Artificial Intelligence Methods for Social Good MI-3 [Optimization]: Combinatorial Optimization and Robust Optimization

> 08-537 (9-unit) and 08-737 (12-unit) Instructor: Fei Fang <u>feifang@cmu.edu</u> Wean Hall 4126

Learning Objectives

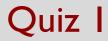
- Understand the concept of
 - Maximin model
 - Minimax regret
- Briefly describe
 - Branch and bound algorithm
- Optional
 - Dual problem of a linear program

Robust Optimization

- How do we deal with uncertainty?
- Expl: Choose location for a party
- When you know the probability of raining (whether forecast): Compute expected utility
- When you don't know: robust optimization
 - Maximin: Maximize the worst case utility (Conservative)
 - Minimax regret (Less conservative)
 - Minimize maximum regret
 - Regret: Gap between my current utility and the best possible utility given the whether condition

Robust Optimization

- Solve Minimax Regret when there are many actions and possibilities
 - Constraint sampling/constraint generation
 - Sample a subset of actions and possibilities
 - Solve the restricted problem
 - Find most "critical" combination of action and possibilities
 - Resolve the restricted problem
 - Repeat until some stopping criteria



Effort Spend

Easy	9-0=9	0.1-0	
	/-0-/	9-1=8	10-3=7
A Medium	5-0=5	7.5-1=6.5	9-3=6
Medium Hard	2-0=3	5-1=4	8-3=5

What is the value of minimax regret?

- ▶ 2
- ▶ 0.5
- ▶ 3

Binary Program with linear objective and constraints

• Key idea:

- Incrementally build a binary search tree
- Prune the tree using lower bound and upper bound
- Expand the most promising leaf node

Branch and Bound

Exp 2: Knapsack (W=10)

ltems		2	3	4	5	
Weight	5	4	2	6	7	
Value	4	3	6	9	5	

Upper bound: LP relaxation (poly-time computable)

Lower bound: Any feasible integer solution

Solving (Mixed) Integer Program with Branch and Bound

- Recall MILP: An optimization problem whose optimization objective is a linear function and feasible region is defined by linear constraints and integer constraints
- Key Ideas of Branch and Bound for MILP
 - If value of all integer variables are determined, then become an LP (can be efficiently solved using e.g., Simplex algorithm!)
 - Substitute the integer variables with binary variables
 - Build a binary search tree for the binary variables
 - Prune the tree using lower bound and upper bound

- An optimization problem whose optimization objective is a linear function and feasible region is defined by linear constraints
- Dual problem of an LP: also a linear program
- Strong duality theorem: LP and its dual have the same value

Exp 3: Maximize Profit

	Price	Labor	Machine
Product I	\$30	0.2 hour	4 hour
Product 2	\$30	0.5 hour	2 hour
Total		<=90	<=800

Write the Dual of an LP (Non-Standard form)

Maximize	Minimize
ith constraint ≤	ith variable ≥ 0
ith constraint ≥	ith variable ≤ 0
ith constraint =	ith variable unrestricted
jth variable ≥ 0	jth constraint ≥
jth variable ≤ 0	jth constraint ≤
jth variable unrestricted	jth constraint =

Other properties of LP Duality

- Complementary slackness
 - > Primal constraint is not tight \rightarrow dual variable=0
 - > Primal variable is not zero \rightarrow dual constraint is tight
 - > Dual constraint is not tight \rightarrow primal variable=0
 - > Dual variable is not zero \rightarrow primal constraint is tight
- Optimality Conditions: If x, y are feasible solutions to the primal and dual problems, respectively, then they are optimal solutions to these problems if, and only if, the complementary-slackness conditions hold for both the primal and the dual problems

Reference and Related Work

- Combinatorial Optimization: Algorithms and Complexity, <u>Chapters 3</u>
 - Christos H. Papadimitriou, Kenneth Steiglitz
- Branch-and-price: Column generation for solving huge integer programs
 - Cynthia Barnhart, Ellis L. Johnson, George L. Nemhauser, Martin W. P. Savelsbergh, Pamela H. Vance
- Robust protection of fisheries with COmPASS
 - William Haskell, Debarun Kar, Fei Fang, Milind Tambe, Sam Cheung, Elizabeth Denicola

Paper Discussion

- (PRA5) <u>Keeping Pace with Criminals: An Extended Study of</u> <u>Designing Patrol Allocation against Adaptive Opportunistic Criminals</u>
 - Chao Zhang, Shahrzad Gholami, Debarun Kar, Arunesh Sinha, Manish Jain, Ripple Goyal, Milind Tambe
- Summary
 - Societal challenge
 - Al method
 - Contributions
- Questions
- Brainstorming Ideas
 - Improvement / future direction / other valid discussions
 - Societal challenge and AI method that can potentially be used to tackle it (not necessarily relevant to the paper)