Problem Set 2. The electric potential
Electromagnetism and Light

Due: 27 January 2023
at 3:00 pm

Reading: *Sternheim and Kane*, chapter 16, sections 3–6;
Electromagnetism and Optics, chapter I, sections 6–8, chapter II.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

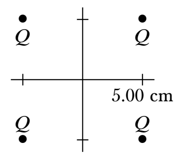
1. Imagine that you have a long, straight line of uniform charge and that you are curious about its charge per unit length. You measure the electric field at a distance of 8.00 cm from the wire and find that it is $1.00 \times 10^5\ \mathrm{N\,C^{-1}}$ and that it points *away from* the wire. What is the charge per unit length on the wire in coulombs per meter?

2. A thin, circular plate of radius 10.0 cm has 10^{11} extra electrons distributed uniformly over its surface. What is the magnitude and direction of the electric field just above the plate near its centre? [Hint: at this location, the field resembles that of an infinite plane of charge with the appropriate surface charge density.]

3. In the Bohr model of the hydrogen atom, an electron 'orbits' the proton at a distance of $a_0 = 5.29 \times 10^{-11}$ m, which is called the *Bohr radius*.

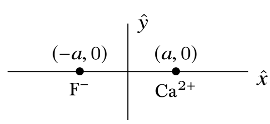
a. What is the electric potential due to the proton alone at this distance? Define the potential to be zero at an infinite distance from the proton.
b. What is the potential energy of the electron, in electron volts, at that distance from the proton?

4. In the first recitation you investigated four equal charges of $Q = 9.00\ \mu$C that were placed on the corners of a square whose sides are 10.0 cm. What is the electric field in the centre of the square? What is the electric potential in the centre? Define the potential so that it vanishes infinitely far from the charges.



5. Suppose that you have arranged a calcium ion Ca^{2+} and a fluoride ion F^- along the $\hat{x}$-axis as shown. If, as usual, we define the electric potential infinitely far away from these charges to be $V(\infty) = 0$, what is the electric potential

a. at the origin $(x, y) = (0, 0)$ and
b. at $(x, y) = (2a, 0)$.
c. Are there any points on the $\hat{x}$-axis where the potential vanishes? In this problem, $2a = 0.2370$ nm (the spacing in a fluorite crystal). [Hint: remember that the potential for a point charge depends on the **distance** from it, and that distances are always **positive**.]



6. Consider a uniform line of charge of length L and total charge Q. In lecture we calculated the electric field in a plane equidistant from the two ends of the line. In this problem we shall instead calculate the electric field at a point above one of the ends of the line. Imagine that the line coincides with the $\hat{z}$-axis, running from the origin to the point $z = L$. Evaluate the electric field at the point $\vec{r} = x\,\hat{x}$ on the $\hat{x}$-axis.

7. **A loop of charge** **[10 points]**

In lecture, I showed you how to find the electric field for a long straight line of charge, but another situation that we can solve is the electric field of a loop. Why would we want to consider a loop of charge? In some molecules, such as benzene C_6H_6, some of the electrons are shared amongst all the carbon atoms, so that the orbitals combine into rings above and below the plane of the carbon atoms. Here we shall look at just a single loop of charge, to keep the problem simple.

Let us start with a thin loop of charge, of total charge Q and radius a. Just so that everyone is using the same coordinates, let the loop be in the xy plane with its centre at the origin, as shown below, and let us only calculate the electric field along the (positive) $\hat{z}$-axis. To solve this problem, we chop the circle into tiny segments, each of length $a\,d\theta'$, where θ' is the angle around the loop.

a. What is the charge per unit length of this loop, λ?

b. Before calculating anything in detail, explain why the electric field at $z > 0$ and $(x, y) = (0, 0)$ should always point exactly in the $\hat{z}$ direction.

c. What is the $\hat{z}$-component of the electric field due to a tiny segment of the loop at this point?

d. Now add up (integrate) the contributions from all of the segments of the loop. What is the electric field on the $\hat{z}$-axis?