Nearest Neighbor and Kernel Survival Analysis

Nonasymptotic Error Bounds and Strong Consistency Rates

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Survival Analysis

	Gluten allergy	Immuno-suppressant	Low resting heart rate	Irregular heart beat	High BMI	Time of death
	✗	✗	✓	✗	✗	Day 2
	✓	✓	✗	✗	✓	Day 10
	✗	✗	✗	✓	✗	Day $\geq$ 6

Feature vector X

Observed time Y

When we stop collecting training data, not everyone has died!

Goal: Estimate $S(t|x) = \mathbb{P}(\text{survive beyond time } t \mid \text{feature vector } x)$

Problem Setup

Model: Generate data point (X, Y, δ) as follows:

1. Sample feature vector $X \sim \mathbb{P}_X$
2. Sample time of death $T \sim \mathbb{P}_{T|X}$
3. Sample time of censoring $C \sim \mathbb{P}_{C|X}$
4. If death happens before censoring ($T \le C$): Set $Y = T, \delta = 1$

   Otherwise: Set $Y = C, \delta = 0$

Borel prob. measure

Continuous r.v. in **time** & smooth w.r.t. **feature space** (Hölder index α)

Estimator (Beran 1981):

Feature space is separable metric space (intrinsic dimension d)

$x \to$ find k training points closest to x $\to$ k data points $\to$ Kaplan-Meier estimator $\to$ $\widehat{S}(t \mid x)$

Kernel variant is similar

Error: $\sup_{t \in [0,\tau]} |\widehat{S}(t|x) - S(t|x)|$ for time horizon τ

Enough of the n training data have Y values $> \tau$

Theory (Informal)

k-NN estimator with $k = \widetilde{\Theta}(n^{2\alpha/(2\alpha+d)})$ has strong consistency rate:

$$\sup_{t \in [0,\tau]} |\widehat{S}(t|x) - S(t|x)| \leq \widetilde{O}(n^{-\alpha/(2\alpha+d)})$$

If no censoring, problem reduces to conditional CDF estimation

→ Error upper bound, up to a log factor, matches conditional CDF estimation lower bound by Chagny & Roche 2014

Proof ideas also give finite sample rates for:

- Kernel Kaplan-Meier estimators
- *k*-NN & kernel Nelson-Aalen *cumulative hazard* estimators $(-\log S(t \mid x))$
- Generalization bound for automatic *k* using validation data

Most general finite sample theory for *k*-NN and kernel survival estimators

Existing kernel results only for Euclidean space
(Dabrowska 1989, Van Keilegom & Veraverbeke 1996, Van Keilegom 1998)

Experiments