Motivation:
- Want to learn which knowledge components depend on which others.
- This "prerequisite structure" would aid curriculum design.
- No data containing natural or experimental variation in topic order.
- But we do have data from tests that cover multiple topics.
- Goal: use test data to learn prerequisite structure.

Structural search algorithm:
- PC algorithm.
  - Input: a set of conditional independences over a set of variables
  - Output: the Markov equivalence class of causal structures consistent with the independences.
  - Structures within the equivalence class are indistinguishable without further evidence.

Measurement:
- Want to know the prerequisite relationships between Knowledge Components (KCs).
- KCS are not measured directly, so no direct measures of dependence between them.
- Need to learn dependencies between latent variables to run PC.
- This is a hard problem; there is currently no general solution.

Structural Model:
- Assumption: Measurement model is known
  - Key assumption: that we know the "Measurement Model" (a.k.a. the Q matrix).
  - I.e. for each test item, we know which set of KCs it measures.
  - Could occur by test design, or could use an existing method for Q matrix learning.

Method to learn independences among KCs:
- E.g.: Say we have four latents, L1-L4. Want to learn whether L1 _||_ L2 | L3.
  - We can test this independence using a Structural Equation Model (SEM) fitter, by fitting the model on the left.
  - If it follows from d-separation that β = 0 if and only if L1 _||_ L2 | L3.
  - SEM fitting provides consistent statistical inference for parameter values.
  - In fact, for any given conditional independence, we can construct a structural model such that an edge coefficient is zero if and only if that independence holds.
  - We developed a construction algorithm that produces such a structural model, and proved its consistency. The details are in the full length version of our paper.

Evaluation on simulated data:
- Simulation tests how well the method recovers the Markov equivalence class when the true model is known.
- Structure: We tested three different structural models. The Markov equivalence classes of these models are on the right.
- Purity: For each structural model, we created two measurement models: one with all pure items, and one with many impure items (i.e. items that load on more than one KC). The six complete SEMs are below right.
- Parameters: For each complete SEM we drew 100 random parameterizations (all variables Gaussian, all relationships linear).
- Sample size: For each parameterization, we generated two datasets: one with 1000 simulated students’ scores, and one with only 150 students’ scores.
- Binary vs. Continuous items: All these scores were continuous-valued, but real test item scores are 0-1. We projected each dataset to a binary copy, thresholding at the mean.

Results of simulation:
- The method performed well in simulation, both for edge adjacencies and edge orientations. The true positive rate for adjacencies and orientations in each condition is shown on the left.
- False positive rates in all conditions were low for both adjacencies (0-5%) and orientations (0.4%). Details are in the full paper.

Conclusions:
- Our method solves a novel problem: learning the dependency structure among knowledge components from test data when the measurement model contains many impure items.
- This is a common situation in educational research.
- Applications could aid curriculum design.
- Room for improvement: extensions to the model constructor could improve efficiency & identifiability.

Special case: Pure measurement models
- "Pure" items load on only one latent
- When the measurement model contains many pure items, build Pure Clusters + MIMIC build can find the structural Markov Equivalence class without knowing the measurement model
- However, in education, most test items load on multiple KCs
- In simulation we varied the # of pure items in the measurement model

Extensions
- Sometimes multiple models test the same independence.
- Our constructor might not produce the most efficient one.
- When the Q matrix is dense or the number of KCs large, the model may be unidentifiable, so the SEM fitter will not give an answer.
- Finding the most efficient model constructor is an open potential extension.