Seeking finer structures in the CMB:
An effective theory for an initial state of inflation

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Prologue
UV problems in quantum field theory

- A fundamental principle in science is the idea of decoupling
  - phenomena at different scales <u>often</u> have tiny effects on each other
  - But in QFT, we often sum over all energy scales
- Addressing the UV problems of such sums has led to great advances
  - renormalization & effective field theory
  - observable effects (e.g. RG running of couplings)
- Inflation has its own UV problem
  - a quantum-mechanical method for structure formation
  - space-time expansion leads to a "trans-Planckian" problem

- <u>This talk</u>:
  - a new effective theory application for this UV problem of inflation
  - But first, we review
    - inflation & structure formation
    - its loose threads & the trans-Planckian problem

Overview:

- inflation and generating structure
- UV problems & opportunities of QFT's
- an effective theory of the initial state
- observations, speculations & conclusions

The cosmic microwave background (CMB)

- Inflation was developed to solve some conceptual/aesthetic problems in cosmology

- If we look backwards in time,
  - lumps of stuff were smoother
  - the universe was much denser & hotter

- Far enough back we reach a turning point:
  - the universe was hot and dense enough to become an opaque plasma
  - this faint glow is the CMB

- Experimental prediction! (COBE)

  we should see the faint relic glow from when

hot plasma (opaque)	→→→	neutral gas (transparent)

  a perfect blackbody to about 1 part in 10^5

Snapshots from the life of a universe

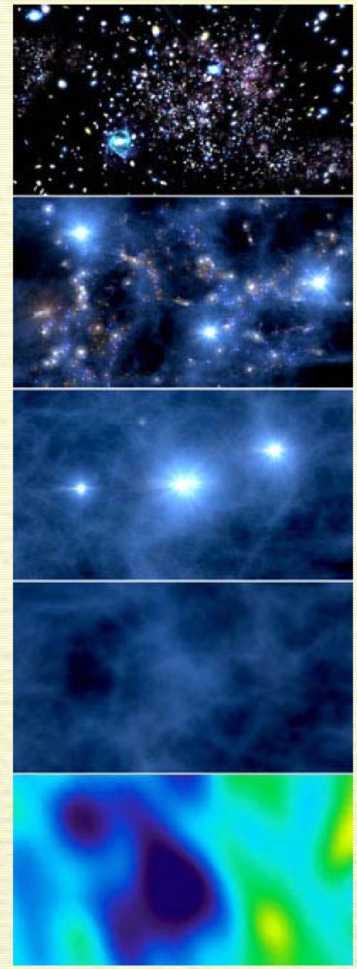
Maturity (ca. today)

Middle age (ca. 10 Gyrs ago)

Young adult (ca. 13.3 Gyrs ago)

Adolescence (ca. 13.5 Gyrs ago)

Childhood (ca. 13.7 Gyrs ago)



A little paradox—a race between two photons

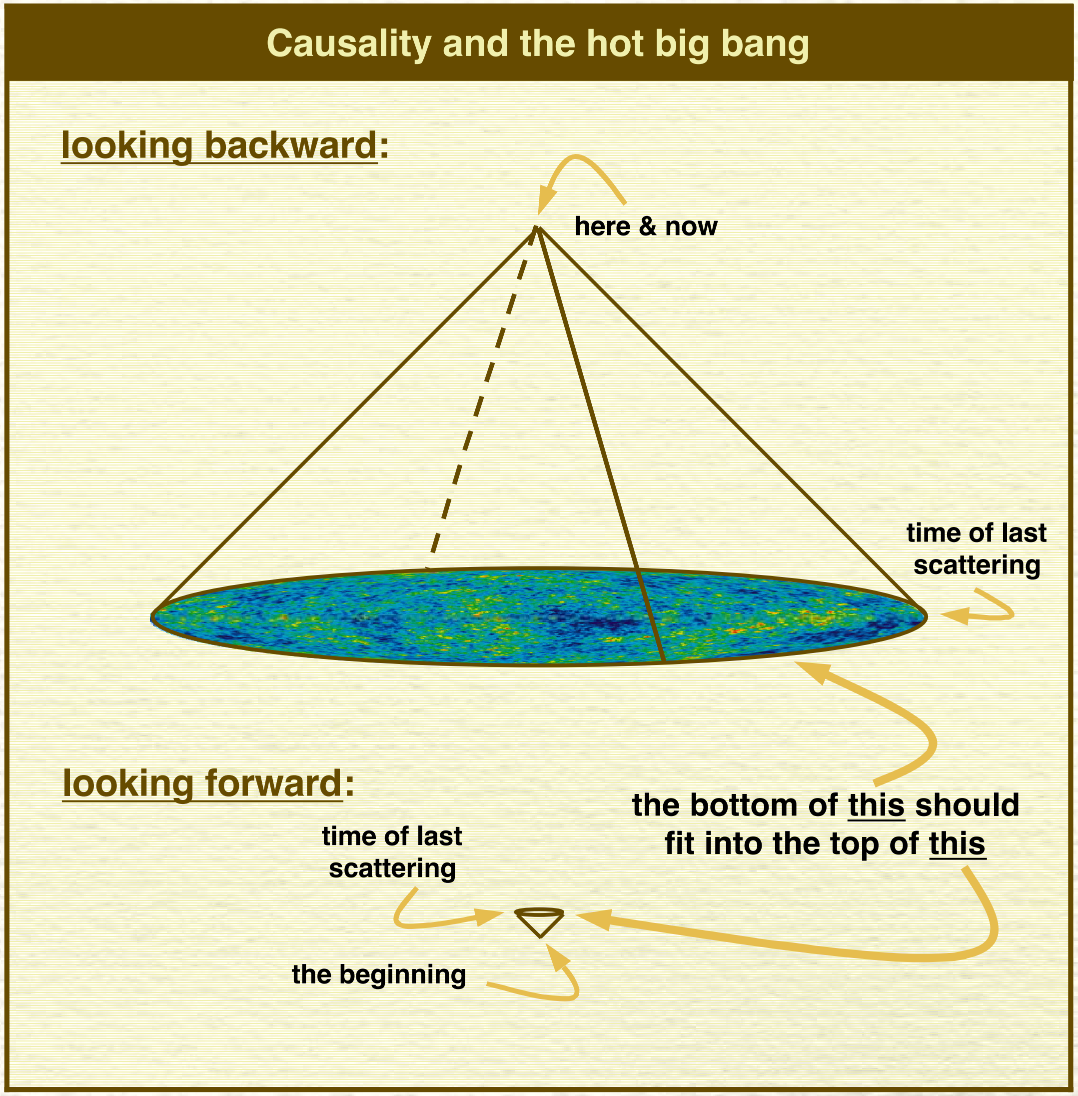
- How could the universe be so smooth at 300,000 years old?

- A thought-experiment:
  a race between two photons

» Photon A
  - starts at the 'beginning'
  - ends when the CMB forms

« Photon B (backwards)
  - starts now
  - goes backwards until the time the CMB forms

- Causality requirement:
- Photon A should travel farther than Photon B



A race between two photons

- In general relativity space is not fixed, but can expand over time
- Locally, a photon moves at c

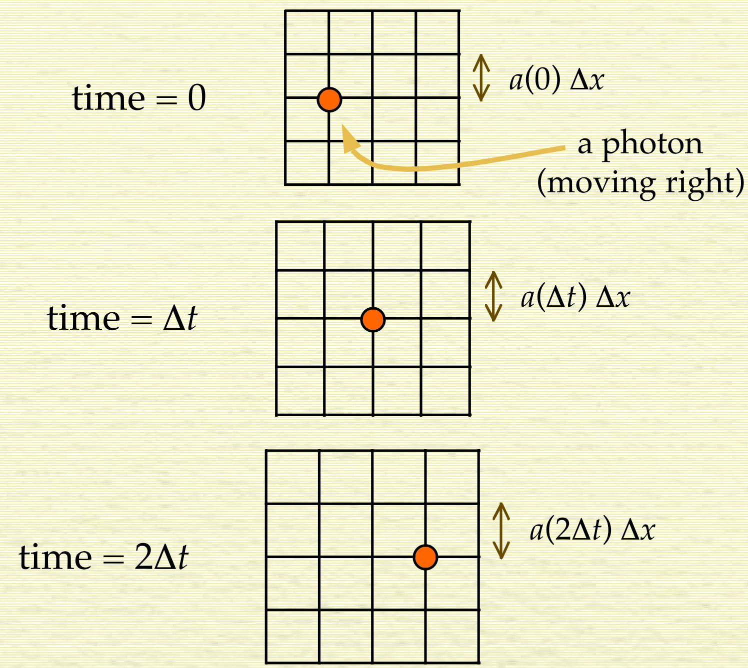
  But globally, general relativity helps:

  the expansion of space adds to how far the photon travels
- If during some early era the universe expanded rapidly enough, Photon A could travel far enough!
- During that era space must expand at an *accelerating* rate
  - this mechanism is called inflation
- Question: How does this help explain the pattern of ripples in the CMB

One way to fix this causality problem

Consider an expanding coordinate grid

The **scale factor** $a(t)$ tells how the grid spacing grows



The total distance traveled over three time-steps is

$$\Delta x_0 + \Delta x_1 + \Delta x_2 = \frac{c\,\Delta t}{a(0)} + \frac{c\,\Delta t}{a(\Delta t)} + \frac{c\,\Delta t}{a(2\Delta t)}$$

A race between two photons

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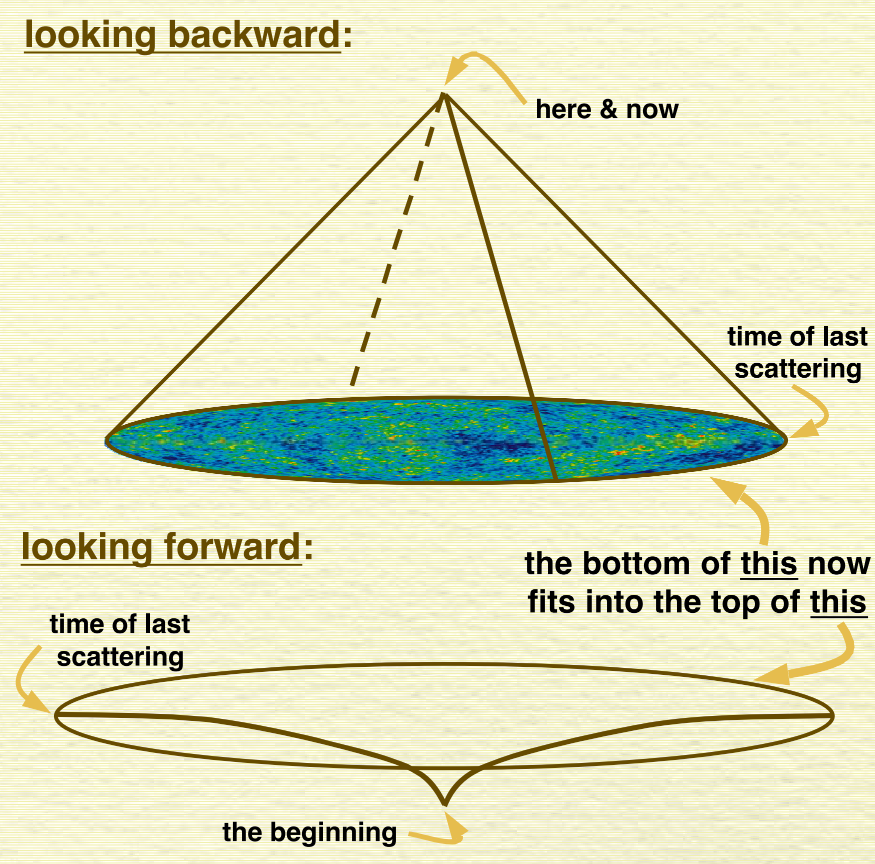
One way to fix this causality problem

If the expansion rate is accelerating, then the photon from the beginning can travel far enough



Inflation—a few preliminaries

- How is inflation implemented?

- Typical ingredients:
  - quantum scalar field(s)
  - moving down a potential, V
  - occurs at large energy, H

- It is usually easier to work with Fourier (momentum) transforms

- Examples:
  - mode functions: $\varphi(t,x) \to \varphi_k(t)$
  - power spectrum: $P_k(t)$

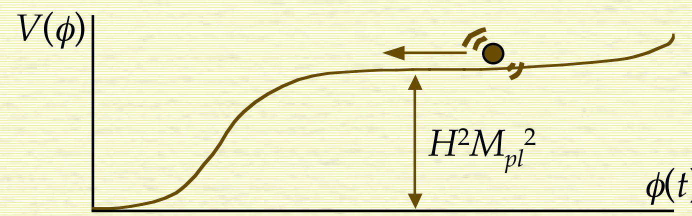
- To understand how inflation works, we shall follow a particular Fourier mode over time

Setting the stage

Divide the scalar field into a

$$\phi(t) = \text{classical zero mode}$$
$$\varphi(t,x) = \text{quantum fluctuation}$$

The quantum part jiggles about as the field rolls down its potential



The **power spectrum** $P_k(t)$ is related to the Fourier transform of a two-point correlator

$$\langle 0 | \varphi(t,x)\varphi(t,y) | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-y)} \frac{2\pi^2}{k^3} P_k(t)$$

How inflation makes structure (I)

- Two basic ingredients:
  - the quantum fluctuations
  - the rapid expansion

- Like everything else, the quantum fluctuations are stretched

- <u>For example</u>:
  a Fourier mode that looks like this

later looks like this

$\lambda(t) = 2\pi / a(t)k$

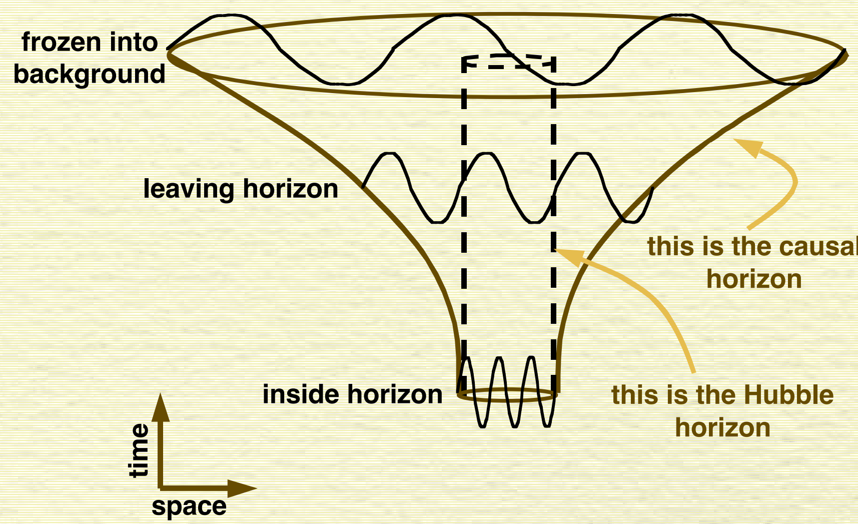
- Stage I: Inflation
  - inside horizon
  - leaves horizon
  - frozen into the background

First Stage: freezing in

Two horizons:

<u>causal horizon</u> = how far a signal can travel
<u>Hubble horizon</u> = the distance between which particles can communicate

Follow a little fluctuation (e.g. a particular P_k) as the universe inflates:



How inflation makes structure (II)

- Stage II: Post-Inflation
  - frozen into the background
  - the Hubble horizon expands
  - fluctuations reenter the horizon

- By freezing in a pattern of primordial fluctuations into the background, inflation provides the initial disturbance to the medium

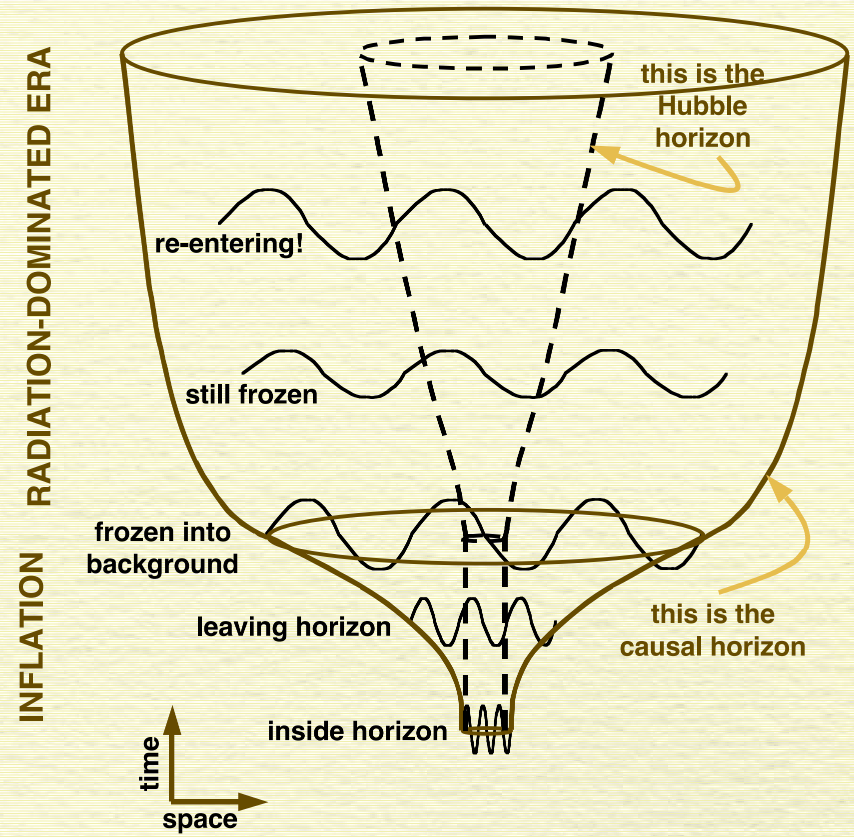
- <u>Analogy</u>:

primordial fluctuations	→	"initial conditions"
matter & radiation fluid	→	how a medium responds

- Together these make the beautiful pattern in the CMB

Second Stage: thawing out

After inflation end, the (Hubble) horizon catches up with the frozen perturbations



The predictions of inflation

- What does inflation predict
  - flat <u>primordial</u> power spectrum
  - nearly Gaussian
  - gravity waves

- These affect the matter/radiation medium to produce
  - correlated structures on all scales
  - synchronized acoustic oscillations
  - gravity waves—not seen (yet?)

What are the c_l's?

$$\delta T(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$

temperature difference

how much of a mode is present

$$c_l = \frac{1}{2l+1} \sum_{m=-l}^{l} \left| a_{lm} \right|^2$$

Predicting the CMB

dark matter

radiation

baryons

neutrinos

primordial perturbations (inflation)

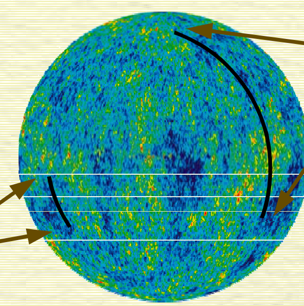
Einstein equations + Boltzmann equations

CMB temperature anisotropies (c_l)

$$c_l = \int d^3k \, P_k \, [T_l^{\mathrm{CMB}}(k)]^2$$

Measuring the CMB

To us, the CMB looks like the inside of a sphere



small l = big angle

big l = small angle

The c_l's tell the likelihood two points are correlated

WMAP (Wilkinson Microwave Anisotropy Probe)

- So, what do we know about the primordial perturbations
  - 2 parameters (so far)

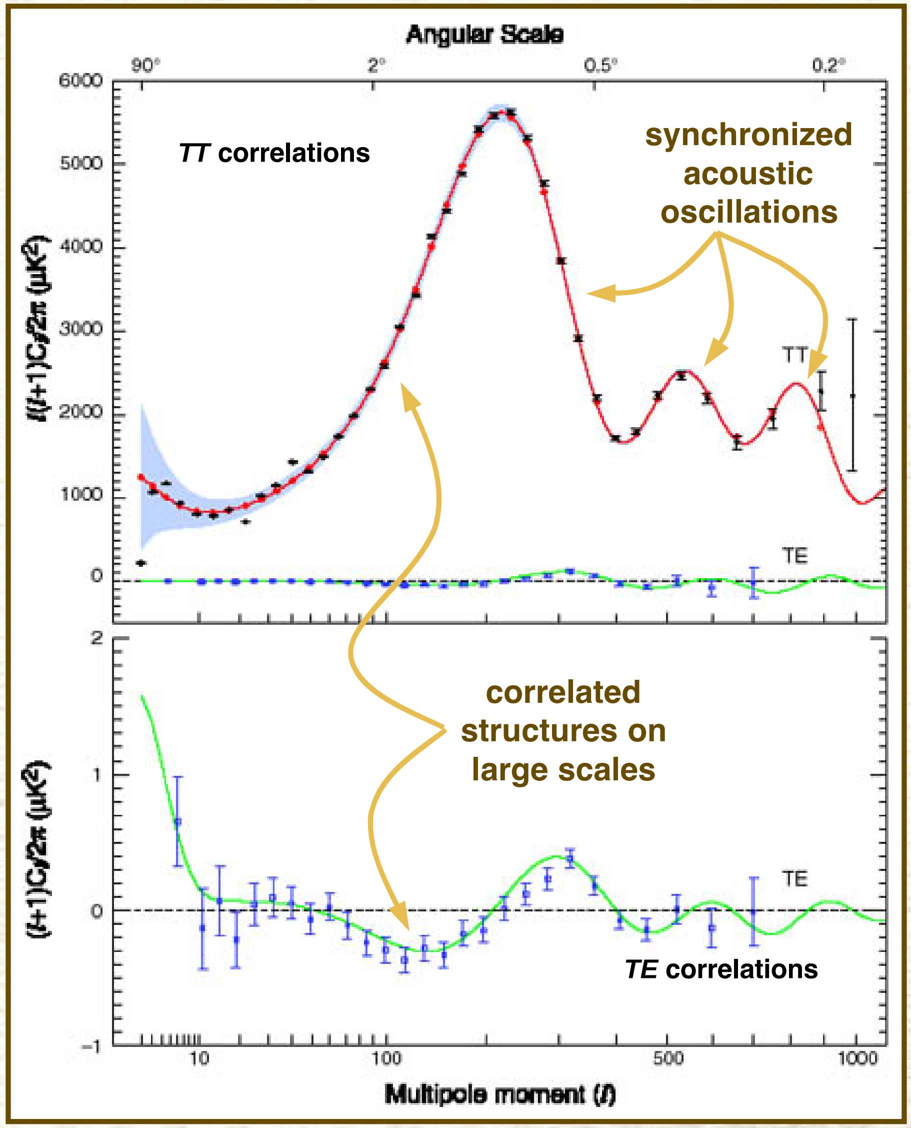
- Precision measurements of the CMB
  - WMAP, Acbar, Boomerang, CBI, VSA, DASI, . . .

6 Parameter Standard Cosmological Model

Ingredients:	$\Omega_b h^2$, $\Omega_m h^2$
Dynamics:	H_0, τ
Initial input:	A_s, n_s

nearly flat primordial power spectrum

$n_s = 0.961 \pm 0.017$
[WMAP 3 year]



from the WMAP/NASA science team

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Tugging at loose threads

- So far, we emphasized inflation's successes
- But what are its shortcomings?
- It is almost always worthwhile to pull at a loose thread in a theory; either
  - the theory falls apart
  - or we learn somthing new and important about the universe

Experimental side

What is the limit on what we can learn about the 'initial' perturbations?

From the CMB? or elsewhere?

Examples of loose threads

No stable cosmological solutions in relativity → the expanding universe

Was the universe hotter & denser long ago? → the cosmic microwave background

Causality (horizon) problem → inflation

Fluctuations smaller than a Planck length? → <u>open question</u>

One loose thread—the trans-Planckian problem

- Unresolved parts of inflation:

  [R. Brandenberger]

  - the trans-Planckian problem
  - what drives inflation?
  - the potential must be finely tuned
  - cosmological constant problem
  - singularity problem
  - the back-reaction problem

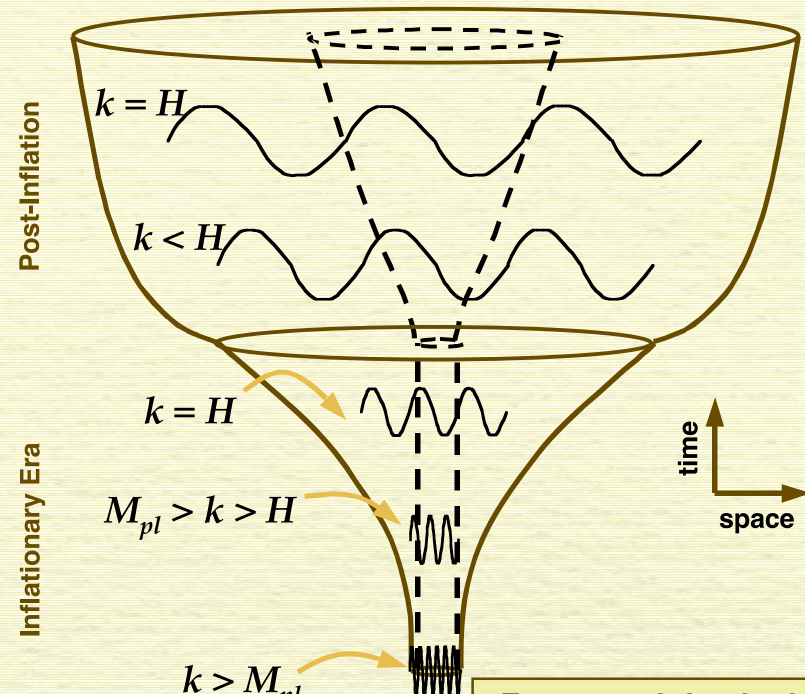
Before we become too optimistic:

"Inflation consists of taking a few numbers that we don't understand and replacing it with a function that we don't understand."

David Schramm (1945–1997)

The trans-Planckian problem

Let us look at a little fluctuation again, following farther back in time,

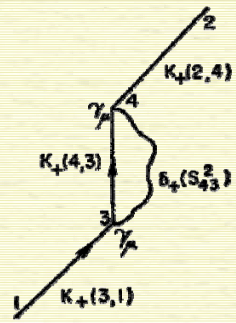
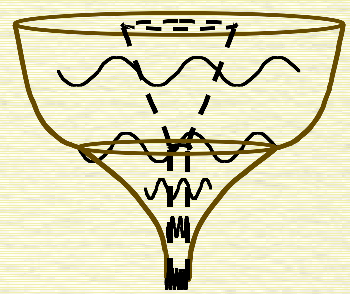


Far enough back, the wavelength is smaller than the Planck length!

Implications for the foundations of quantum field theory

- The trans-Planckian problem —a new opportunity for quantum field theory
  - structure formation in inflation is inherently a quantum process
- UV problems in QFT's and their resolution has led to new insights
  - infinites → renormalization & running of couplings
  - irrelevant operators → effective field theories
- <u>New structures in QFT's</u>:
  - effective initial states
  - *boundary* renormalization

A few of the UV problems of the past

radiative corrections		→	renormalization, RG flows, running couplings
nonrenormalizable operators	4-fermion operators, $V - A$ theory	→	effective field theory, chiral Lagrangians, HQEFT, SCET, . . .
trans-Planckian problem		→	effective initial states, ???

The old trans-Planckian problem (ca. 1940's)

- A much older incarnation of the trans-Planckian problem
- Interactions produce radiative corrections
- Integrate over all momenta in a loop
  - including trans-Planckian momenta
  - also, these 'perturbative' corrections were infinite!
- Why did Feynman not need to worry about quantum gravity in looking at $e^{-}e^{+}$ scattering?
- The answer: Renormalization
  - large momentum → short distance
  - cancel infinities with local operators

Radiative or loop corrections

From Feynman's original paper on QED,

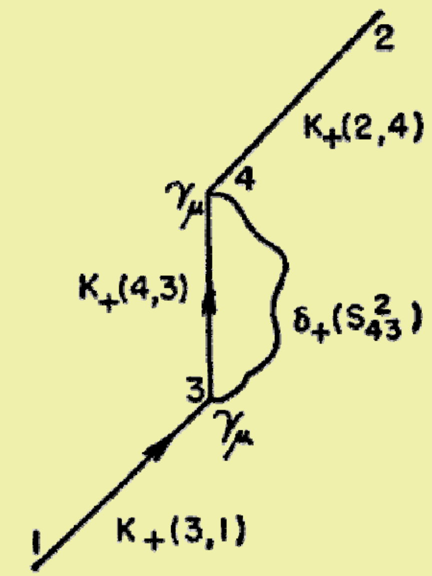


FIG. 2. Interaction of an electron with itself, Eq. (6).

'self-energy' correction to electron mass

$$m_0 \rightarrow m_R$$

The effective theory idea

- A basic tenet of physics:

  do not need a theory of all scales to understand a system at a particular, limited range of scales

- Applies to both approaches:
  - emergent (e.g. mesons)
  - reductionist (e.g. thermodynamics)

- In quantum field theory, the effective theory idea has a very precise formulation

Can we apply something of this philosophy to the trans-Planckain problem?

The recipe for an effective field theory

1. Choose the relevant degrees of freedom

Fields, particles, etc.

Examples:

Chiral Lagrangians, HQEFT, SCET, Standard Model, . . .

2. Choose the symmetries

space-time, gauge, approximate

3. Construct all the interactions consistent with #1 and #2.

Two classes of interactions

- Renormalizable (long-distance)
- Nonrenormalizable (short-distance)

new physics hides among these!

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Approaching the trans-Planckian problem

- What is the trouble with smaller than Planck length fluctuations?

$$l_{pl} = \sqrt{hG_N / 2\pi c^3} \approx 1.6 \times 10^{-33}\,\mathrm{cm}$$

gravity strong

need a quantum theory of gravity!

- Connection between the large & small [David Schramm]
  - the CMB as a cosmic microscope [Easther,Greene, Kinney, Shiu]

- Can is this be understood rigorously, in a controlled framework?

- Experimental Question:
  Can these 'trans-Planckian' signals be seen? What is their signature?

The "bottom-up" perspective

Start with an effective theory up to the Planck scale

↑

Determine the signal in the CMB

Our Group
H. Collins & R. Holman

ISCAP Group
Greene, Schalm, Shaar, Shiu + Kinney, Easther

'Adiabatic' Group
Anderson, Molina, Mottola

Stanford Group
Kaloper, Kleban, Lawrence, Shenker, Susskind

The "top-down" perspective

Start with an idea for physics beyond the Planck scale

↓

Determine the signal in the CMB

Many, many ideas
stringy uncertainty
minimal length prescription
modified dispersion relation
a-states (de Sitter space),etc.

The effective theory of an initial state

- In quantum field theory (or just QM)
  We look at matrix elements:

expectation values of *operators* in a *state*

$$\langle 0 | \varphi(t,x)\varphi(t,y) | 0 \rangle$$

- In ordinary EFT choose a state
  - new physics can affect how it evolves

- Something is missing:
  - what if new physics appears in the state?

Collins & Holman, PRD 71:085009 (2005)

- The effective theory of an initial state is a method for adding new short-distance structures in the state

Collins & Holman, hep-th/0501158, hep-th/0507081, hep-th/0605107, hep-th/0609002

An important matrix element

In inflation, one example is the two-point function (power spectrum)

$$\langle 0 | \varphi(t,x)\varphi(t,y) | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-y)} \frac{2\pi^2}{k^3} P_k(t)$$

But we do not really know the state at all scales; so replace

$$|0\rangle \Rightarrow |0_{\mathrm{eff}}\rangle$$

An effective initial state structure

State is fixed in terms of modes

$$\varphi_k - U_k^{\mathrm{vac}} \neq 0 \quad \text{for } k > M$$

Describe a general state by how it differs from a standard 'vacuum'

The effective state short-distance structure

- The basic idea of the effective state,
  - standard vacuum to a point, ($k < M$)
  - add general structures beyond, ($k > M$)
  - M is the scale of new physics

- Write a general mode by how it differs from a standard mode

  standard vacuum modes ⇔ relevant & marginal operators

- In an expanding background, use the Bunch-Davies 'vacuum'
  - flat vacuum as $k \to \infty$

- Write a general mode by how it differs from a standard mode

  standard vacuum modes ⇔ relevant operators

Mode expansion of a quantum field

Expand in the eigenmodes,

$$\varphi = \int \frac{d^3k}{(2\pi)^3}\left[\varphi_k e^{ik\cdot x} a_k + \varphi_k^* e^{-ik\cdot x} a_k^{\leq}\right]$$

Klein-Gordon equation – normalization constraint = one free parameter

$$\varphi_k = \frac{U_k + f_k U_k^*}{\sqrt{1 - f_k f_k^*}}$$

General modes: φ_k Bunch-Davies modes: U_k

An effective initial state structure

An ansatz with new UV structures:

$$f_k = \sum_{n=1} i^n d_n \frac{k^n}{a_0^n M^n}$$

Vanishes as $k << M$

The effective state propagator

- A further subtlety in choosing the propagator
- If we impose an boundary condition on the modes at $\eta = \eta_0$, then check the compatibility of time-ordering
- $\exists$ a unique choice of the propagator
  - compatible with the initial condition
  - avoids non-renormalizable divergences (c.f. de Sitter α-states)

Collins, Holman & Martin, hep-th/0306028
Einhorn & Larsen, hep-th/0209159
Goldstein & Lowe, hep-th/0302050

- Interpretation:
  the new term is for the *forward* propagation of the information in the initial state

The effective state propagator

The propagator in a momentum represenation

$$G(\eta, x; \eta', y) = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-y)} G_k(\eta, \eta')$$

Initial-time compatibility ⇒

$$\begin{aligned} G_k(\eta, \eta') &= \Theta(\eta - \eta') U_k(\eta) U_k^*(\eta') \\ &+ \Theta(\eta' - \eta) U_k^*(\eta) U_k(\eta') \\ &+ f_k^* U_k(\eta) U_k(\eta') \end{aligned}$$

Minkowski space limit—image sources

In flat space, the propagator has two sources, one real source and a second ficticious one to fix the initial condition

$$G_k(t, t') = \underbrace{G_k^F(t, t')}_{\text{The Feynman propagator}} + f_k^* G_k^F(\underbrace{2t_0 - t}_{\text{the image time}}, t')$$

Renormalization of the initial state

- The basic idea of the effective state is to use
  - standard vacuum up to a point ($k < M$)
  - add general structures beyond it ($k > M$)

- Renormalization:
  radiative corrections sum over of the new short-distance structures
  - a new class of infinities

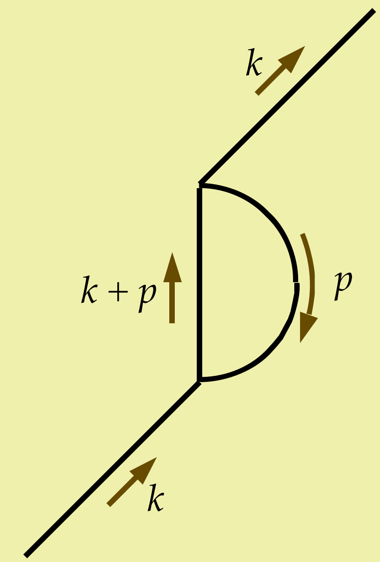
- Infinites are confined precisely to initial time
  - add boundary counterterms to Lagrangian

hep-th/0501158, hep-th/0507081

- Experimental prediction:
  For measurements made at scale Λ ($= k$ or H?) trans-Planckian effects suppressed by

$$\left(\frac{\Lambda}{M}\right)^n$$

Loop corrections for the inflaton



Interaction of an inflaton with itself

Boundary renormalization of Lagrangian, L

$$L = L_0 + L_I + L_{\text{bct}}\,\delta(t - t_0)$$

this part tells how operators evolve

this part tells how states evolve

this part cancels the boundary divergences

A brief example: $\frac{1}{2}\nabla_\mu\Phi\,\nabla^\mu\Phi - \frac{1}{2}m^2\Phi^2 - \frac{1}{24}\lambda\Phi^4$

- Follow the prescription:
  - evaluate n-point function to some order in λ
  - 'bulk' renormalization proceeds as usual
  - evaluate state-dependent (f_k) terms
  - cancel their divergences with local boundary counterterms

- The renormalized matrix elements are completely finite away from the boundary

$$\boxed{\left(\frac{H}{M}\right)^n, \left(\frac{k}{M}\right)^n < \varepsilon \qquad \text{experimental precision}}$$

  - for k, $H < M$, only a finite set is needed

- There is a beautiful correspondance,

large-distance structures	$\Leftrightarrow$	relevant (dim ≤ 3) boundary counterterms
short-distance structures	$\Leftrightarrow$	irrelevant (dim > 3) boundary counterterms

A flat-space example

Divide the field:

$$\Phi\,(t,x) = \underbrace{\phi(t)}_{\text{zero mode}} + \underbrace{\varphi(t,x)}_{\text{fluctuation}}$$

Evaluate the one-point function,

$$\left\langle 0_{\rm eff}(t)\middle|\varphi(t,x)\middle|0_{\rm eff}(t)\right\rangle$$

$$= \int_{t_0}^{t} dt'\,\frac{\sin[m(t'-t)]}{m}\left\{\frac{d^2}{dt^2}\phi + \frac{1}{6}\lambda\phi^3\right.$$

$$+\,\phi(t')\left[m^2 + \frac{1}{4}\lambda\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\right]$$

$$\left.\boxed{+\,\frac{1}{4}\lambda\phi(t')\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}f_k^* e^{-2i\omega_k(t'-t_0)}} + \cdots\right\}$$

New terms diverge only at boundary, e.g.

$$d_n^*\int\frac{d^3k}{(2\pi)^3}\frac{\omega_k^n}{M^n}\frac{1}{\omega_k}e^{-2i\omega_k(t'-t_0)}$$

$$\approx H_{n+2}^{(2)}[2m(t-t_0)] \approx \frac{1}{(t-t_0)^{n+2}}$$

hep-th/0501158, 0507081, 0605107, 0609002

Trans-Planckian ripples in the CMB

- What is the generic form of the trans-Planckian correction?
  - 'hyperfine' modulation of the CMB

- Two standard classes of corrections (top-down & effective theory: H/M)

$$P_k = \frac{H^2}{4\pi^2}\left[1 - 2x\frac{H}{M}\cos\left(\frac{2M}{H} + \phi\right) + \cdots\right]$$

(effective theory: k/M)

model-dependent order one constants

$$P_k = \frac{H^2}{4\pi^2}\left[1 - 2\frac{k}{M}d_1\cos(2k\eta + \phi) + \cdots\right]$$

Natural UV cutoffs in expanding space-times
Perimeter Institute, Sept. 2006

- A clear prediction for effects of new physics in the inflationary state!

Corrections to the power spectrum

New short-distance effects in the state modify the power function slightly

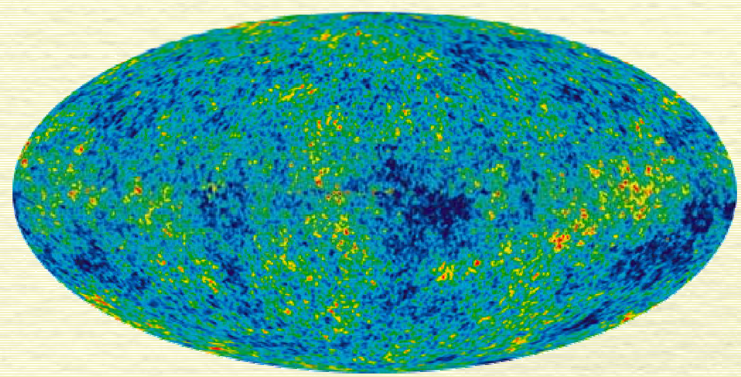
$$P_k = \frac{H^2}{4\pi^2}\left[1 - 2\frac{k}{M}d_1\cos(2k\eta + \phi) + \cdots\right]$$

(order one term)

The power spectrum sets the initial data that makes the CMB ripples

$$c_l = \int d^3k\, P_k\, [T_l^{\mathrm{CMB}}(k)]^2$$

From the WMAP/NASA Science Team



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Observations

- Can we observe a trans-Planckian signal,
  - CMB or elsewhere?

- New effects suppressed by H/M
  - M = scale of new physics
  - Planck scale ($M = M_{pl}$)?
  - something in between ($H < M < M_{pl}$)?

- CMB experiments: (nearer future)
  - WMAP, Planck, . . .
  - precision: one part in 10^3 or so

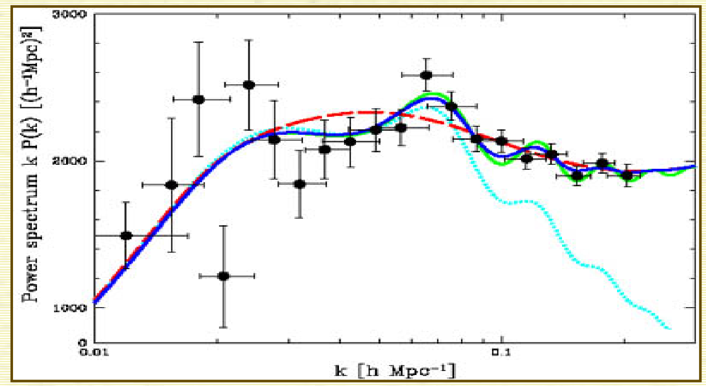
- Large scale structure experiment: (10–15 yr)
  - SKA, 21 cm high-z gas, cosmic inflation probe, etc.
  - precision: one part in 10^5 or 10^6 !

numbers from David Spergel's ISCAP talk

The same ripples appear in the LSS

The primordial perturbations influence the large scale structure (LSS) as well

SDSS has also seen the first acoustic peak,



astro-ph/0608632

The galaxy survey future (next 10–15 years)

The square kilometer array (SKA) will look at a 10^9 Mpc^3 volume of the universe

Speculations

- Here, we looked at just one observational signal from the effective initial state
- But it opens new possibilities and many, many new questions!

What is their connection to the invariant states of de Sitter space?

How is their energy-momentum renormalized?

How are specific models (composite inflaton, shortest distance, etc.) realized in the effective theory?

How readily are various UV-completions distinguishable?

What current bounds does the LSS place on trans-Planckian physics?

What is their connection to symmetry-breaking terms in an effective field theory?

Do they provide new sources for non-Gaussianities in the CMB?

Is there an initial time RG flow?

Are their other settings in which they could be applied, where boundary effects are important (CM)?

What is the trans-Planckian signal large scale structure?

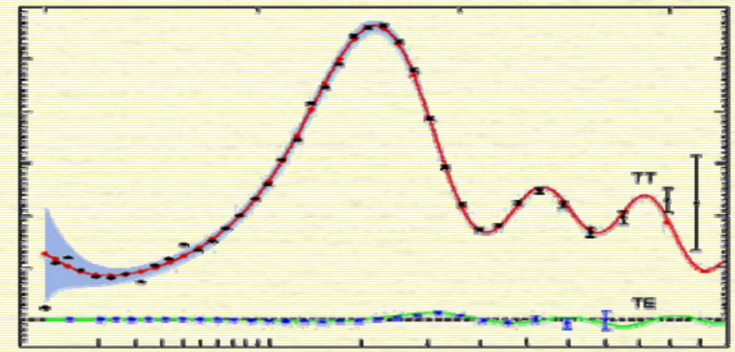
What does a particular initial state imply for inflation?

- Inflation presents new, unique challenges to our foundation for quantum field theory
  - UV effects in an expanding background
  - answers to formal questions ⇒ observable effects!

Conclusions

- Cosmology is now a precision experimental science
  - mysterious cosmological 'standard model' [inflation + ΛCDM]
- Inflation, explains the origin of structure but has some loose threads
  - e.g. its trans-Planckian problem
- The effective state approach to the trans-Planckian problem has uncovered
  - a model-independent prediction for the signals from physics above the inflationary scale
  - fascinating new renormalizable structures for quantum field theories
- The long-range prospects for observing such effects are extremely good

WMAP precision measurements



Trans-Planckian corrections to the primordial power spectrum

$$P(k) = \frac{H^2}{4\pi^2}\left[1 - 2\frac{k}{M} d_1 \cos\left(2k\eta + \phi\right) + \cdots\right]$$

Precision measurements of the CMB & LSS