Problem Set 6. Circuits and magnetic forces — Due: 3 March 2023
Electromagnetism and Light — at 3:00 pm

Reading: *Sternheim and Kane*, chapter 19, sections 1–4, 9–10;
Electromagnetism and Optics, chapters VIII.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

1. A 1,000 Ω resistor and a 1.00×10^{-5} F capacitor are connected in series to a 100 V battery.

a. What is the time constant for this RC circuit?

b. What is the final charge on the capacitor, Q_0?

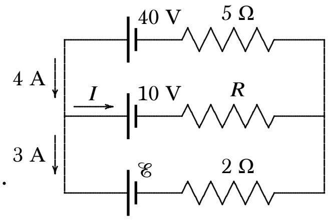
c. How long does it take to charge this capacitor to within one electron of the final charge, $Q(t) = Q_0 - e$?

2. Consider the circuit shown to the right.

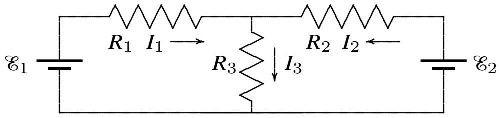
a. Determine the current I.

b. Determine the resistance R.

c. Determine the electromotive force of the lowest battery, $\mathscr{E}$.



3. As I mentioned in lecture, we can apply Kirchhoff's rules to more complicated circuits. For example, here is a circuit with two batteries and three resistors,



a. Use the fact that $\Delta V = 0$ around a closed loop to obtain two independent equations.

b. What is the relation between the currents I_1, I_2, and I_3?

c. You now have three equations and three unknown quantities, the currents. Solve these equations to determine I_1, I_2, and I_3 in terms of the emfs and the resistances.

4. The resting potential difference across an axon membrane is −70 mV.

a. What is the charge per square metre for a myelinated and an unmyelinated axon?

b. Next, consider an axon that has roughly the shape of a long cylinder. Its resistivity is approximately 2 Ωm. What then is the resistance of an axon 30 cm long and 10 μm in diameter? Are nerves 'good' conductors?

5. The potential difference across the membrane of an axon is $\Delta V = V_i - V_o = -70$ mV. To have a sense of how the concentrations of the sodium (Na^+) and chlorine (Cl^-) ions differ from their passive equilibrium values, use the Nernst equation to find the expected *relative* concentrations of these ions outside and inside the axon, c_o/c_i, if there were no sodium-potassium pump.

6. A chlorine ion Cl^- moves with a velocity $\vec{v}$ in a uniform magnetic field, $\vec{B} = B\,\hat{x}$. Find the direction of the force on the ion when

a. $\vec{v} = v\,\hat{x}$,

b. $\vec{v} = v\,\hat{y}$,

c. $\vec{v} = -v\,\hat{y}$,

d. $\vec{v} = v\,\hat{z}$, and

e. $\vec{v} = -v\,\hat{z}$.

7. A long wire with a 50.0 A current runs along the $\hat{y}$-axis with the current flowing in the $+\hat{y}$ direction. An electron travels along the $\hat{x}$-axis in the $-\hat{x}$ direction at $1.00\times10^5\ \mathrm{m\,s^{-1}}$. If the magnetic field due to the wire in the right half of the xy plane is

$$\vec{B}(x) = -\frac{2k'I}{x}\,\hat{z}, \qquad k' = 10^{-7}\ \mathrm{N\,A^{-2}},$$

find the

a. force and

b. acceleration

of the electron when it is 0.500 m from the wire.

8. Consider a *magnetic* version of the Millikan oil drop experiment. A drop of oil with 4 extra electrons moves horizontally with a velocity $\vec{v} = v\,\hat{x}$. In the absence of any other forces, it would fall because of the gravitational force, $\vec{F}_g = -mg\,\hat{z}$.

a. If the oil drop is moving through a constant magnetic field $\vec{B} = -B\,\hat{y}$, at what speed should it be moving in order not to fall?

b. Now evaluate your result for the specific case when $B = 2.50$ T, the oil drop has a diameter of 1.10 μm, and the oil has a density of $0.850\ \mathrm{g\,cm^{-3}}$.