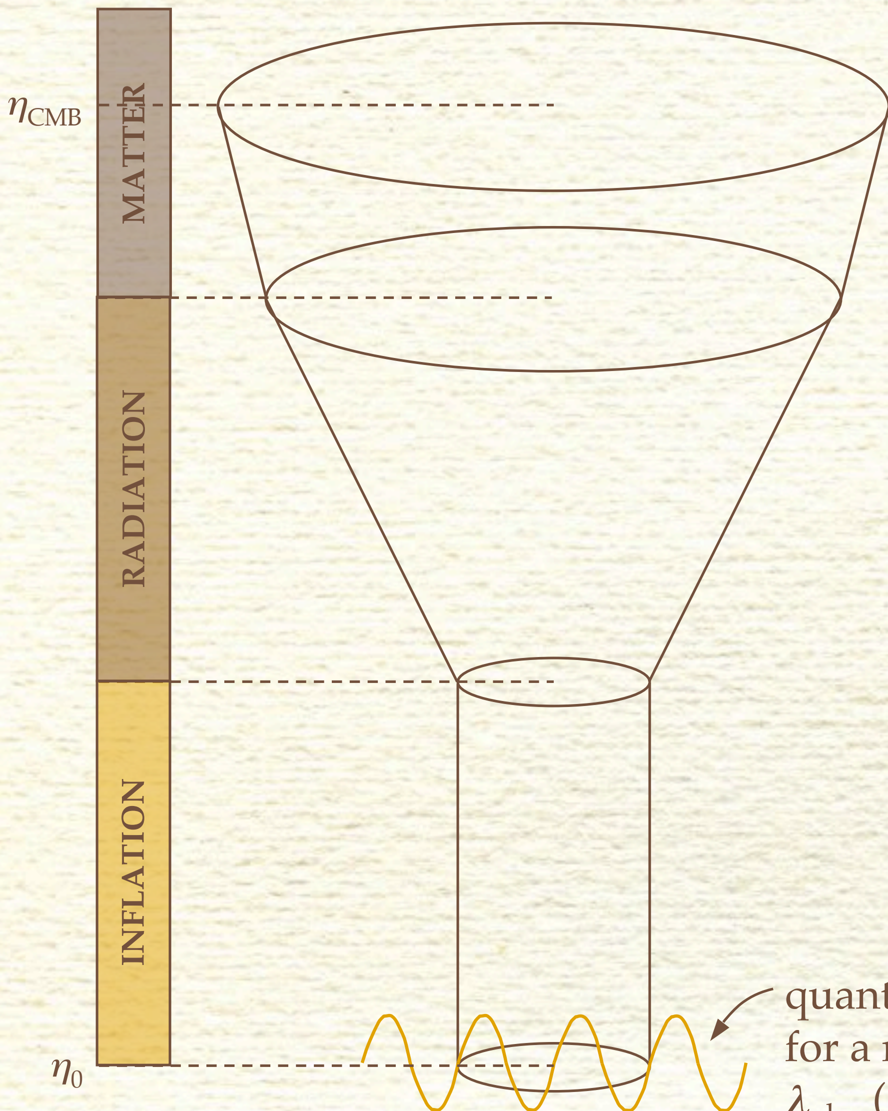

quantum fluctuations of the inflaton, for a mode $\varphi_k(\eta)$ of wavelength $\lambda_{\rm phys}(\eta) = 2\pi\, a(\eta)/k$

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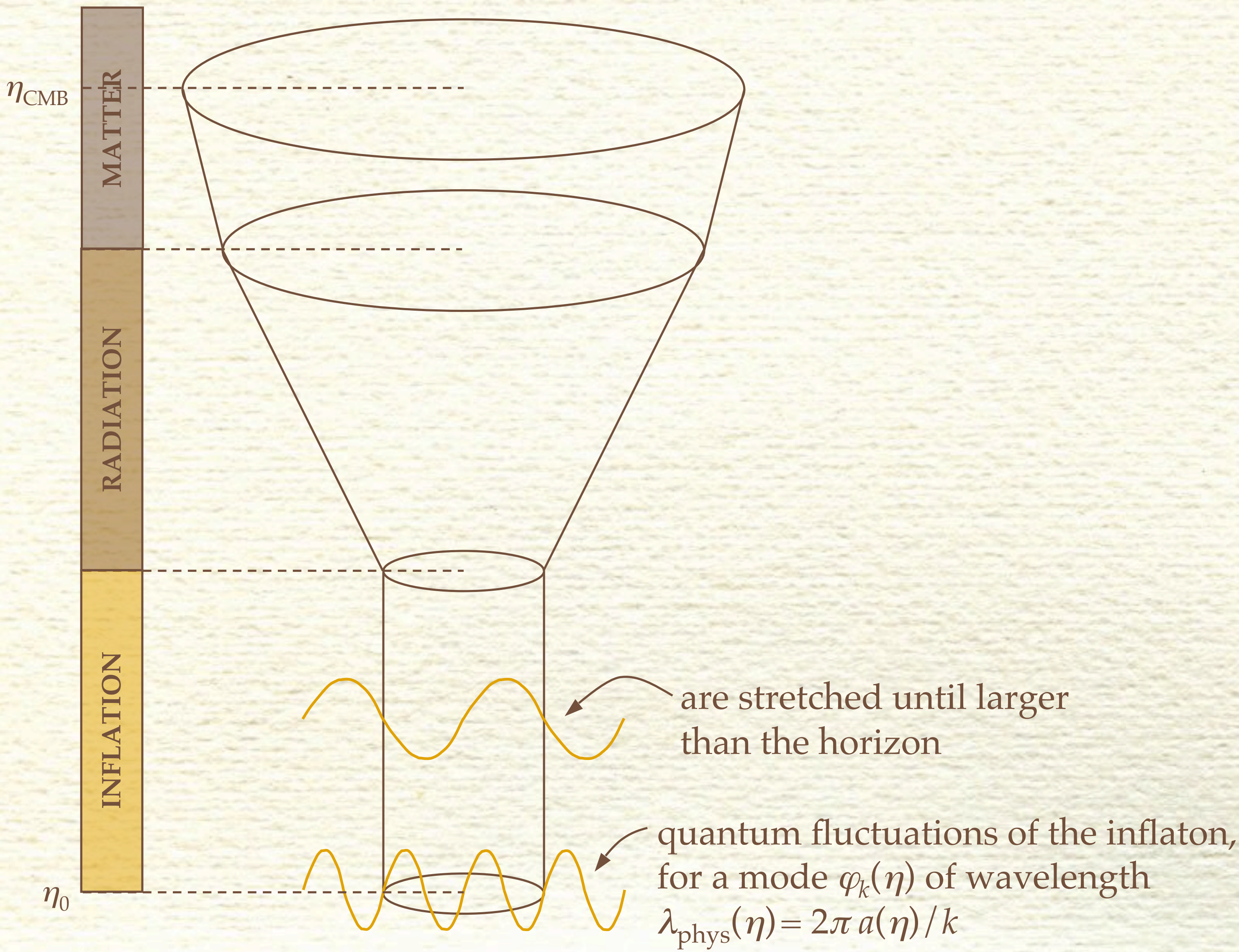

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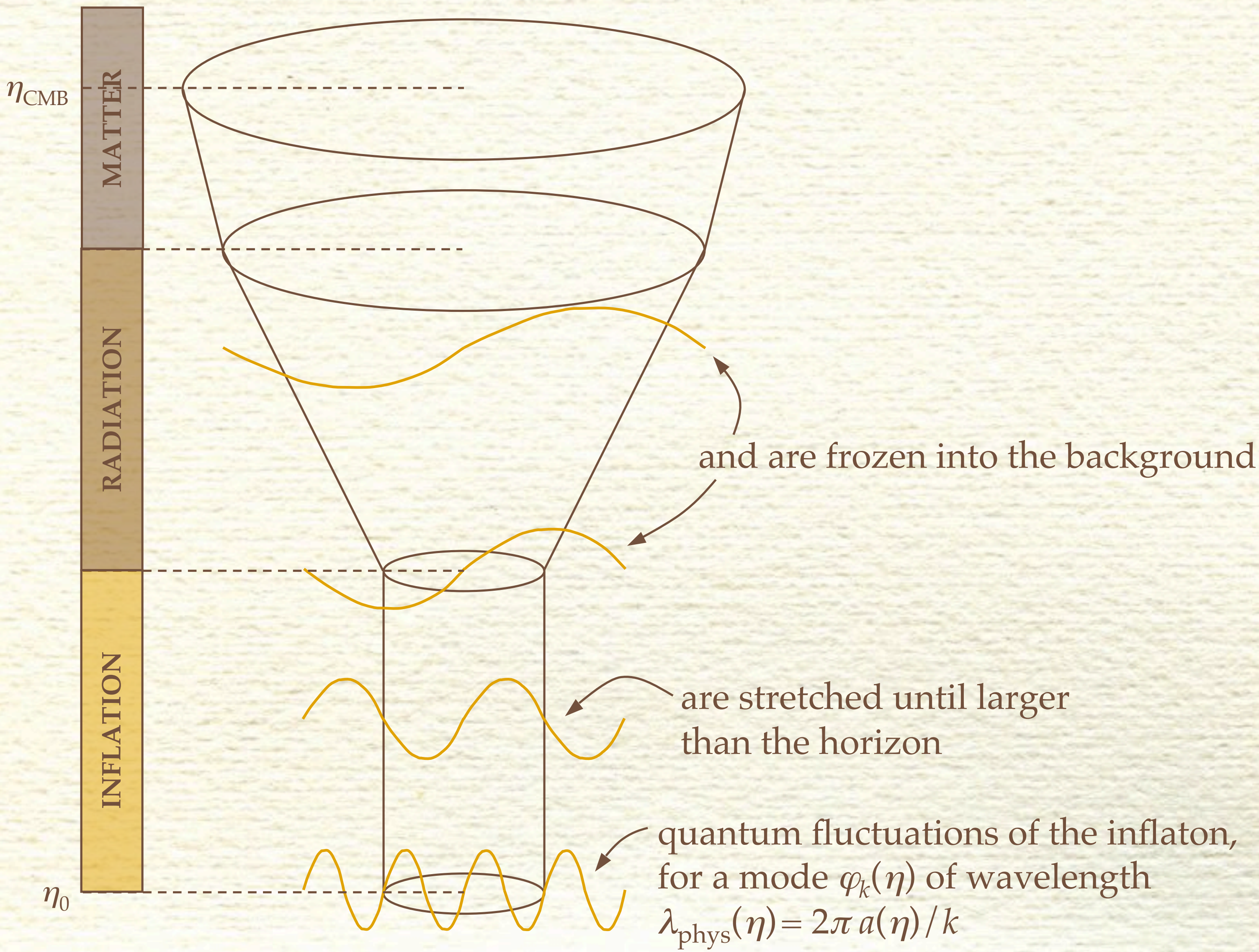

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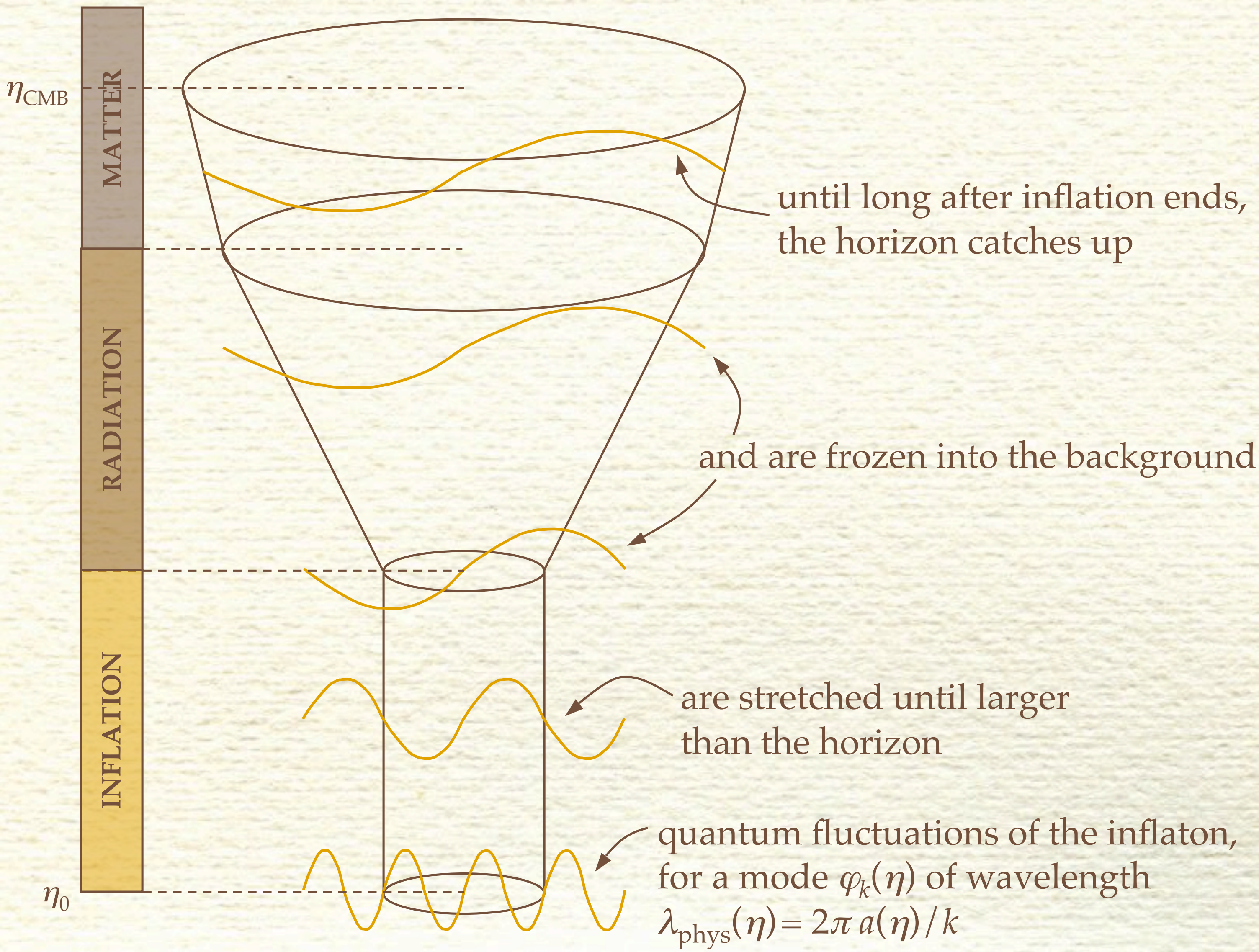

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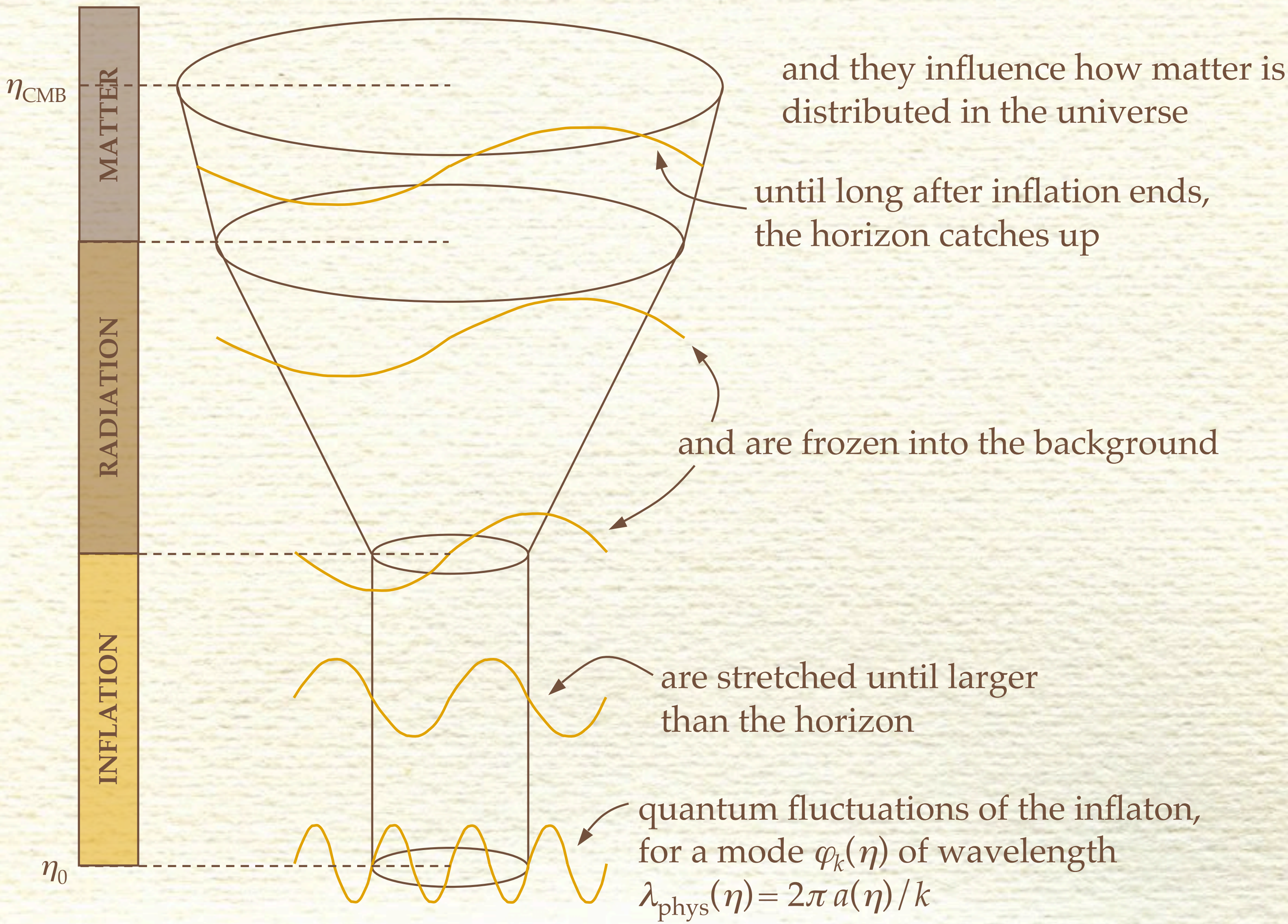

Unparticles as inflatons

The simplest measure of the pattern of these 'primordial perturbations' to the background is the two-point function

$$\boxed{\langle 0 | \sigma(\eta, \vec{x}) \sigma(\eta, \vec{y}) | 0 \rangle = \int \frac{d^3 \vec{k}}{(2\pi)^3} e^{i \vec{k} \cdot (\vec{x} - \vec{y})} \frac{2\pi^2}{k^3} P_k(\eta)}$$

or its Fourier transform, the *power spectrum* $P_k(\eta)$,

just the Wightman function again

$$P_k(\eta) = \frac{k^3}{2\pi^2} \Gamma_k^{>}(\eta, \eta) = \frac{1}{(16\pi^2)^d} \frac{4}{\sqrt{\pi}} \frac{\Gamma(\frac{3}{2} - d)}{\Gamma(d)} \left(\frac{k}{a(\eta)} \right)^{2d}$$

diverges unless $d < \frac{3}{2}$

Since $1 < d < \frac{3}{2}$, the power spectrum is not very flat

Unparticles as inflatons

Experimentally, one assumes $P_k \propto k^{n_s - 1}$, and finds

$$n_s = 0.960^{+0.014}_{-0.013}$$

WMAP, 5yr data, 2008

In de Sitter space,

$$a(\eta) = -\frac{1}{H\eta}$$

so the power spectrum is

$$\boxed{P_k(\eta) = \frac{H^{2d}}{(16\pi^2)^d} \frac{4}{\sqrt{\pi}} \frac{\Gamma(\frac{3}{2} - d)}{\Gamma(d)} (-k\eta)^{2d}}$$

So, at least for slowly rolling inflation,
an unparticle is not an especially good inflaton

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Low energy degrees of freedom

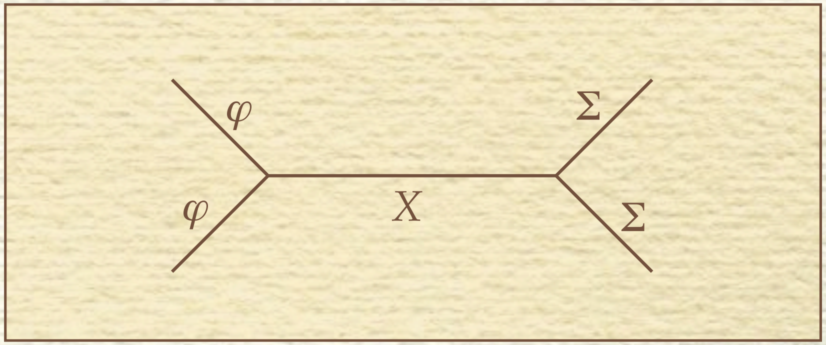
Until now, we have not said how unparticles might arise

Consider the following scenario with 3 sorts of fields

– some very massive (M) fields	X
– some 'proto-unparticles' (particles)	Σ
– the inflaton	φ

Start at very high energies: $M < E$

Σ-φ interact by exchanging an X



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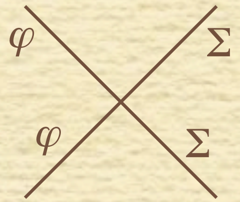
Integrate out the X: $E < M$

generates an effective Σ-φ interaction



$$\sim \frac{1}{M^2}\,\varphi^2\Sigma^2$$

Low energy degrees of freedom

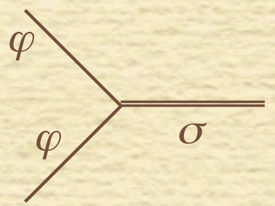
Until now, we have not said how unparticles might arise

Consider the following scenario with 3 sorts of fields

– some very massive (M) fields	X
– some unparticles	σ
– the inflaton	φ

The Σ's have a non-trivial IR fixed point: $E < \Lambda < M$

dimensional transmutation
$\Sigma \rightarrow \sigma$



$$\sim \Lambda^{2-d} \frac{\Lambda^2}{M^2} \varphi^2 \sigma$$

Low energy effective interactions

From this basic picture, we can write the general form for the possible unparticle-inflaton couplings

dimensional transmutation scale

integers

$$c\ \Lambda^{4-N-N'd}\left(\frac{\Lambda}{M}\right)^{n}\varphi^{N}\sigma^{N'}$$

dimensionless coupling

heavy mediator mass

The natural energy scale is set by the expansion: $E \to H$

$$H < \Lambda < M$$

The leading effect is from the most relevant operator

$$c\ \Lambda^{2-d}\left(\frac{\Lambda}{M}\right)^{n}\varphi^{2}\sigma$$

Radiative unparticle corrections

So, even if the inflaton is not itself an unparticle,

any unparticles around during inflation
can affect the power spectrum

$$\boxed{\langle 0(\eta) | \, \varphi(\eta,\vec{x})\varphi(\eta,\vec{y}) \, | 0(\eta)\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \frac{2\pi^2}{k^3} P_k(\eta)}$$

A technical point:

- we do not use an S-matrix
- time-evolve the *entire* two-point function

Schwinger, 1961; Keldysh, 1964; Mahanthappa, 1962

Time-evolve with Dyson's equation

$$|0(\eta)\rangle = T\, e^{-i\int_{\eta_0}^{\eta} d\eta'\, H_I(\eta')} |0(\eta)\rangle$$

with

$$H_I(\eta) = -\,c\,\Lambda^{2-d}\left(\frac{\Lambda}{M}\right)^n a^4(\eta) \int d^3x\, \vec{\varphi}^2 \sigma$$

Tree-level prediction

The 0^{th} order prediction is just the standard one

$$P_k(\eta) = \frac{k^3}{2\pi^2} |\varphi_k(\eta)|^2$$

$$\boxed{\varphi(\eta,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} [\varphi_k(\eta)\, e^{i\vec{k}\cdot\vec{x}}\, a_{\vec{k}} + \varphi_k{}^*(\eta)\, e^{-i\vec{k}\cdot\vec{x}}\, a_{\vec{k}}{}^\dagger]}$$

Again, a few simplifying assumptions,

– work in de Sitter space

– neglect the inflaton mass m (it will reappear later)

– use the Bunch-Davies vacuum

so that

$$\varphi_k(\eta) = \frac{H}{2^{1/2}} \frac{(i - k\eta)\, e^{-ik\eta}}{k^{3/2}}$$

The tree-level power spectrum is flat

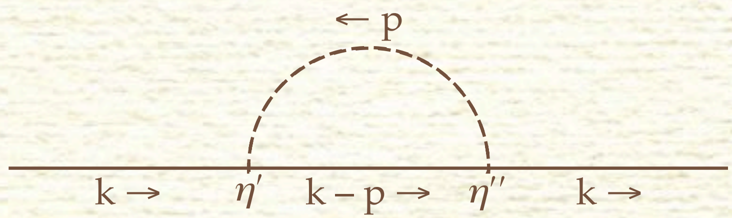
$$P_k(\eta) = \frac{H^2}{4\pi^2} [\, 1 + k^2\eta^2 \,] \rightarrow \frac{H^2}{4\pi^2}$$

The loop

The 1st order prediction is more complicated

$$P_k(\eta) = P_k^{\text{tree}}(\eta) + P_k^{\text{loop}}(\eta) + \ldots$$

Where the loop contribution comes from the graph



The *general* structure of the loop is

$$P_k^{\text{loop}}(\eta) = \frac{H^2}{4\pi^2}\,\frac{c^2}{\Gamma(d)}\,\frac{2^{3/2}\pi^{1/2}}{(8\pi^2)^d}\left[\frac{\Lambda}{H}\right]^{4-2d}\left[\frac{\Lambda}{M}\right]^{2n}\Pi(k\eta,k\eta_0)$$

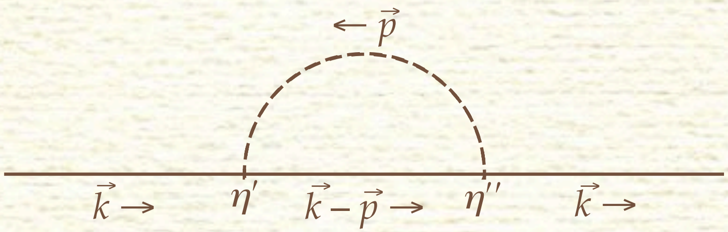
$\Pi(k\eta,k\eta_0)$ is a dimensionless loop integral

Collins & Holman, hep-ph/0802.4416

How does it scale in k?

UV divergences?

The loop has three places where UV divergences could potentially occur,



1. short distances ($p \rightarrow \infty$)
2. coincident times ($\eta'' \rightarrow \eta'$)
3. "trans-Planckian" times ($\eta', \eta'' \rightarrow -\infty$)

All of these limits turn out to be safe

$\Rightarrow$ no renormalization is needed

Collins & Holman, hep-ph/0802.4416

#1. safe, even in flat space

#2. safe, provided $d < \frac{5}{2}$

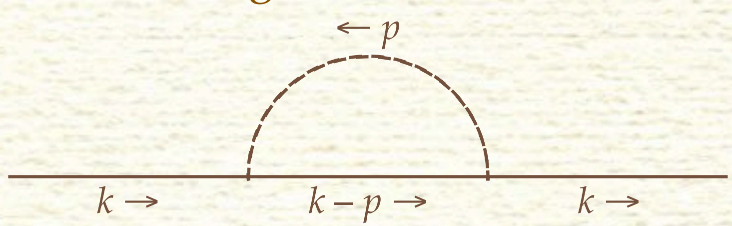
#3. safe, provided $d < 2$

An unparticle loop in flat space

The UV behavior is much milder for unparticles

An example, let us look at the same graph, except now in flat space and returning to the *S*-matrix formalism

$iA(k^2) =$



Feynman amplitude,

$$A(k^2) = c^2\,\Lambda^{2(2-d)}\left(\frac{\Lambda}{M}\right)^{2n}\frac{1}{(16\pi^2)^d}\,\frac{\Gamma(1-d)}{\Gamma(d)}\int_0^1 dx\,\left(\frac{xm^2 - x(1-x)k^2 - i\varepsilon}{1-x}\right)^{d-1}$$

On-shell,

$$A(m^2) = \frac{c^2\Lambda^2}{(16\pi^2)^d}\left(\frac{m}{\Lambda}\right)^{2(d-1)}\left(\frac{\Lambda}{M}\right)^{2n}\frac{\Gamma(1-d)\Gamma(2-d)\Gamma(2d-1)}{\Gamma(d)\Gamma(d+1)}$$

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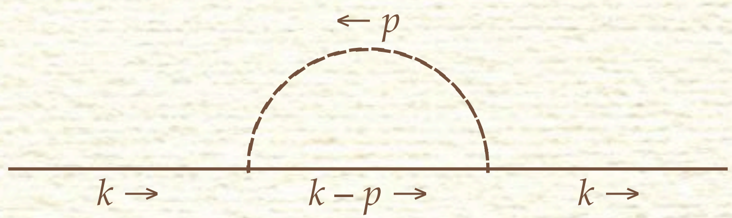


Feynman amplitude,

UV divergence at $d = 1$

$$A(k^2) = c^2\, \Lambda^{2(2-d)} \left[\frac{\Lambda}{M}\right]^{2n} \frac{1}{(16\pi^2)^d}\, \frac{\Gamma(1-d)}{\Gamma(d)} \int_0^1 dx \left[\frac{xm^2 - x(1-x)k^2 - i\varepsilon}{1-x}\right]^{d-1}$$

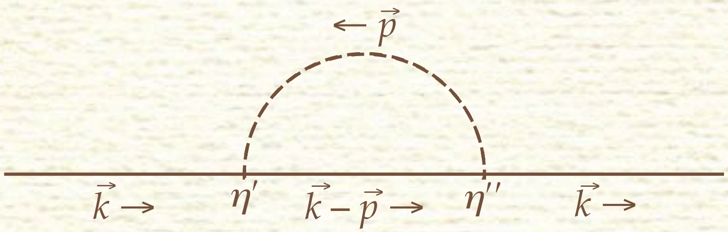
On-shell,

$$A(m^2) = \frac{c^2\Lambda^2}{(16\pi^2)^d} \left[\frac{m}{\Lambda}\right]^{2(d-1)} \left[\frac{\Lambda}{M}\right]^{2n} \frac{\Gamma(1-d)\Gamma(2-d)\Gamma(2d-1)}{\Gamma(d)\Gamma(d+1)}$$

If the dashed line were a particle, we would have a UV divergence (requiring some renormalization)

IR divergences?

The loop also has three places where IR divergences could potentially occur,



1. soft unparticle ($p \rightarrow 0$)
2. soft virtual inflaton ($p \rightarrow k$)
3. late times ($\eta', \eta'' \sim \eta \rightarrow 0$)

The first is completely finite, but the other two are not

Collins & Holman, hep-ph/0802.4416

#2. the $p \rightarrow k$ divergence is cured when $m \neq 0$

#3. this divergence means only that $P_k(\eta)$ has a non-trivial scaling in k

IR divergences

The loop also has two places where IR divergences actually occur,

$$P_k{}^{\mathrm{loop}}(\eta) = \frac{H^2}{4\pi^2}\,\frac{c^2}{\Gamma(d)}\,\frac{2^{3/2}\pi^{1/2}}{(8\pi^2)^d}\left[\frac{\Lambda}{H}\right]^{4-2d}\left[\frac{\Lambda}{M}\right]^{2n}\Pi(k\eta,k\eta_0)$$

The first is completely finite, but the other two are not

Collins & Holman, hep-ph/0802.4416

#2. the $p \to k$ divergence is cured when $m \neq 0$

$$\Pi(k\eta) \sim \ln\frac{m}{k} + \cdots$$

#3. this divergence means only that $P_k(\eta)$ has a non-trivial scaling in k

$$\Pi(k\eta) \sim (-k\eta)^{2\mathrm{d}-3}$$

Predictions

If we define some dimensionless integrals, Π_i, which are finite as $m \to 0$ and $k\eta \to 0$, we find

$$P_k^{\text{loop}}(\eta) = \frac{H^2}{4\pi^2} \frac{c^2}{\Gamma(d)} \frac{2^{3/2}\pi^{1/2}}{(8\pi^2)^d} \left[\frac{\Lambda}{H}\right]^{4-2d} \left[\frac{\Lambda}{M}\right]^{2n}$$
$$\times \left\{ \Pi_0 + \Pi_1 \ln\frac{m}{k} + \Pi_2 (-k\eta)^{2d-3} + \Pi_3 (-k\eta)^{2d-3} \ln\frac{m}{k} \right\}$$

So we learn that if the unparticle scaling dimension is

$$d < \frac{3}{2}$$

then there is an enhancement at the “red” end of the power spectrum

Recall that the most recent experimental bounds are

WMAP, 5yr data, 2008

$$n_s = 0.960^{+0.014}_{-0.013}$$

Conclusions

Can unparticles play any role during inflation?

Unparticle inflatons do not produce
a realistic power spectrum

—

A unparticle coupled to the usual inflaton produces an
enhancement at the red end of the power spectrum
provided $d < \frac{3}{2}$

—

Future bounds on the acoustic oscillations:

CMB precisions: 0.1% LSS precisions: 0.001%

—

Some more general observations about unparticles:

1. Wightman function diverges for $d > \frac{3}{2}$,
2. loops free of UV divergences,
3. "unparticle regularization", . . .

the end

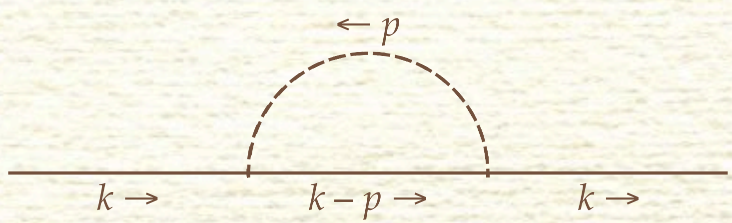
An aside: Unparticle regularization

Since unparticle loops are freer of divergences, we can use them to regularize massless scalar field theories

without altering the space-time symmetries!

Example: $c\Lambda\varphi^2\chi$ coupling in flat space

$iA(k^2) =$



Evaluate in $4 - 2\delta$ dimensions:

$$A_{\mathrm{dr}} = \frac{c^2\Lambda^2}{16\pi^2}\left[\frac{1}{\delta} - \gamma + \ln 4\pi - \int_0^1 dx\ \ln\frac{xm^2 - x(1-x)k^2 - i\varepsilon}{\mu^2}\right]$$

Evaluate with a $d = 1 + \delta$ unparticle: $(c\Lambda\mu^{1-d}\varphi^2\chi)$

$$A_{\mathrm{un}} = \frac{c^2\Lambda^2}{16\pi^2}\left[-\frac{1}{\delta} - 2\gamma - 1 + 2\ln 4\pi - \int_0^1 dx\ \ln\frac{xm^2 - x(1-x)k^2 - i\varepsilon}{\mu^2}\right]$$

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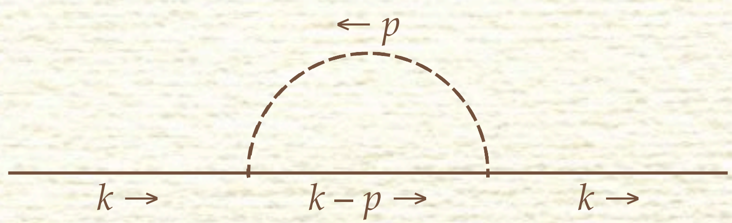
$iA(k^2) =$ 

Evaluate in $4 - 2\delta$ dimensions:

$$A_{\rm dr} = \frac{c^2\Lambda^2}{16\pi^2}\left[\frac{1}{\delta} - \gamma + \ln 4\pi - \int_0^1 dx\ \ln\frac{xm^2 - x(1-x)k^2 - i\varepsilon}{\mu^2}\right]$$

same finite parts

Evaluate with a $d = 1 + \delta$ unparticle:

$$A_{\rm un} = \frac{c^2\Lambda^2}{16\pi^2}\left[-\frac{1}{\delta} - 2\gamma - 1 + 2\ln 4\pi - \int_0^1 dx\ \ln\frac{xm^2 - x(1-x)k^2 - i\varepsilon}{\mu^2}\right]$$