The Nucleus

What is the nucleus? Of what is it made? What are its properties and why does it have these properties? Is it stable or dynamical?

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Near the beginning of the last century, it was finally realized that the atoms---the ‘indivisible’ bits of matter that still retain distinctive chemical properties---themselves have a structure and that moreover this structure is extremely non-uniformly distributed. Most of the chemical properties are determined by a very light, diffuse cloud of electrons; but most of the mass of the atom resides in a tiny, dense core called the nucleus. Since atoms in isolation tend to be electrically neutral, and since each electron carries a single negative unit of charge, this nucleus was assumed to be an object made of bound positive charges, called protons. But what forces held these protons together against their electromagnetic repulsion and were protons the only ingredients of the nucleus?

The neutron

The neutron, an electrically neutral particle as its name suggests, is the other constituent of the nucleus. It has a role in the stability of nuclei, although one that is ultimately limited---all nuclei beyond a point are unstable.

Aside from its different electromagnetic properties, the neutron is otherwise quite similar to the proton. To compare,

	charge	mass	spin	mag. moment
proton (p)	+1	938.3 MeV	1/2	2.79 μ_N
neutron (n)	0	939.6 MeV	1/2	-1.91 μ_N

(where $\mu_N = e\hbar/2m_p$)

There are two details of this table to notice especially. First, the magnetic moments suggest that the proton and the neutron---which despite being electrically neutral, does exhibit a magnetic character beyond what its spin would suggest---must have some substructure. Second, the neutron is the slightly heavier of the two; so _in isolation_ it is at least energetically allowed to decay---as in fact it does,

$$n \to p + e^- + \bar{\nu}_e$$

$$\Delta Mc^2 = m_n c^2 - m_p c^2 \approx 1.3 \text{ MeV} \approx 2.53 \, m_e$$

The mean life for this decay is 885.7 ± 0.8s, or about 14.8 minutes.

An aside: The interaction that produces this decay is the _weak force_.

If the neutron is unstable, does it also decay when it is a part of a nucleus? The answer depends critically on the environment in which it exists---if the decay of the neutron into a proton results in a less energetically stable nucleus, then it will not. Otherwise it does decay and this process is called _β-decay_.

Nuclei

Having introduced the constituents of the nucleus and described a few of their properties in isolation, we next build more complex structures and describe how nucleons behave in aggregate. Like most things, the properties of a nucleus are a bit more than the sum of the parts that compose it---we must also account for the mutual interactions of these parts, which is why, for example, a bound state of a proton and a neutron has a slightly smaller mass than the sum of their separate masses.

We shall specify a particular nucleus by how many protons and neutrons it contains,

Z = # of protons (= # of electrons, if neutral)
A = # of nucleons (protons + neutrons)
N = # of neutrons = A - Z

Z is called the atomic number and A is called the atomic weight. Because the numbers of electrons and protons are equal in a neutral atom, Z also determines a particular chemical element, even for different values of A.

Notation: Although not entirely standard, the notation we shall adopt will be to write a particular nucleus of the element X as

$$^{Z}X^{A}$$

As an example, the most common form of oxygen is $^{8}O^{16}$.

All nuclei with the same atomic number share the same basic chemical properties and therefore correspond to the same elements. Such nuclei are called isotopes. As an illustration, consider the simplest three nuclei we can build---the three isotopes of hydrogen

	content	name	terrestrial abundance	mean life
${}_1H^1$	p	---	99.985	stable
${}_1H^2$	pn	deuteron	0.015	stable
${}_1H^3$	pnn	triton	trace	17.8 yrs

Notice that while all of these elements have similar chemical properties, their nuclear properties can be quite different---most dramatically, tritium is radioactively unstable.

In principle, it would seem that we could assemble any combination of protons and neutrons into ever larger nuclei. However, because of the properties of the force that binds the nucleus together---it is strong but of a very limited range---and the electromagnetic repulsion of the protons and the inherent instability of the neutron, at some point large nuclei very quickly fall apart. Even before this point, any nucleus with more than 83 protons (bismuth) eventually decays, though elements up to uranium (Z = 92) still occur naturally since one of its isotopes, ${}_{92}U^{238}$ has a very long half-life (4.47 Gyr).

Elements heavier than uranium can still be produced in the laboratory and over the years, heavier and heavier nuclei have been made. Most recently, element-118 was made by scientists at the Lawrence Livermore National Laboratory (CA) and the Joint Institute for Nuclear Research (Dubna); the particular isotope was ${}_{118}118^{297}$ [PRC 74, 044602 (2006)].

Size and density

To a reasonable degree, nucleons are relatively incompressible so that the volume occupied by a nucleus scales nearly linearly with the number of nucleons that it contains. Thus, the radius of a particular nucleus scales as the one-third power of A, $r \propto A^{1/3}$, the constant of proportionality being determined empirically. By scattering sufficiently energetic particles off of the nucleus, its radius was found to be about 10^{-5} that of an atom. If the natural scale for atomic sizes is the Angstrom (Å), the natural scale for the nucleus is a femtometer (fm), which is often called a fermi.

atom:	1 Å	=	10^{-10} m	=	10^{-8} cm
nucleus:	1 fm	=	10^{-15} m	=	10^{-13} cm

In these units, the approximate relation between the size and atomic weight of nucleus is

$$r \approx 1.2\ A^{1/3}\ \text{fm} \qquad \text{[Nuclear radius]}$$

From this relation we can estimate the typical density of the material inside a nucleus

$$\begin{aligned}\rho_{nucl} &= M/V \approx Am_p/(\ (4\pi/3)\ r^3\ ) \\ &= (3m_p/4\pi)\ (1.2\cdot10^{-15}\ \text{m})^{-3} \\ &= (3/4\pi)\ 1.67\cdot10^{-27}\text{kg}\ (1.2\cdot10^{-15}\ \text{m})^{-3} \\ &\approx 3\cdot10^{17}\ \text{kg m}^{-3}\end{aligned}$$

For comparison,

$$\rho_{earth} \approx 5.5\cdot10^{5}\ \text{kg m}^{-3}$$

$$\rho_{\text{white dwarf}} \approx 10^{11}\ \text{kg m}^{-3}$$

$$\rho_{\text{neutron star}} \approx 10^{16}\ \text{kg m}^{-3}$$

To acquire a sense of what such great densities mean, if the earth had the same density as a nucleus, its diameter would be about one-fifth of a mile, or about 336 m.

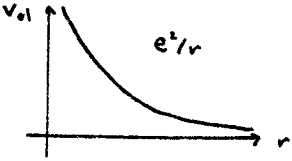
Nuclear binding

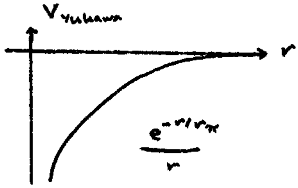
How do we determine whether a particular assemblage of protons and neutrons forms a stable nucleus? To address this question requires understanding some of the forces that act among the nucleons. Our presentation here will remain mostly qualitative, since the interactions are in practice extremely complicated to calculate at all precisely, even though the basic character of this force is very well known. Therefore after introducing four basic features of how pairs of nucleons interact, we discuss specific cases more empirically. Also, while the weak interaction---that which gave rise to the neutron decay---is important for the ultimate stability of some nuclei, we shall neglect it and a few other subtleties for the moment, returning to them a little later.

To begin, there are four basic features of the interactions between nucleons that determine how tightly a particular nucleus is bound together.

(1) The first is the most familiar: the electric repulsion between protons

$$V_{pp}^{EM}(r) = e^2/r$$
$$\approx 1.4\ \text{MeV}\ (1\ \text{fm}/r)$$





(2) The second is a strong, but short-ranged attractive interaction between nucleons, described by a Yukawa potential

$$V_{p\uparrow p\uparrow}(r) = V_{n\downarrow n\downarrow}(r)$$
$$\approx -3.7 \text{ MeV} \exp(-r/r_\pi)/(r/r_\pi)$$

where $r_\pi \approx 1.4$ fm.

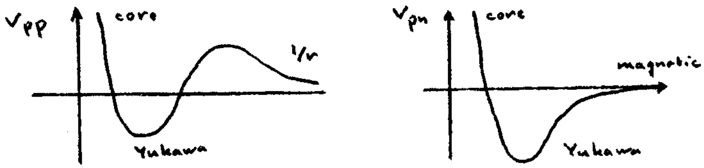
(3) At very short distances, there is a ‘repulsive core’ interaction which basically means that the nucleons do not overlap much; we already saw this repulsion in the fact that the volume of a nucleus scaled with the number of nucleons.

(4) Finally, both protons and neutrons have spin 1/2 and must satisfy the Pauli exclusion principle; so with many of either together, they cannot all be placed in the same lowest energy state. This behavior should be familiar from how electrons fill out successively higher orbitals in atoms.

The Yukawa potential quite overwhelms the electric repulsion when nucleons are close, although the exponential factor causes it diminish very rapidly. This property is important for the binding energy of large nuclei---every proton feels to an appreciable extent the repulsion of every other, but it only feels the attraction of the nucleons that are close to it.

To appreciate this point better, let us calculate both potentials at 1 fm and at 5 fm:

$$V_{Yuk}(1fm) \approx -2.5 \text{ MeV} \qquad V_{Yuk}(5fm) \approx -29 \text{ keV}$$
$$V_{EM}(1fm) \approx 1.4 \text{ MeV} \qquad V_{EM}(5fm) \approx 290 \text{ keV}$$



Including all but the Pauli exclusion, the general form of the potential between two nucleons is as shown in the two figures. At very short distances, the hard core repulsion dominates and the potential is positive and grows rapidly as the separation diminishes. At larger distances, of the order of a fermi, the Yukawa potential produces the dominant effect creating an attractive interaction which overpowers any electromagnetic effects. However, the range of this nuclear Yukawa force is very limited and soon the electromagnetic force becomes the leading effect.

Thus far, we have considered in some detail the interactions between only two nucleons at a time, for more complicated nuclei we shall only state how tightly they are bound by appealing to the experimentally measured values of their binding energies, though guided by what we have just learned from their pair-interactions.

Binding energy

The binding energy of a particular nucleus is equal to the difference between its energy and the sum of the masses of all its nucleons measured in isolation,

$$BE({}^{Z}X^{A}) = Z\, m_p + N\, m_n - m(Z,A)$$

It is usually convenient to choose a measure appropriate for the objects we are studying, so for nuclei we should apply units where the mass of a proton or a neutron is about one. One such measure is the atomic mass unit (u), defined to be 1/12th the mass of a carbon-12 nucleus,

$$\text{mass}({}^{6}C^{12}) = 12\ \text{u}$$

In terms of the more conventional units of mass,

$$1\ \text{u} = 1.66054\cdot 10^{-17}\ \text{kg} = 931.494\ \text{MeV}/c^2$$

Since carbon-12 is more stable than a set of six separate protons and six separate neutrons, the mass of the proton and the neutron are each a little larger than one,

$$m_p = 1.007825\ \text{u} \qquad m_n = 1.008665\ \text{u}$$

A good indication of the relative stability of a nucleus is its binding energy per nucleon,

$$\overline{BE}(A) = BE(Z,A)/A$$

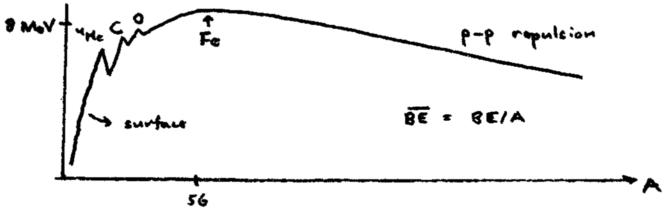
Let us calculate a few examples,

$$\begin{aligned} BE({}^{2}He^{4}) &= 2\, m_p + 2\, m_n - 4.002603\ \text{u} \\ &= 0.03038\ \text{u} = 28.30\ \text{MeV} \\ \overline{BE} = BE/A &= (28.30/4)\ \text{MeV} = 7.074\ \text{MeV/nucleon} \end{aligned}$$

$$\begin{aligned} BE({}^{10}Ne^{20}) &= 10\, m_p + 10\, m_n - 19.99244\ \text{u} \\ &= 0.1724\ \text{u} = 160.6\ \text{MeV} \\ \overline{BE} = BE/A &= (160.6/20)\ \text{MeV} = 8.03\ \text{MeV/nucleon} \end{aligned}$$

$$BE({}^{92}U^{238}) = 92\ m_p + 146\ m_n - 238.050783\ u$$
$$= 1.934\ u = 1802\ MeV$$
$$\overline{BE} = BE/A = (1802/238)\ MeV = 7.57\ MeV/nucleon$$

$$BE({}^{26}Fe^{56}) = 26\ m_p + 30\ m_n - 55.934942\ u$$
$$= 0.5285\ u = 492.3\ MeV$$
$$\overline{BE} = BE/A = (492.3/56)\ MeV = 8.79\ MeV/nucleon$$



Iron-56 is the most stable of all of the elements. This stability results from two competing effects among the forces acting within a nucleon. For very large nuclei, as we showed earlier, there is a growing role for the electric repulsion among the protons. As we add a proton to a nucleus, only the nucleons in the immediate vicinity experience the strong attraction to it---which for a big nucleus is only a small fraction of the total number of nucleons---since the Yukawa force is very short-ranged. But every other proton repels it to some appreciable extent. In contrast, for small nuclei there is a distinct advantage in adding more nucleons---even protons---since each new nucleon will automatically be close to every other one, so the contribution from the Yukawa attraction per nucleon is very high.

That iron-56 should be the most stable isotope depends on when one of these two competing effects gives way to the other, which in turn depends on the relative strengths of the two forces and the range of the Yukawa attraction.

However, there is a limit to how many nucleons we can put together---eventually the electric repulsion overcomes the nuclear binding. It might be thought that we could evade this limit by adding more and more neutrons, and this idea might have succeeded but for the inherent instability of the neutron. Isotopes with N>>Z tend to decay to nuclei with N and Z more similar---although still N will typically be larger than Z, especially for heavy nuclei. So let us consider a slightly idealized case where Z≈N and where the nuclear binding energy per nucleon is about 8 MeV. Let us further assume that the electric charge is uniformly distributed over a spherical volume. Under these conditions, the largest nucleus will be one where the electric energy just cancels the nuclear binding energy,

$$E_{tot} = E_{el} - \overline{BE}\, A_{max} \approx 0$$

For a uniform sphere of charge

$$E_{el} = (3/5)\ (Z^2e^2/4\pi\varepsilon_0 r) \qquad Z = A_{max}/2$$

$$r = r_0\, A_{max}^{1/3}$$

Thus,

$$A_{max} = (1/\overline{BE})\ (3/5)\ (1/4)\ (A_{max}{}^2 e^2/4\pi\varepsilon_0 r_0 A_{max}{}^{1/3})$$

$$= (1/\overline{BE})\ (3/20)\ (e^2/4\pi\varepsilon_0 r_0)\ A_{max}{}^{5/3}$$

$$A_{max} = (\ \overline{BE}\ (20/3)\ (4\pi\varepsilon_0 r_0/e^2)\ )^{3/2}$$

$$= (\ (20/3)\ 1/(9\cdot 10^9)$$

$$\cdot\ 1.2\cdot 10^{-15}/(1.602\cdot 10^{-19})^2$$
$$\cdot\ 8\cdot 10^6 \cdot 1.602\cdot 10^{-19}\ )^{3/2}$$
$$\approx 296$$

which is quite close to ${}^{118}118^{297}$. In nature, we might go a bit higher since $Z_{max} < A_{max}/2$. Note also that the answer depends upon $r_0^{3/2}$, as well.

Regardless of its precise value, this idea represents a limit to how large an element we can construct---beyond it, nuclei simply fall apart. Even well before this limit a nucleus might not be stable---we already saw an indication of this from the instability of the neutron---but might decay through purely quantum mechanical effects. We discuss three of the most common of such decays next.

β-decay

When we first introduced the neutron, we remarked that not only in isolation, but even as a part of a nucleus it sometimes might be unstable to decaying into a proton accompanied by an electron and its antineutrino,

$$n \to p + e^- + \bar{\nu}_e$$

or more generally,

$$^{Z}X^{A} \to {}^{Z+1}D^{A} + e^- + \bar{\nu}_e$$

Whether this decay occurs depends on whether the mass of the original nucleus is more than the sum of the masses of the products. Since the electron and neutrino are light, we shall often ignore them, so whether this decay occurs depends upon whether the following quantity is positive,

$$KE = [\, M(Z,A) - M(Z+1,A)]\, c^2$$

Aside from the mass of the leptons, KE corresponds to the kinetic energy available to the products.

This process is called β-decay, based on an early classification of various types of radioactivity observed about a century ago, before it was realized that the emitted particle was an electron (the neutrino is very difficult to detect). Actually this basic process occurs in a set of closely related radioactive processes, all involving leptons. Leptons are particles which also have spin 1/2 but which do not feel the nuclear force (the strong force). Some of these, such as the electron, do interact electro-magnetically, whereas others---the neutrinos---do not. The neutrinos do experience the weak force, which is the interaction responsible for β-decays.

An aside: Three distinct families of leptons have been observed in nature, differing in their masses and the relative strengths of their interactions with other particles. The charged leptons are the electron (e), the muon (μ) and the tau (τ). Each of these has an antiparticle and an associated neutrino: ν_e, ν_μ and ν_τ. Whether or not the neutrinos have distinct antiparticles is still not yet known. Each lepton is assigned a lepton number +1 and its antiparticle as lepton number -1.

	mass		mass		mass
e	0.511 MeV	μ	106 MeV	τ	1777 MeV
ν_e	small	ν_μ	small	ν_τ	small

In addition to the basic neutron decay, there are other closely related versions of this same basic process---rearranged slightly---that occur in some nuclei. For example, a nucleus with too many protons, relative to the number of neutrons, might favor the decay of a proton into a neutron, although such a decay does not occur in isolation. But in a nucleus, we sometimes gain a little energy by reducing the electrostatic repulsion

$$p \rightarrow n + e^+ + \nu_e$$

$$_ZX^A \rightarrow {}_{Z-1}D^A + e^+ + \nu_e$$

$$KE = [\ M(Z,A) - M(Z-1,A)]\ c^2$$

A final common possibility occurs when highly elliptical electron orbitals (such as the K or L shells) bring electrons frequently near the nucleus. Then a process called <u>electron capture</u> can occur,

$$p + e^- \rightarrow n + \nu_e$$

$$_ZX^A + e^- \rightarrow {}_{Z-1}D^A + \nu_e$$

$$KE = [\ M(Z,A) - M(Z-1,A)]\ c^2$$

<u>Examples</u>

β^--decay frequently occurs when a nucleus has too many neutrons; for example,

$$_6C^{14} \rightarrow {}_7N^{14} + e^- + \bar{\nu}_e \qquad (t \approx 8270 \text{ yr})$$

β^+-decay frequently occurs when a nucleus has too few neutrons; for example,

$$_6C^{10} \rightarrow {}_5B^{10} + e^+ + \nu_e \qquad (t \approx 28 \text{ s})$$

α-decay

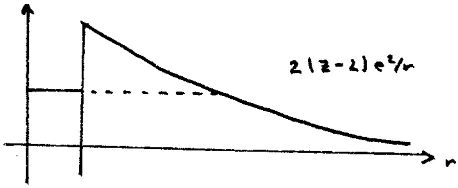
Another common form of radiation is α-decay and the particle emitted from the nucleus was identified by Rutherford as early as 1908 to be nothing more than a Helium-4 nucleus. As we saw earlier, the p-p repulsion in heavy nuclei reduces the binding energy for larger and larger Z. It is therefore sometimes possible that the removal of an α-particle (${}^{2}He^{4}$)---a particularly tightly bound combination---results in a more stable configuration,

$$ {}^{Z}X^{A} \rightarrow {}^{Z-2}D^{A-4} + {}^{2}He^{4} $$

$$ KE = [M(Z,A) - M(Z-2,A-4)] c^2 - 28.3 \text{ MeV} $$

that is, this decay can occurs when this KE is positive. We might wonder why such heavy nuclei should exist at all if such a decay is allowed. The reason lies in the structure of the potential that is experienced by the α-particle---classically the particle is confined by a potential barrier, but quantum mechanically, the wave function extends through and beyond it. Thus, with a sufficient amount of time---depending on the height and width of the barrier---there is a probability that the α-particle will be outside the barrier and can then escape.

We can understand this decay a little better by recalling the shape of the potential we described for the p-p interaction; in the setting here, both objects are still positively charged, but now with charges Z-2 and 2, so the potential is qualitatively what we saw before,



By suitably modeling the potential and solving for the wave function of the ${}^{2}He^{4}$ particle, we can estimate the life-time for this decay.

<u>Examples</u>

As we mentioned, α-decay is especially common for large, heavy nuclei and quite often the resulting nucleus of the decay is itself unstable to a further α-decay, as happens in the case of uranium-234,

$$ {}^{92}U^{234} \rightarrow {}^{90}Th^{230} + {}^{2}He^{4} \rightarrow {}^{88}Ra^{226} + {}^{2}He^{4} \rightarrow {}^{86}Rn^{222} + {}^{2}He^{4} \rightarrow \ldots $$

$3.56\cdot10^{5}$ yr $1.13\cdot10^{5}$yr $2.31\cdot10^{3}$yr   5.52 d

<u>γ-decay</u>

The final common type of radiation involves high energy photons, called γ-rays when their energies are in the MeV range. Although we have spoken as though the nucleon content is completely sufficient to specify the state of a nucleus, it is not. Just as for the electron orbitals, it is possible for nucleons to be in excited states associated with other eigenstates of the nuclear potential. A nucleus might be in such a state if it was struck by sufficiently high energy particles or if it is the result of a fission process that left it with some extra internal energy. As with the electrons, nucleons in these excited states will relax---possibly in a cascade of decays---into the ground state, emitting photons with characteristic energies associated with the transitions. Since these occur at precise energies, they are quite useful for peering inside the nucleus to learn about its potential and its states.

$$^{Z}X^{*A} \rightarrow {}^{Z}X^{A} + \gamma$$