Chapter 10
Counterexamples to Commonly Held Assumptions on Unit Commitment and Market Power Assessment

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10.1 Centralized Versus Decentralized Unit Commitment (UC)

This first subsection disproves the commonly held assumption that, in theory and under the condition of perfect information, decentralized and centralized UC would lead to the same power quantities traded and, hence, to the same optimal social welfare [1]. We see that, even in the absence of any uncertainties, independent optimization of the individual performance objectives by the decentralized market participants can lead to lower efficiency than centralized minimization of total operating cost.[1]

10.1.1 The Standard Argument: Centralized UC is identical to Decentralized UC

Mathematically, a *centralized economic dispatch* is the problem of minimizing the total generation cost, using the quantities produced by each of the possible generators as decision variables such that total generation equals total demand Q_D [1]. Using the variables Q_i and C_i for the quantities produced and the cost incurred

[1] This result concerns short-term supply optimization for a given demand and does not consider long-term investment issues.

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M. Ilić et al. (eds.), *Engineering IT-Enabled Sustainable Electricity Services*, Power Electronics and Power Systems 30, DOI 10.1007/978-0-387-09736-7_10,

by generator i, respectively, and the variable n for the total number of available generators, we can write the problem as

$$\min_{\mathbf{Q}} \sum_{i=1}^{n} C_i(Q_i) \quad \text{s.t.} \quad \sum_{i=1}^{n} Q_i = Q_D.$$

This basic version of an unconstrained economic dispatch remains indeterminate. An economically motivated condition for solving this problem is the equal incremental condition

$$\frac{\delta C_1}{\delta Q_1} = \ldots = \frac{\delta C_n}{\delta Q_n} = \lambda.$$

The term λ is known as the short-run marginal cost (SRMC) and, at the optimum, all unit marginal costs are equal to it. Using u_i as a binary variable that determines whether the generation unit i is turned on or off at a given moment, the basic *centralized unit commitment problem* without start-up costs or minimum up/down time constraints is

$$\min_{\mathbf{u},\mathbf{Q}} \sum_{i=1}^{n} u_i C_i(Q_i) \quad \text{s.t.} \quad \sum_{i=1}^{n} Q_i = Q_D.$$

Following the Lagrangian relaxation method, we first form the Lagrangian function

$$L(\mathbf{u},\mathbf{Q},\lambda) = \sum_{i=1}^{n} u_i(C_i(Q_i) - \lambda Q_i) + \lambda Q_D.$$

By minimizing this last equation over $\mathbf{Q}$, we obtain the conventional economic dispatch equal incremental condition from above which permits us to solve for Q in terms of λ, the system incremental cost. Rewriting the Lagrangian as

$$L(\mathbf{u},\lambda) = \sum_{i=1}^{n} u_i(C_i(Q_i(\lambda)) - \lambda Q_i(\lambda)) + \lambda Q_D$$

and using the Lagrangian method to minimize $L(\mathbf{u},\lambda)$ with respect to $\mathbf{u}$ gives us the switching curve law or the average cost rule

$$u_i = \begin{cases} 1 & \text{if } C_i - \lambda Q_i < 0 \\ 0 & \text{if } C_i - \lambda Q_i > 0, \end{cases}$$

that is, the unit i is turned on if the average cost $\frac{C_i}{Q_i} < \lambda$ and off otherwise. Once on, a conventional economic dispatch is used to adjust to demand changes if these are monitored more frequently [1].

With competitive bilateral transactions taking place in a *decentralized economic dispatch* and each party's objective being the maximization of its individual profit, the decentralized problem is

$$\max_{Q_i} \pi_i(Q_i).$$

Here $\pi_i(Q_i) = PQ_i - C_i(Q_i)$ stands for the profit made by the market participant i through some sort of trading process, given price P. Thus, under perfect conditions, when the market converges to a single electricity price, one can maximize π_i by setting marginal cost equal to price:

$$\frac{\delta C_1}{\delta Q_1} = \ldots = \frac{\delta C_n}{\delta Q_n} = P.$$

The process of bilateral decisions will stabilize P at the system-wide economic equilibrium under a perfect information exchange among all market participants. This result is simply obtained by each market participant optimizing its own profit for the assumed exogenous market price P [1]. In the *decentralized unit commitment* setting, all generator owners are assumed to be price takers in a competitive market place. Each participant makes a unit commitment decision typically for each hour one day ahead, before knowing the actual spot price. After the spot price of a respective hour is known, the generator decides how much power to sell in order to maximize profit. The only control for generator i is u_i, whether to turn on or off at a given hour. The expected generation level $\hat{Q}_i$ may be regarded as a function of the control u_i and the expected price $\hat{P}$. In the case of deterministic prices and ignored start-up costs and must-run time constraints, a generator's profit while on is $\hat{\pi}_{i,\text{on}} = \hat{P}\hat{Q}_i - C_i(\hat{Q}_i)$. The generator will turn on only if $\hat{\pi}_{i,\text{on}} > 0$, which is equivalent to $\frac{C_i(\hat{Q}_i)}{\hat{Q}_i} < \hat{P}$, which is exactly the average cost rule used for coordinated unit commitment [1].

Based on this derivation, current teaching is that, under perfect market assumptions and when neglecting start-up costs and intertemporal time constraints, individual power producers would schedule the same power units in a decentralized market as would a central system operator in a coordinated market. *Thus, both centralized and decentralized UC should lead to the same power quantities traded, the same minimum operating cost, and, with given inelastic demand, to the same total social welfare optimum.* The performance objectives of the individual market participants (to maximize profits) and the objective of a centralized entity (to maximize social welfare by minimizing operating cost) would then be equivalent [1].

10.1.2 The Counterexample: The "Tragedy" of Decentralized UC

We next prove that centralized and decentralized UC are, in general, not economically equivalent. Under certain conditions, some generators would not self-schedule to prevent loss; those units, however, would be scheduled by a PoolCo-type market (or a "social planer") to minimize overall operating cost and, in turn, would receive fixed operating costs to prevent them from loss. These situations are missed by the previous argument, as *the average cost rule does not always lead to the total social welfare optimum that a centralized operator strives for.*

To prove this claim by example, we derive the strict conditions under which this situation takes place in the case of two generators with quadratic operating cost $C_i(Q_i) = a_i Q_i^2 + b_i Q_i + c_i$. We consider only one particular hour for which demand Q_D is given. In the case of inelastic demand, the total social welfare optimum is attained by minimizing total operating cost. Without loss of generality, we consider generator 2 to be the one that would not self-schedule in order to avoid loss when bidding marginal costs. Three conditions must hold simultaneously in order to produce the specific situation:

1. Generator 1 makes profit, independent of generator 2 participating during the hour or not: $\frac{C_1(Q_1)}{Q_1} < P$.
2. Generator 2 incurs loss if it is scheduled and receives no extra payment: $\frac{C_2(Q_2)}{Q_2} > P$.
3. The total cost for satisfying the given load is smaller if both generators operate instead of only generator 1: $C_1(Q_1) + C_2(Q_2) < C_1(Q_D)$.

Performing some mathematical operations that are described in more detail in [2] and [3], we obtain the following *three conditions on the demand which, if fulfilled, lead to different units scheduled and, hence, different economic outcomes from either centralized or decentralized UC:*

$$Q_D > Q_{\min \pi_1} := \max \left[\frac{(a_1 + a_2)\sqrt{\frac{c_1}{a_1}} + \frac{b_1 - b_2}{2}}{a_2}, \sqrt{\frac{c_1}{a_1}} \right],$$

$$Q_D < Q_{\max \pi_2} := \frac{(a_1 + a_2)\sqrt{\frac{c_2}{a_2}} + \frac{b_2 - b_1}{2}}{a_1},$$

$$Q_D > Q_{\min C} := \frac{\sqrt{(a_1 + a_2)c_2} + \frac{b_2 - b_1}{2}}{a_1}.$$

Table 10.1 and Fig. 10.1 provide an illustrative example with numbers and graphs. Parameters for generator 2 stem from a best quadratic fit to heat rate data

Table 10.1 Example generator parameters (**a**), resulting demand conditions (**b**), and higher prices and total operating cost with decentralized instead of centralized UC for example demand of 800 MW (**c**)

	G_1	G_2
a [\$/MW^2/h]	0.0092	0.0026
b [\$/MWh]	9.3	17
c [\$/h]	1200	390

(a)

$Q_{\min \pi_1}$ [MW]	361
$Q_{\max \pi_2}$ [MW]	915
$Q_{\min C}$ [MW]	652

(b)

	D.UC	CUC		
P [\$]	**24.02**		**18.55**	
	G_1	G_1+G_2	G_1	G_2
Q_i [MW]	800	800	503	297
C_i [\$/h]	**14,528**	**13,874**	8,197	5,677
π_i [\$/h]	4,688	963	1,123	-160

(c)

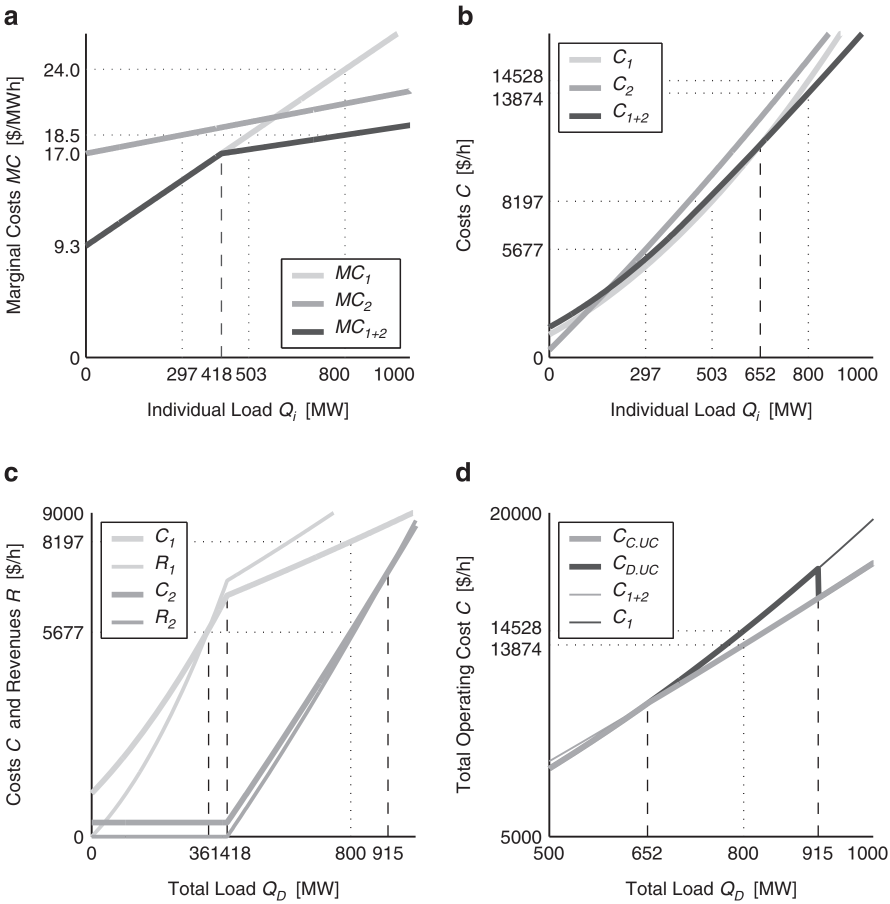

Fig. 10.1 Situation with centralized UC leading to lower operating cost than decentralized UC: Market supply functions (**a**), generator operating cost function (**b**), individual operating costs and revenues following marginal cost bids (**c**), and total operating cost (**d**)

of the thermal plant “Morro Bay 4” published in [4] and an assumed fuel price of 2 $/MBtu. The parameters were slightly changed for generator 1 in order to create a sample situation.

10.1.3 Discussion

The literature gives several examples of cases in which individual objective functions are not aligned with those of the overall social welfare. The most often cited example was given by Hardin in *The Tragedy of the Commons* [5]. Another one is Braess’ article on traffic networks [6] in which he gives an example in which drivers’ attempt to minimize their transit times leads to increased congestion and increased traffic times for all participants. Braess’ paradox has become an important issue in the context of queuing networks [7]. In power systems, however, the commonly held assumption is still that, at least in theory, a centralized and a decentralized UC should lead to the same power quantities traded and to the same optimal social welfare. The performance objectives of the individual market participants are considered equal to the one of minimizing total operating cost [1, 8, 9].

The important implication of the example given before is that, even in the absence of load uncertainties and intertemporal constraints, *decentralized UC does not necessarily lead to the same maximized welfare as centralized decision making.* The reason is that, under certain circumstances, several generators can supply the load at a lower overall cost than the subset of generators that would make positive profits in a market setting if switched on during the hour.

In the Pennsylvania–New Jersey–Maryland and the New York electricity markets, the ISO (Independent System Operator) offers a voluntary unit commitment service, based on three-part bids, allowing generators to bid actual operating costs more precisely and permitting a more efficient unit commitment. Generators may also self-schedule their own units, but they may also allow the ISO to determine the most economic unit commitment for their plants. Participating generators are guaranteed recovery of their start-up and minimum generation costs in the event they fail to recover these costs from the prices received in the ISO-coordinated markets [10, 11]. This mechanism eliminates the uncertainty of whether a generator will be committed only to lose money, and it allows for a more efficient dispatch. The quadratic cost curve example shows how a PoolCo-type market would work more efficiently than a power exchange (for which the one-part bids result in some inefficiency).

It is important to note that the conclusions here focus on the short run, in that they do not take into account the long-term motivational effects of a decentralized commitment on investment decisions and the possible entry of new firms or generating plants. The literature gives several qualitative arguments why a decentralized commitment process might be preferable despite the better overall efficiency of the centralized process [11, 12].

10.2 Marginal Cost Bidding and Market Power

This second subsection illustrates that a generator owner's optimum bid sequence for a centralized wholesale market under a decentralized UC regime is generally above marginal cost even in the complete absence of market power. This result challenges economic literature stating that market prices above marginal cost would unambiguously indicate gaming and the abuse of market power.

10.2.1 An Illustrative Model

We will use a simple model to show how prices above marginal cost arise in a decentralized UC scheme as a natural consequence of the decentralized decision process and intertemporal constraints. We deploy the dynamic programming formulation from [13] for calculating a generator's optimal bidding strategy in the presence of a price forecast with given standard variation. For simplification, we consider a generator whose marginal operating cost is constant over the output range: $\mathrm{MC} = b$. The owner can offer electricity by submitting a bid to a centralized market for each hour and is scheduled if the bid price turns out to be lower than or equal to the market price. We neglect the case of the generator being the marginal unit and scheduled for less than full output. Because of the constant marginal cost, the most efficient way to operate the generator is to either produce full output Q_G or nothing and to use a flat bid curve. In addition to variable costs, the generator incurs fixed operating cost c for every hour of operation regardless of whether it is producing electricity or not and also start-up cost c_u and shutdown cost c_d. As intertemporal constraint, once the generator is switched on, it has to remain in that state for at least 2 hours, during which it incurs the fixed operating cost. If the generator gets scheduled for 1 hour but not for the other, it still incurs the fixed operating cost c for the second hour as well. Hence, the generator has to internalize these intricacies when it is bidding into an hourly market. The generator does not know the market prices when bidding but has some knowledge about the probability distribution of the prices, which are considered to be exogenous variables, not influenced by the behavior of the generator.

We now consider the specific situation in which only 2 successive hours have price distributions above MC (Fig. 10.2a). The problem of finding the optimal bids is drastically simplified and can be solved in a closed form. In this special example, the sum of the fixed costs can be united into one term total incurred fixed cost $c_\mathrm{tot} = c_\mathrm{u} + c_\mathrm{d} + 2c$ which will be incurred once the generator starts up. This aggregation does not change the optimal strategy but simplifies the formulation. Fixed nonoperating costs, such as sunk capital costs, which are incurred regardless of the generator producing output or not during 1 h, do not affect the optimal decision and can be disregarded. For the numerical calculation, we assume that prices can have only a

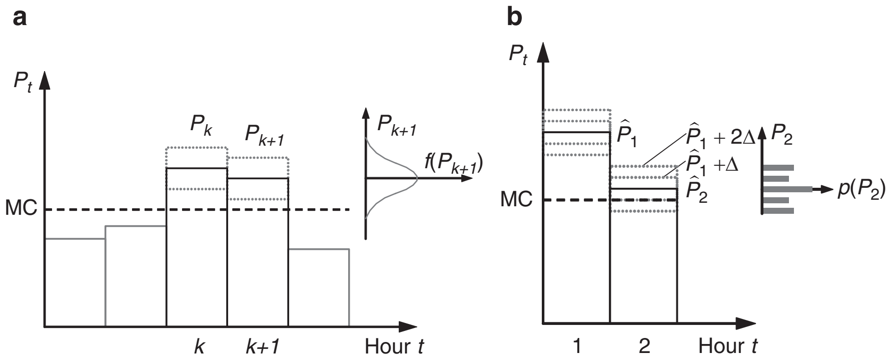

Fig. 10.2 Marginal cost and hourly predicted prices for the next day (**a**) and assumed discrete price distribution of two relevant hours (**b**)

limited number of discrete values during the two hours and are uncorrelated: $P_k \in \{\widehat{P}_k - 2\Delta, \widehat{P}_k - \Delta, \widehat{P}_k, \widehat{P}_k + \Delta, \widehat{P}_k + 2\Delta\}$ (Fig. 10.2b).

10.2.2 Profit Optimization in a Competitive Market

In order to find the optimal bidding sequence, the profits for all possible combinations of bid heights have to be compared:

$$J = \max_{x_1, x_2} [J(x_1, x_2)]$$

with $\{(x_1, x_2) \,|\, (x_1, x_2) = (P_i, P_j)\}$ and (P_i, P_j) being possible prices for the 2 hours. In order to calculate the expected profit for the bid combinations, all possible price outcomes have to be compared:

$$\begin{aligned} J(x_1, x_2) = & \sum_{P_i | P_i \geq x_1} \sum_{P_j | P_j \geq x_2} \mathrm{p}(P_1 = P_i)\, \mathrm{p}(P_2 = P_j)\, ((P_i + P_j - 2b)\, Q_G - c) \\ & + \sum_{P_i | P_i \geq x_1} \sum_{P_j | P_j < x_2} \mathrm{p}(P_1 = P_i)\, \mathrm{p}(P_2 = P_j)\, ((P_i - b)\, Q_G - c) \\ & + \sum_{P_i | P_i < x_1} \sum_{P_j | P_j \geq x_2} \mathrm{p}(P_1 = P_i)\, \mathrm{p}(P_2 = P_j)\, ((P_j - b)\, Q_G - c)\,. \end{aligned}$$

Whereas finding the optimal bid sequence in our example is still possible, this task becomes increasingly intractable when optimizing for more periods. The time for calculation increases exponentially with the number of periods.

Table 10.2 shows an illustrative example. The optimum bids are not only higher than the marginal cost but also higher than the average operating cost. In addition,

Table 10.2 Example of optimal bids being above MC: Prices (a), generator cost (b), expected profits for different bid sequences (c).

Price distributions	
$\widehat{P}_1$	60
$\widehat{P}_2$	50
Δ	2
$\mathrm{p}(P_k = \widehat{P}_k \pm 2\Delta)$	0.19
$\mathrm{p}(P_k = \widehat{P}_k \pm \Delta)$	0.16
$\mathrm{p}(P_k = \widehat{P}_k)$	0.30

(a)

Generator	
MC	**50**
Q_G	1
c	4
c_u	1
c_d	1
c_{tot}	10

(b)

Bid Sequence	Exp. Profit
(**58**,**52**) or (**60**,**54**)	1.172
(58,54)	1.154
(56,50) or (56,52)	1.080
(60,56) or (62,56)	
(60,52)	0.927

(c)

we see that the optimum bids vary between different hours and are dependent on the assumed forecast prices and related price dynamics.

10.2.3 Discussion

Many of the recent papers on assumed market power abuse in deregulated electricity markets assume that market participants bid their true marginal costs in a competitive market if no market power is exerted. However, in the context of bidding decisions of power plants, which not only incur MC but also start-up and shutdown costs and minimum commitment constraints, these assumptions are not valid. Generators bid higher than MC, not because they can exercise market power, but because of intertemporal constraints and uncertainties about prices of consecutive hours.

The literature disagrees as to what exactly constitutes market power but generally agrees that it has to do with actively raising the prices at which one is willing to sell output (one's price offer) above MC in order to change the market price [14] ("If suppliers exercise market power, prices could be higher than marginal costs."). MC include both the variable costs due to fuel and the other variable operating and maintenance costs. For example, [15] states that "The fundamental measure of market power is the margin between price and the marginal cost of the highest cost unit necessary to meet demand. (. . .) if no firm were exercising market power, then all units with marginal cost below the market price would be operating."

In the formulation of this chapter, the *power producer is modeled as a price taker*. He has assumptions about the probability distributions of prices for certain hours. His bidding decision does not affect the prices and, hence he has no market power. Nevertheless, his optimum bids deviate from MC. *It is, therefore, not market power that creates prices above MC but the necessity to incorporate start-up and shutdown constraints in the presence of uncertain prices*. The generator in the example responds to the simple economic incentive of maximizing profits given uncertain prices. As a result, the competitive price does not equal marginal cost at peak periods under competition, and therefore simple price-cost margin studies cannot confirm the exercise of market power. We thus conclude that above MC bids of generators do not necessarily indicate the exercise of market power. Especially in times when prices are very volatile, generators have to bid above marginal costs in order to take account of the possibility of being scheduled for one hour and not the following one.

10.3 Summary and Take-Aways

By adopting the perspective of an individual market participant in a decentralized UC regime and simulating economically optimal behavior, we could draw the following two conclusions:

1. *Coordination Value*: Decentralized UC does not lead to the same short-time efficient economic outcome as centralized UC, even in the theoretical case of perfect information. The non-obvious reason is that, under certain load and cost conditions, the overall minimum operating cost to supply a given demand could be achieved with some generators switched on which would not operate in a completely decentralized market. The consequence of this observation does not limit itself to electricity markets only, but can be considered a more general situation where the performance objectives of the individual market participants are not equal to the one of minimizing total production cost, quite analogous to the *Tragedy of the Commons*.
2. *Market Power*: Above marginal costs in electricity markets are a necessary consequence of the decentralized decision making under uncertainty and do not necessarily indicate gaming in the presence of market power. The reason is that generators face intertemporal time constraints not common in other industries. As a consequence, and in contrast to economic teaching, price taking generators have to bid above marginal cost in order to internalize the uncertainties of being on but not selling any power into the market into their bids.

The results of this chapter were published in [2]. More details and an extended optimization algorithm that also considers intertemporal correlation between price forecast errors can be found in [3].

Acknowledgments This work was partially supported by a Rotary Ambassadorial Scholarship, a fellowship from the MIT graduate student office, a DOC scholarship from the Austrian Academy of Sciences, and by NSF IIS-0915054.

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