Technical Appendix to "Has Moral Hazard Become a More Important Factor in Managerial Compensation?"

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I. Identification

The identification of this model is an application of Gayle and Miller (2008). For the reasons given in the text, we proceed as if true compensation, w_t, and excess returns, x_t, are observed for the purposes of establishing identification of the other parameters. Identification of the remaining parameters, namely the risk-aversion parameter (ρ), tastes for shirking over diligence (α_2/α_1), tastes for diligence over the value of quitting (α_2/α_0), and the signalling function ($g(x)$) proceeds in two steps. First, we prove (α_2/α_1), (α_2/α_0), and $g(x)$ are identified if ρ is known. Then we give sufficient conditions for identifying ρ.

Defining $v_t(x, \rho)$ as

$$v_t(x, \rho) \equiv \left(\frac{\alpha_0}{\alpha_2}\right)^{1/(b_t-1)} \exp\left[\frac{\rho w_t(x)}{b_{t+1}}\right], \tag{1}$$

it follows from the optimal contract for diligent work,

$$w_t = \frac{b_{t+1}}{\rho(b_t - 1)} \ln\left(\frac{\alpha_2}{\alpha_0}\right) + \frac{b_{t+1}}{\rho} \ln\left[1 + \eta_t \left(\frac{\alpha_2}{\alpha_1}\right)^{1/(b_t-1)} - \eta_t g(x_t)\right], \tag{2}$$

that for a given value for ρ, a transformation of the optimal compensation, depending only on (observed) bond prices, is a linear mapping of $g(x)$. Namely,

$$v_t(x,\rho) = 1 + \eta_t \left[(\alpha_2/\alpha_1)^{1/(b_t-1)} - g(x)\right]. \tag{3}$$

So, if the values of the intercept and the slope of the mapping could be found, and the value of ρ were known, then $g(x)$ could be simply determined. Taking the expectation with respect to x conditional on the price of bonds at time t yields

$$E\left[v_t(x,\rho)\right] = 1 + \eta_t(\alpha_2/\alpha_1)^{1/(b_t-1)} - \eta_t \equiv \underline{v}_t(\rho). \tag{4}$$

We now impose a regularity condition on $g(x)$, satisfied by our parameterization, that says $g(x) \to 0$ as $x \to \infty$. Intuitively this condition states that the shareholders attach negligible probability to a manager shirking if the firm's excess returns are extraordinarily high. The condition implies

$$\lim_{x\to\infty} v_t(x,\rho) = 1 + \eta_t(\alpha_2/\alpha_1)^{1/(b_t-1)} \equiv \overline{v}_t(\rho). \tag{5}$$

Solving for the signaling function $g(x)$, the nonpecuniary benefit ratio (α_2/α_1), and the tastes for participation (α_2/α_0) given ρ, using equations (3), (4), and (5), proves the following.

Proposition 1. *For any* $\rho^* > 0$,

$$\begin{aligned}
\frac{\alpha_2^*}{\alpha_0^*} &= \left[E_t\left\{\exp\left[-\frac{\rho^* w_t(x)}{b_{t+1}}\right]\right\}\right]^{1-b_t} \\
\frac{\alpha_2^*}{\alpha_1^*} &= \left(\frac{\overline{v}_t(\rho^*) - 1}{\overline{v}_t(\rho^*) - \underline{v}_t(\rho^*)}\right)^{b_t-1} \\
g^*(x) &= \frac{\overline{v}_t(\rho^*) - v_t(x,\rho^*)}{\overline{v}_t(\rho^*) - \underline{v}_t(\rho^*)}.
\end{aligned}$$

Proof of Proposition 1. The expression for α_2^*/α_0^* follows directly from rearranging the participation constraint (7). Subtracting equation (5) from (3), we obtain

$$\eta_t g(x) = \overline{v}_t(\rho^*) - v_t(x, \rho^*).$$

Subtracting equation (4) from (5) we obtain

$$\eta_t = \overline{v}_t(\rho^*) - \underline{v}_t(\rho^*). \tag{6}$$

Substituting for η_t using (6) in the previous equation and making $g(x)$ the subject of the resulting equation yields:

$$g(x) = \frac{\overline{v}_t(\rho^*) - v_t(x, \rho^*)}{\overline{v}_t(\rho^*) - \underline{v}_t(\rho^*)}.$$

Finally, making (α_2/α_1) the subject of equation (4) and then substituting for η_t using (6), we obtain

$$\frac{\alpha_2^*}{\alpha_1^*} = \left(\frac{\overline{v}_t(\rho^*) - 1}{\eta_t}\right)^{b_t - 1} = \left(\frac{\overline{v}_t(\rho^*) - 1}{\overline{v}_t(\rho^*) - \underline{v}_t(\rho^*)}\right)^{b_t - 1},$$

as required.

Proposition 1 establishes that if ρ^* is known then (α_2^*/α_1^*), (α_2^*/α_0^*), and $g^*(x)$ are identified since they can be written as a mapping of the data, because consistent estimates of the mappings $\overline{v}_t(\rho)$ and $\underline{v}_t(\rho)$ can be obtained from the data. See Gayle and Miller (2008) for details on constructing nonparametric consistent estimates of these quantities. A natural place to begin investigating the identification of ρ^* is the participation constraint. When $(\alpha_2/\alpha_0) > 1$, meaning the nonpecuniary benefits of working do not fully compensate the manager for the total benefits of his alternative, and thus expected compensation is positive, the data imply a lower bound for the risk-aversion parameter, ρ. To picture this, define the mapping

$$\psi_t(\rho) \equiv E_t\left[\exp\left(-\frac{\rho w_t(x)}{b_{t+1}}\right)\right].$$

From its definition, $\psi_t(0) = 1$, while the assumption above implies

$$\psi_t'(0) = \frac{\partial}{\partial\rho} E\left[\exp\left(-\frac{\rho w_t(x)}{b_{t+1}}\right)\right]_{\rho=0} = -E\left[\frac{w_t(x)}{b_{t+1}}\right] < 0.$$

Also $\psi_t(\rho)$ is convex in ρ because

$$\frac{\partial^2}{\partial\rho^2}\left[\exp\left(-\frac{\rho w_t(x)}{b_{t+1}}\right)\right] = \left(\frac{w_t(x)}{b_{t+1}}\right)^2 \exp\left(-\frac{\rho w_t(x)}{b_{t+1}}\right) > 0$$

and the expectations operator preserves convexity. Assuming $\alpha_2/\alpha_0 > 1$, it now follows that $\psi_t(\rho)$ crosses the unit level from below just once at say ρ_t, which implies $\psi_t(\rho) > 1$ for all $\rho > \rho_t$. This rules out the possibility that $\rho^* \leq \rho_t$. Intuitively, the participation equation is satisfied by different combinations of ρ and α_2/α_0 satisfying $\rho > \rho_t$ and $\alpha_2/\alpha_0 = \psi_t(\rho)^{1-b_t}$ as we see in Figure 1.

Along this line, as ρ increases, the person becomes more risk averse, the expected utility from $w_t(x)$ declines along with its certainty equivalent, but this is just offset by nonpecuniary amenities from the job. Consequently an observer with cross-sectional data on a homogeneous set of firms and managerial compensation paid out in that period cannot distinguish between a sample of managers with a high risk tolerance and unpleasant working conditions, versus a sample with lower tolerance but more nonpecuniary benefits. The remaining parameters are then inferred from the value ascribed to ρ, the slope of the contract with respect to abnormal returns determining $g(x)$ and thence the probability distribution of abnormal returns under shirking.

Accordingly, we now suppose there are data on at least two states $s \in S$, that is dates with distinct bond prices, or sectors where the nonpecuniary benefits of the job and the alternative opportunities for work are the same. More formally, the two states have different compensation plans $w_r(x)$ and $w_s(x)$ but the same nonpecuniary benefits from diligent work α_2. In this case $w_r(x) \neq w_s(x)$ because the probability density function of abnormal returns from working diligently differs by state, that is $f_{2r}(x) \neq f_{2s}(x)$, or the density from shirking differs, that is $f_{1r}(x) \neq f_{1s}(x)$.

The existence of multiple states provides a means of identifying ρ. Since the participation condition holds for each state $s \in S$ separately, we can in principle solve moment conditions of the form

$$\left[\int \exp\left[-\rho b_{r+1}^{-1} w_r(x)\right] f_{2r}(x)\mathrm{d}x\right]^{\varkappa(r)} = \left[\int \exp\left[-\rho b_{s+1}^{-1} w_r(x)\right] f_{2s}(x)\mathrm{d}x\right]^{\varkappa(s)}$$

in ρ, where $\varkappa(r) = \varkappa(s) = 1$ when they are sectors and $\varkappa(r) = 1 - b_r$ and $\varkappa(s) = 1 - b_s$ when they are dates. Figure 1 illustrates how identification would be achieved with two states, ρ^* determined by a unique intersection of $\psi_s(\rho)$ with $\psi_t(\rho)$. Although there may be multiple roots in ρ to the equations defined by the separate states $r \in S$ and $s \in S$, if there is a unique root common to all possible pairs, then ρ is identified.

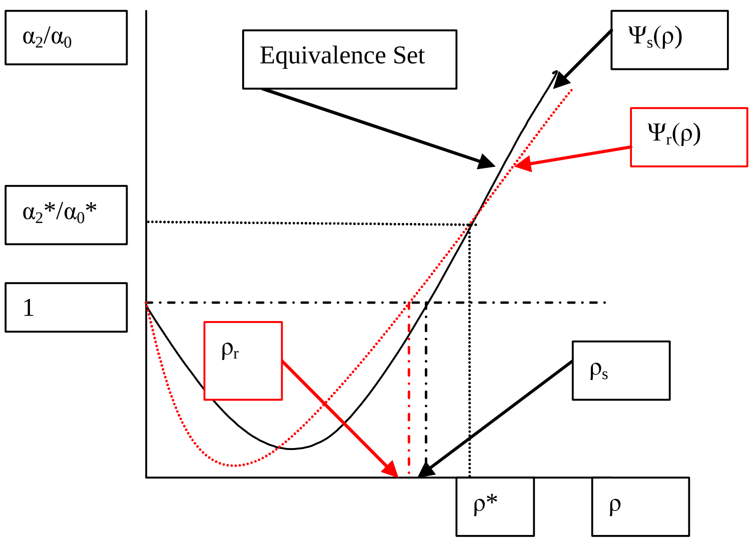

Figure 1: Equivalence Set

REFERENCES

Gayle, George-Levi, and Robert A. Miller. 2008. "Identifying and Testing Generalized Moral Hazard Models of Managerial Compensation." Tepper school of Business, Carnegie Mellon University.