Problem Set 8. Magnetic sources
Electromagnetism and Light

Due: 24 March 2023
at 3:00 pm

Reading: *Sternheim and Kane*, chapter 19, section 12; chapter 20, sections 1–3
Electromagnetism and Optics, chapter XI; chapter XII, sections 1–4.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

1. A singly charged ion moves with a constant velocity $v\,\hat{x}$ through a region with a magnetic field 1.25 T in the $\hat{z}$ direction and an electric field $5.00\times10^{3}\ \mathrm{V\,m^{-1}}$ in the $\hat{y}$ direction.

a. What is the speed v of this particle?

b. If this ion has a charge $-e$ and was first accelerated from rest through a 1.334 V potential before reaching the speed that you found in part a, what is its mass?

c. What is its mass in atomic mass units, $u = 1.67\times10^{-27}$ kg? What element do you think this ion might be?

2. A mass spectrometer has an electric field of $1.00\times10^{5}\ \mathrm{N\,C^{-1}}$ and a magnetic field 0.600 T in the velocity selector and a magnetic field of 0.800 T in the bending region.

a. What is the velocity of the ions that pass unbent through the velocity selector?

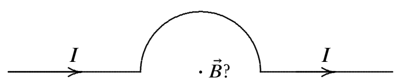
b. Find the spatial separation of singly ionised neon-20 and neon-22 isotopes with charges of e after they have travelled through half a circle in the bending region. Approximate the mass of a proton or a neutron as $m = 1.67\times10^{-27}$ kg.

3. Three parallel wires are lying in a plane. A current of $I = 10.0$ A flows in the same direction along each of them and the distance between adjacent wires is 20.0 cm. Find the net force per unit length

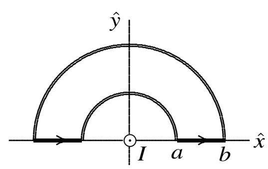
a. on the central wire and

b. on one of the outer wires.

4. Find the magnetic field at the centre of a semicircular wire, of radius a, through which a current I is flowing, as shown. Do the straight parts of the wire contribute to the magnetic field at that point?



5. Without using Ampère's law, show explicitly that the integral $\oint d\vec{\ell}\cdot\vec{B}$ for the following path made of straight segments and semi-circular paths in the xy plane vanishes when $\vec{B}$ is the magnetic field generated by a wire running along the $\hat{z}$-axis with a current I.



6. A thin conducting plane has a current flowing uniformly along it. If this plane is at $z = 0$ and the current is flowing in the $+\hat{y}$ direction, what is the magnetic field everywhere? Here you can treat the plane as though it were infinite, with a finite *current density* j measured in amperes per metre.

7. A *coaxial cable* is a commonly used type of cable. It has a special property that you will discover for yourself in this problem. It consists of a solid central conductor of radius a separated by an insulating material from a solid conducting shell of inner radius b and outer radius c. Both conductors carry uniformly distributed currents, but in opposite directions parallel to their common axis. The total current flowing in either direction is I. Choose your coordinates so that the axis of this wire is the $\hat{z}$-axis. If r is the distance to the $\hat{z}$-axis, find the magnetic field $\vec{B}$ in the regions (a) $r < a$, (b) $a < r < b$, (c) $b < r < c$, and (d) $r > c$. (e) What is the advantage of this cable, compared with just running separate wires to and from a device?

8. A *toroid* is a coil of wire wrapped evenly around a torus—for example, with a rectangular cross-section, with N turns of the wire in total. Imagine that the wrapping is uniform enough to be able to treat the current as purely radial on the top and bottom, and purely up or down on the inner and outer cylindrical surfaces. In the drawing we have only shown a few of the wrappings of the wires. The current I is flowing *downward* on the outer cylindrical surface and *upward* on the inside surface. Use Ampère's law to find the magnetic field in a plane perpendicular to the $\hat{z}$-axis and crossing through the toroid for (a) $r < a$, (b) $a < r < b$, and (c) $r > b$. Here r is the distance from the $\hat{z}$-axis.