Problem Set 7. Magnetic forces and dipoles
Electromagnetism and Light

Due: 17 March 2023
at 3:00 pm

Reading: *Sternheim and Kane*, chapter 19, sections 5, 7–8
Electromagnetism and Optics, chapters IX–X.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

1. When a mass m moves in a circle, it is accelerating even though its speed v is not changing. Its *centripetal* acceleration is always in the direction perpendicular to the direction of the velocity, $\vec{v}$, towards the centre of the circle, and it is given by $a = v^2/r$, where r is the radius of the circle. Suppose that an electron is moving in the xy plane in a constant magnetic field, $\vec{B} = B\,\hat{z}$, where $B = 1$ T.

a. What is the radius of the electron's orbit if it is moving at a speed of $1.00\times 10^7\ \mathrm{m\,s^{-1}}$?

b. Is it circulating in a clockwise or counterclockwise direction about $\hat{z}$?

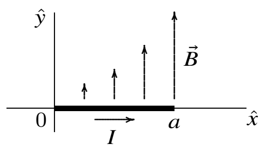
c. How long does it take the electron to complete a single orbit?

2. The magnetic force on a horizontal wire is upwards with a magnitude of 10.0 N when a 5.0 A current is flowing in the wire to the right. What is the force on the wire when the current's magnitude is doubled and its direction is reversed?

3. A wire is placed on the $\hat{x}$-axis with one end at $x = 0$ and the other at $x = a$. A current I is flowing along it in the $\hat{x}$ direction. A magnetic field $\vec{B} = B_0(x^2/a^2)\,\hat{y}$ fills all of space.

a. What is the magnetic force on the wire?

b. What is the magnetic torque on the wire about the origin?

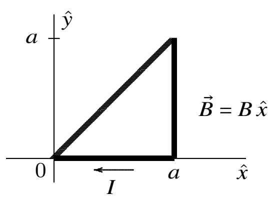


4. Suppose that you are given a wire of length L and resistance R and a small battery with an electromotive force $\mathcal{E}$. What is the largest magnetic dipole moment that you could achieve using these ingredients if you bend the wire into a simple loop?

5. A right isosceles triangular loop of wire, located in the xy plane with a 10.0 A current flowing in a clockwise direction, is placed in a magnetic field $\vec{B} = (0.200\ \mathrm{T})\,\hat{x}$, as shown. The lengths of the shorter sides of the triangle are both 1.00 m long.

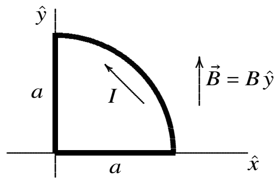
a. What is the magnetic force on each side of the wire?

b. What is the total magnetic force on the wire?



6. A 3.00 mA current flows around a rectangular loop whose sides are 2.50 cm and 4.00 cm long. If the current is initially circulating in a counterclockwise direction about a 65.0 μT magnetic field, how much energy is required to turn the loop so that the magnetic field is along the area enclosed by the loop? What is the magnitude of the torque in the initial and final configurations described?

7. Imagine that a quarter-circle current loop is placed in the xy plane as shown, along which a current I circulates in a counterclockwise direction. The loop is placed in a uniform magnetic field $\vec{B} = B\,\hat{y}$.



Calculate the torque on the loop by evaluating the torque on each of its sides. The full expression for the torque about the point $\vec{r}_0$ is

$$\vec{\tau} = \int d\vec{\tau} = \int \left(\vec{r}' - \vec{r}_0\right) \times \left(I\, d\vec{\ell}' \times \vec{B}\right).$$

Here $\vec{r}'$ is a point on the side you are treating—so you will be integrating over $\vec{r}'$. So that everyone is using the same point, choose the origin, $\vec{r}_0 = \vec{0}$, so that the torque calculated about this point would be

$$\vec{\tau} = I \oint \vec{r}' \times \left(d\vec{\ell}' \times \vec{B}\right).$$

Just to get you started, the integrals over each of the sides are

$$\begin{aligned}\vec{\tau} &= I\int_{\text{left side}} \vec{r}' \times \left(d\vec{\ell}' \times \vec{B}\right) + I\int_{\text{bottom side}} \vec{r}' \times \left(d\vec{\ell}' \times \vec{B}\right) + I\int_{\text{curved side}} \vec{r}' \times \left(d\vec{\ell}' \times \vec{B}\right) \\ &= I\int_0^a dy'\, \vec{r}' \times \left((-\hat{y}) \times \vec{B}\right) + I\int_0^a dx'\, \vec{r}' \times \left(\hat{x} \times \vec{B}\right) + I\int_0^{\pi/2} d\theta'\, \vec{r}' \times \left((-a\sin\theta'\,\hat{x} + a\cos\theta'\,\hat{y}) \times \vec{B}\right).\end{aligned}$$

You still need the appropriate parametrisation for $\vec{r}'$ on each of the sides. For example, on the bottom it would be $\vec{r}' = x'\,\hat{x}$.

8. Now let us treat the last problem again using the magnetic dipole moment instead—you should find the calculation to be much, much shorter.

a. What is the magnetic dipole moment of the current loop in the previous problem, $\vec{\mu}$?

b. Do you find the same torque as before if you calculate $\vec{\tau} = \vec{\mu} \times \vec{B}$, using $\vec{B} = B\,\hat{y}$?