Artificial Intelligence Methods for Social Good

Lecture 2-4: Mechanism Design with Money

08-537 (9-unit) and 08-737 (12-unit)
Instructor: Fei Fang
feifang@cmu.edu
Wean Hall 4126

Quiz 1: Recap: Dominant Strategy Solution (on Piazza)

- Is there a dominant strategy solution in the following game?
  - A: Yes
  - B: No

		Player 2	
		c	d
Player 1	a	2,1	4,0
	b	1,0	3,0

Quiz 2: Recap: Nash Equilibrium

- How many Pure Strategy Nash Equilibria are there in Battle of Sexes?
  - A: 0
  - B: 1
  - C: 2
  - D: 4

		Berry	
		Football	Concert
Alex	Football	2,1	0,0
	Concert	0,0	1,2

Quiz 3: Recap: Optimization Problem

- For the following optimization problem, which value is closest to the optimal value of a?
  - A: 1.0
  - B: 2.0
  - C: 2.1
  - D: 1.9

x_i	1.0	2.0	3.5
y_i	2.01	3.98	7.0

$$\min_a \sum_{i=1}^{3} (y_i - ax_i)^2$$
$$\text{s.t. } a \in \mathbb{R}$$

Let's Play!

- 1 bonus point in HW7 per person, one piece of paper per person
- 1 box of milk chocolate to be purchased using your bonus point
- Write down your name and how much you are willing to pay to get the chocolate on the paper. Fold it and give it to me.
- I will give the chocolate to the one with the highest value written on the paper. The person needs to pay a price that equals the second highest value written by anyone.

Outline

- Mechanism Design with Money (Non-Bayesian Case)
  - Basic Concepts
  - Desirable Properties
  - Goals
- Mechanism Design with Money (Bayesian Case)
  - Basic Concepts
  - Desirable Properties
  - Goals
- Case Study: Second-Price Auction

Learning Objectives

- Understand the concept of
  - Transferable utility
  - Mechanism
  - Induced game
  - Bayesian Game
  - Direct Mechanism
- Check properties for a given mechanism
  - Truthfulness
  - Incentive Compatibility
  - Budget Balance
  - Individual Rationality
  - Tractability
- Compute objective values given a mechanism
  - Revenue Maximization
  - Maximin Fairness
  - Price of Anarchy Minimization

Recap: Game Theory

- A game: (N, A, u)

Mechanism Design with Money (Non-Bayesian Case)

- Outcome
- Def 1: Transferable Utility
  - An outcome is divided into nonmonetary part (who gets the chocolate) and a monetary part (how much each player pays)
  - Utility = utility of nonmonetary part – monetary part
  - Note
    - "money" can be virtual currency
    - Quasilinear utility function: linear in one argument (virtual currency)

Mechanism Design with Money (Non-Bayesian Case)

- A game setting: (N, O, u) where $u_i: O \rightarrow \mathbb{R}$
- Def 2: Mechanism (A, M)
  - Designer of a mechanism (auctioneer in the chocolate auction)
    - Know possible outcomes and agents' preferences
    - Specify action sets for the agents
    - Specify mapping from joint actions to outcomes
  - Agent/Participant
    - Know what the designer knows
    - Know what designer specifies
    - Cannot change outcomes
    - Utility based on outcome

Mechanism Design with Money (Non-Bayesian Case)

- Def 3: Induced game
  - Once mechanism is announced, then agents play in a multi-player game
  - Solution concepts: dominant strategy solution, Nash Equilibrium

Mechanism Design with Money (Non-Bayesian Case)

- With transferable utility
- Split a deterministic mapping in a mechanism into
  - Def 4: Choice rule
  - Def 5: Payment rule

Mechanism Design with Money (Non-Bayesian Case)

- Mechanism designer's task
  - Pick a mechanism that can will cause rational agents to behave in a desired way, i.e., the solution of the induced game
    - Satisfy some constraints
    - Optimize certain goals
  - An optimization problem

Quiz 4: Mechanism Design with Money (Non-Bayesian Case) (on Piazza)

- What do you think are desired properties in the chocolate auction
  - A: The person who wants the chocolate the most gets it
  - B: Everyone can afford the required payment
  - C: If someone does not get the chocolate, he does not pay
  - D: Everyone is willing to write down a price that equals his true valuation of the chocolate

Quiz 5: Mechanism Design with Money (Non-Bayesian Case)

- What do you think can be a reasonable goal for the auctioneer?
  - A: Maximize total payment he collects from the participants
  - B: Minimize the maximum difference among the participants' payments
  - C: Maximize the value to the person who gets the chocolate
  - D: Make everyone hate the auctioneer

Mechanism Design with Money (Non-Bayesian Case)

- Some common desirable properties
- Def 6: Efficiency
  - If include the mechanism as an agent whose utility is just the total payments he collects, then efficiency ensures the total utility of all agents is maximized
  - Also called economic efficiency

Mechanism Design with Money (Non-Bayesian Case)

- Def 7: Budget Balance
  - The mechanism disburses and collects same amount of money to and from the participants
  - How to make the chocolate auction budget balanced?
  - Def: Weak budget balance: the mechanism may make a profit

Mechanism Design with Money (Non-Bayesian Case)

- Def 8: Individual rationality
  - No agent loses by participating in the mechanism

Mechanism Design with Money (Non-Bayesian Case)

- Def 9: Tractability
  - The mechanism can compute choice and payment efficiently

Mechanism Design with Money (Non-Bayesian Case)

- Some common optimization goals
- Def 10: Revenue maximization
  - Maximize the total payment the mechanism collects
- Def 11: Revenue minimization
  - Minimize the total payment the mechanism collects

Quiz 6: Mechanism Design with Money (Non-Bayesian Case) (on Piazza)

- Which of the following do you think is the fairest outcome in the chocolate auction?
  - A: Everyone pays 0, give the chocolate to the one who wrote the lowest price on paper
  - B: Everyone pays 1/N of the actual cost of buying the chocolate, give the chocolate randomly
  - C: Everyone pays 1/N of the actual cost of buying the chocolate, give the chocolate to the one who wrote the highest price on paper
  - D: The one who wrote the highest price on paper gets the chocolate, and pays the amount he wrote. Everyone else pays 0.

Mechanism Design with Money (Non-Bayesian Case)

- Def 12: Maximin fairness
  - Make the least-happy agent the happiest

Mechanism Design with Money (Non-Bayesian Case)

- Keep in mind that one the mechanism is specified, it is a game among participating agents
- Def 13: Price of Anarchy Minimization
  - Minimize the ratio between optimal social welfare and the social welfare achieved in equilibrium given the mechanism

Learning Objectives

- Understand the concept of
  - Transferable utility
  - Mechanism
  - Induced game
  - Bayesian Game
  - Direct Mechanism
- Check properties for a given mechanism
  - Truthfulness
  - Incentive Compatibility
  - Efficiency (Social Welfare Maximization)
  - Budget Balance
  - Individual Rationality
  - Tractability
- Compute objective values given a mechanism
  - Revenue Maximization/Minimization
  - Maximin Fairness
  - Price of Anarchy Minimization

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Recap: Game Theory

- Def 14: Bayesian Game
  - A player's utility function depends on his "type"
  - In security games, a terrorist may have different utility function from a smuggler
  - In chocolate auction, a person on diet may have different valuation of the chocolate from a person who loves chocolate and is not on diet, leading to different utility functions. Each participant knows his own type but only knows a prior distribution of other players' type. (Keep in mind that one the auction mechanism is specified, it is a game among participating agents)

Mechanism Design with Money (Bayesian Case)

- Let's revisits all the concepts, properties and goals in the Bayesian case

Mechanism Design with Money (Bayesian Case)

- Def 1b: Transferable Utility
  - An outcome is divided into nonmonetary part (who gets the chocolate) and a monetary part (how much each player pays)
  - Utility = utility of nonmonetary part – monetary part

Mechanism Design with Money (Bayesian Case)

- A Bayesian game setting: (N, O, Θ, p, u) where $u_i: O \times \Theta \rightarrow \mathbb{R}$
- Def 2b: Mechanism
  - Designer of a mechanism (auctioneer in the chocolate auction)
    - Know possible outcomes, the agents' type space and distribution over type space, and agents' preferences for each type (how much I value the chocolate when I had or didn't have breakfast)
    - Specify action sets for the agents
    - Specify mapping from joint actions to outcomes
  - Agent/Participant
    - Know what the designer knows
    - Knows his own type (e.g., whether I had breakfast this morning, called private information)
    - Know what designer specifies
    - Cannot change outcomes
    - Utility based on outcome

Mechanism Design with Money (Bayesian Case)

- Def 3b: Induced game with independent private type and incomplete information
  - Agent knows his own type and the prior distribution of other agents' type
  - Agent's utility only depends on his own type and the joint action of all agents, does not depend on other agents' types
  - Once mechanism is announced, then agents play in a multi-player incomplete information game
  - Solution concepts: dominant strategy solution, Bayes-Nash Equilibrium

Mechanism Design with Money (Bayesian Case)

- With transferable utility
- Split a deterministic mapping in a mechanism into
  - Def 4b: Choice rule
  - Def 5b: Payment rule

Mechanism Design with Money (Bayesian Case)

- Definition 15: Direct mechanism
  - Instead of specifying a mapping from the joint actions of players to outcomes, just specifying a mapping from the joint type profile to outcomes
  - What is the action set for the players?
    - Declare his type to the mechanism (he may lie)
  - Direct mechanism leads to a simple normal form game
  - Indirect mechanism may lead to an extensive form game, e.g., Dutch auction

Mechanism Design with Money (Bayesian Case)

- Definition 16: Valuation Function
  - With transferrable utility
  - A mapping from nonmonetary outcomes to real values
  - Agent type ⇔ valuation function
  - Declaring agent's type is equivalent to declaring agent's valuation function
  - Denote true valuation function as v_i and declared valuation function as $\widehat{v_i}$

Mechanism Design with Money (Bayesian Case)

- Mechanism designer's task
  - Pick a mechanism that can will cause rational agents to behave in a desired way, i.e., the solution of the induced game
    - Satisfy some constraints
    - Optimize certain goals
  - An optimization problem

Mechanism Design with Money (Bayesian Case)

- Def 17. Truthfulness
  - Every agent will declare his true valuation function in dominant strategy equilibrium
  - If everyone telling the truth is a dominant strategy equilibrium: Dominant Strategy Incentive Compatible (DSIC), Strategyproof, Truthful
  - If everyone telling the truth is a Bayes-Nash equilibrium: Bayesian-Nash Incentive Compatible (BNIC)

Mechanism Design with Money (Bayesian Case)

- Some common desirable properties
- Def 6b: Efficiency
  - If include the mechanism as an agent whose utility is just the total payments he collects, then efficiency ensures the total utility of all agents is maximized
  - Also called economic efficiency, social-welfare maximization

Mechanism Design with Money (Bayesian Case)

- Def 7b: Budget Balance
  - The mechanism disburses and collects same amount of money to and from the participants
  - Def: Weak budget balance: the mechanism may make a profit
  - May only hold ex ante

Mechanism Design with Money (Bayesian Case)

- Def 8b: Ex interim Individual rationality
  - No agent loses by participating in the mechanism
  - For every possible valuation for agent i, in expectation of other agents' valuations
- Def 8c: Ex post Individual rationality
  - No agent loses by participating in the mechanism
  - For every possible valuation for all agents

Mechanism Design with Money (Bayesian Case)

- Def 9b: Tractability
  - The mechanism can compute choice and payment efficiently given any declared valuation function

Mechanism Design with Money (Bayesian Case)

- Some common optimization goals
- Def 10b: Revenue maximization
  - Maximize the expected total payment the mechanism collects
- Def 11b: Revenue minimization
  - Minimize the worst case total payment the mechanism collects

Mechanism Design with Money (Bayesian Case)

- Def 12b: Maximin fairness
  - Make happiness level of the least-happy agent highest in expectation

Mechanism Design with Money (Bayesian Case)

- Keep in mind that one the mechanism is specified, it is a game among participating agents
- Def 13b: Price of Anarchy Minimization
  - Minimize the worst-case ratio between optimal social welfare and the social welfare achieved in equilibrium given the mechanism

Mechanism Design with Money (Bayesian Case)

- Revelation principle
  - A social choice function is a mapping from true joint type profile to outcome
  - We say a mechanism implements a social choice function if in equilibrium (dominant strategy solution or BNE) of the induced game, the outcome is the same as what is specified by the social choice function
  - If there exists a mechanism that implements a social choice function, then there exists a truthful direct mechanism that implements the social choice function
  - Reasoning: Let the new mechanism lie for each player

Learning Objectives

- Understand the concept of
  - Transferable utility
  - Mechanism
  - Induced game
  - Bayesian Game
  - Direct Mechanism
- Check properties for a given mechanism
  - Truthfulness
  - Incentive Compatibility
  - Efficiency (Social Welfare Maximization)
  - Budget Balance
  - Individual Rationality
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Case Study: Second-Price Auction

- Understand the concept of
  - Transferable utility?
  - Mechanism: Allocation rule? Payment rule?
  - Induced game: Players? Actions? Payoffs?
  - Bayesian Game: Bayesian?
  - Direct Mechanism?
- Check properties for a given mechanism
  - Truthfulness?
  - Incentive Compatibility: Dominant Strategy Incentive Compatible? BNIC?
  - Efficiency (Social Welfare Maximization)?
  - Budget Balance?
  - Individual Rationality?
  - Tractability?
- Compute objective values given a mechanism
  - Revenue Maximization/Minimization: value=?
  - Maximin Fairness: value=?
  - Price of Anarchy Minimization: value=?

Summary

- Mechanism Design with Money (Non-Bayesian Case)
- Mechanism Design with Money (Bayesian Case)
- Case Study: Second-Price Auction

- Key takeaways:
  - The mechanism decides the rule of the game
  - If you are asked to compare two mechanisms, check all these properties and goals
  - Select the right mechanism to achieve a specific goal or properties (e.g., maximize social welfare)

Mechanism Design: Additional Resources

- Text book
  - *Algorithmic Game Theory 1st Edition, Chapter 9*
  - *by Noam Nisan (Editor), Tim Roughgarden (Editor), Eva Tardos (Editor), Vijay V. Vazirani (Editor)*
  - http://www.cs.cmu.edu/~sandholm/cs15-892F13/algorithmic-game-theory.pdf
- Online course
  - https://www.youtube.com/user/gametheoryonline

Acknowledgment

- The slides are prepared based on course slides of "Game Theory Online II" (Matt Jackson, Kevin Leyton-Brown, Yoav Shoham)