Problem Set 3. Electric dipoles — Due: 3 February 2023
Electromagnetism and Light — at 3:00 pm

Reading: *Sternheim and Kane*, chapter 16, sections 7 & 9–11;
Electromagnetism and Optics, chapter III.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

1. An ammonia molecule NH_3—just like a water molecule—has a permanent electric dipole moment. Its value is 5.00×10^{-30} C m. If this dipole moment arises from net charges $+3e$ and $-3e$ in the distributions of the positive and negative charges, what separation in meters between these charges would produce this dipole moment?

2. The neutron is electrically neutral, but it is actually composed of electrically charged quarks; so in principle it could have an electric dipole moment, p. The current *bounds* on the dipole moment of the neutron is that $p < 5\times10^{-47}$ C m.

a. If the charge separation were the largest that it could be—the size of the neutron, 10^{-15} m—estimate the largest value of q that is consistent with this bound.

b. Now suppose instead that $q = e$ (quarks actually have charges $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$). What would be the largest possible separation ℓ in the averages of the positive and negative charge distributions in a neutron?

3. How do dipoles interact? For example, do two dipoles like to align? Consider two dipoles whose electric dipole moment vectors are $\vec{p}_1$ (the left dipole) and $\vec{p}_2$ (the right dipole), where $\|\vec{p}_1\| = \|\vec{p}_2\| = p$. You can assume that the distance *between* the dipoles is much, much larger than the separation of the charges *within* each dipole.

(a) Which of the following two configurations has the lower energy? Please explain why.

$\uparrow \quad \uparrow \qquad$ or $\qquad \uparrow \quad \downarrow$

(b) Which of the following two configurations has the lower energy? Please explain why.

$\longrightarrow \quad \longrightarrow \qquad$ or $\qquad \longrightarrow \quad \longleftarrow$

4. To get a sense of the energy scales for real dipoles, imagine two water molecules 3.00×10^{-10} m apart. The dipole moment of water is 6.19×10^{-30} C m.

a. What is the magnitude of the electric field *at* the right dipole *due to* the left dipole for the configuration shown in the first part of problem 3?

b. If the left dipole is fixed, what is the energy difference between the two configurations shown in the first part of problem 3?

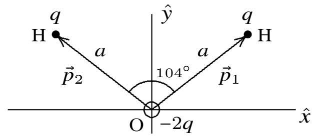
5. How do we know the charge of the electron? Here is a way to find out: imagine that you have made a uniform electric field in a region that points *downwards* (towards the ground), $\vec{E} = -E\,\hat{y}$. Spraying oil through an atomiser produces a cloud of tiny spherical droplets of oil, some of which will have a tiny charge (usually just a few extra electrons) due to friction with the atomiser. You carefully measure the drops and find that they have a diameter of 1.10 μm, and the oil has a density of 0.850 g cm^{-3}.

a. By adjusting the value of the field, E, you discover that some of the droplets can be made to 'levitate'—they are suspended in the air without falling. What is happening?

b. If a particular droplet has four extra electrons, what value should you choose for E to make this droplet 'levitate'?

Note: By measuring the values of E needed to suspend many different droplets, you would find that the charges of the droplets come in multiples of a basic unit of charge e. This is roughly how the physicist Robert Millikan determined the charge of the electron.

6. One way to model a water molecule is as a pair of dipoles that are joined together. Each hydrogen atom has a charge of $+q$ and the oxygen atom has a net charge of $-2q$. The positive charge of each hydrogen atom, together with *half* the charge of the oxygen atom, form a dipole.



a. Write the $\hat{x}$ and $\hat{y}$ components of the two dipoles thus formed, $\vec{p}_1$ and $\vec{p}_2$, based on the coordinates shown in the figure.

b. The total molecular dipole is the sum of these two dipole moment vectors, $\vec{p} = \vec{p}_1 + \vec{p}_2$. What is the direction of $\vec{p}$? If $p = 6.19 \times 10^{-30}$ Cm, and the distance between the hydrogen and oxygen atoms is $a = 9.65 \times 10^{-11}$ m, what is q in multiples of e? What does this result mean?

7. An important lesson from physics is to learn how to understand *whether* an approximation can be safely used or not. To illustrate this idea, we shall look at a dipole made from a charge e at $\vec{r}_+ = \frac{1}{2}a\,\hat{z}$ and a charge $-e$ at $\vec{r}_- = -\frac{1}{2}a\,\hat{z}$. In lecture, I said that *far away* from the dipole, and along its axis, the electric field can be approximated by

$$\vec{E}(y) = \frac{2kp}{z^3}\,\hat{z}.$$

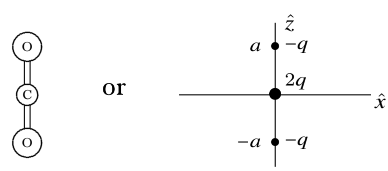
For a HF molecule $a = 39.8$ pm and $p = 6.37 \times 10^{-30}$ Cm.

a. At $z = 50.0$ pm, is this formula a good approximation, *i.e.* calculate $\vec{E}$ using both this approximation and the exact Coulomb expression?

b. At $z = 5.0000$ nm, is this a good approximation?

8. Why do some molecules have permanent dipole moments while others do not? An important factor is the *symmetry* of the molecule. H_2O or NH_3 are clearly lopsided—the hydrogen atoms are only on one side of the oxygen or nitrogen atom.

Let us analyse a different example—carbon dioxide. In a CO_2 molecule, the atoms are arranged in a row. In the figure to the right we have simplified our picture for a CO_2 molecule slightly by modelling it as a charge $2q$ at the origin and two charges $-q$ both at a distance a away along the $\hat{z}$-axis as shown.



a. First write the exact expression for the electric field along the $\hat{z}$-axis for $z > a$ due to these three point charges.

b. Rewrite and simplify your expression for the total electric field by combining all three terms into one fraction over a common denominator.

c. Now imagine that $z \gg a$. Keep only the largest term in this limit in both the numerator and denominator for your fraction from part b. How does this expression scale in z? Does this molecule have a dipole moment?