Problem Set 11. Waves and optics — Due: 28 April 2023
Electromagnetism and Light — at 3:00 pm

Reading: *Sternheim and Kane*, chapter 24, sections 3–5;
Lecture Notes, chapter XIX, sections 3–6.

Please show all of the necessary steps in solving the following problems. Full credit will only be given for complete solutions.

1. The lowest frequency G string of a violin is 33.0 cm long and its fundamental frequency is 196 Hz.

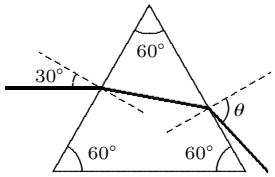
a. What is the wave speed of the string?
b. The A string, which is also 33.0 cm long, has a fundamental frequency of 440 Hz. How far from the end of the string should one press this string to obtain the same fundamental frequency as the E string, 659 Hz?

2. Imagine that you would like to rotate the plane of polarisation of some light by $90°$. The light starts by being linearly polarised in the vertical direction with an intensity of $100\ \mathrm{W\,m^{-2}}$.

a. What is the intensity of the light if it first passes through a filter with an axis at $45°$ to the vertical and then a second with an axis at $90°$ to the vertical?
b. What is the intensity of the light if it passes through three filters whose axes are successively at $30°$, $60°$, and then $90°$ to the vertical?
c. What is the intensity of the light if it passes through nine filters whose axes are each rotated $10°$ further from the vertical than the previous one?

3. A ray of light enters a prism from its left side, as shown below, at an angle of $30°$ to the normal.

a. If the index of refraction is n, at what angle θ does the ray leave the prism on its other side? That is, determine an expression for θ in terms of n.



For a prism made from crown glass, the index of refraction is 1.517 for red light (656 nm), 1.520 for yellow light (589 nm), and 1.527 for blue light (484 nm).

b. At what angles θ do the different colours leave the prism?
c. What is the difference between the angles for the red and blue light?

4. A diffraction grating with 5,000 lines per centimetre is illuminated with yellow light that has a wavelength of 589 nm.

a. What is the angular position of the $n = 1$ line?
b. How many yellow lines can be seen in total?

5. Suppose that you are h metres tall. What is the minimum height that a mirror should be for you to be able to see your entire body at once, standing parallel to the mirror?

6. A lens is made from a plastic with an index of refraction $n = 1.50$. One side is convex with a radius of curvature of 10.0 cm. Find the radius of curvature of the other side of the lens, and make a sketch of the lens, if

a. $f = 15.0$ cm,

b. $f = 10.0$ cm, and

c. $f = -15.0$ cm.

7. Suppose that you are giving a presentation using a projector, where its lens is 3.00 m from a screen and it has a focal length of 8.00 cm.

a. Where is the 'object' (a bright, internal screen) located when the projector is in focus?

b. If a letter has a height of 10.0 cm on the external screen, what is its height on the screen inside the projector?

c. Someone in the audience complains that the text on your image is too small. Where should you put the projector to double the size of the images on the screen?

d. ernatively, if you did not wish to move the projector, what focal length lens would you need to use to double the size of the image on the screen?

8. A person with severe myopia (nearsightedness) is often advised to choose lenses made from a high-index plastic ($n = 1.74$) rather than from ordinary polycarbonate plastic ($n = 1.586$) when ordering glasses. Suppose that the power of the lenses is -15.0 diopters, and that the lenses have a radius 2.00 cm (they are circular) and are flat on one side. How much thinner would the high-index lenses be at their edge (in millimetres) than the ordinary lenses?