Problem Set 5. Gauss's law and currents — Due: 17 February 2023
Electromagnetism and Light — at 3:00 pm

Reading: *Sternheim and Kane*, chapter 17; chapter 18
Electromagnetism and Optics, chapters VI and VII.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

1. A *solid* sphere of radius a is uniformly charged with a total charge $Q > 0$.

a. Use Gauss's law to determine the electric field everywhere.
b. Where is the magnitude of the electric field the largest?
c. What is its value there?
d. Find *two* distances from the centre of the sphere where the electric field has half of its maximum value.
e. If the electric potential infinitely far from the sphere is defined to be zero, find the electric potential at the centre of the sphere.

2. In problem 2 of recitation 5, I claimed that the electric field *inside* an infinitely long uniformly charge cylindrical shell, with a charge per unit length of λ, vanishes and that the electric field *outside* the cylindrical shell is exactly the same as it is for a *line* of charge with the same charge per unit length. Verify both of these statements using Gauss's law.

3. Silver has 5.80×10^{22} free electrons per cubic centimeter. If a 10.0 A current is running through a silver wire that has a diameter of 1.00 cm, what is the drift speed, $\bar{v}$, of the electrons? Would you be able to walk faster than this?

4. Let us compare two wires made from different materials.

a. What is the resistance of a 10.0 m long aluminium wire with a radius of 2.00 mm?
b. What would be the radius of a 10.0 m long wire made of copper that would have the same resistance?
c. What would be the masses of each of these wires? Which would be the heavier of the two? **Note**: the density of aluminium is 2.70 $\mathrm{g\,cm^{-3}}$ and the density of copper is 8.90 $\mathrm{g\,cm^{-3}}$; you can assume that the wires are at 20° C.

5. We use electrical appliances every day, but how much energy do they use?

a. A toaster typically uses 1500 W when plugged into a 120 V line. If it takes 1.00 minute to toast a bit of bread and the electrical energy costs 12 cents per kilowatt-hour, how much did it cost to make the toast?
b. An air conditioner uses 2100 W when plugged into a 230 V line. How much does it cost to run the air conditioner for 12.00 hours each day throughout the month of August?

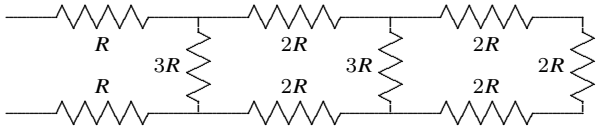
6. The energy that is stored in electrical devices—in batteries or circuits or capacitors—can be used to perform mechanical work. Later in the course we shall see how mechanical work can also be converted into electrical currents. A motor is a familiar device for performing mechanical tasks. Imagine that you have a motor being driven by a 12.0 V battery and that has a effective resistance of $0.200\ \Omega$.

a. What is the current in the motor?

b. How much power is the battery supplying to the motor?

c. If 75.0% of this power can be used to lift a 50.0 N weight, at what constant speed can it lift the weight?

7. Find the equivalent resistance of the following network of resistors.



8. Find the potential difference across each of the resistors in the following circuit.