1. Define the following terms. (4 points each)

(a) tautology

A wff that evaluates as true under any truth-assignment to its propositional variables.

(b) inconsistency

A wff that evaluates as false under any truth-assignment to its propositional variables.

(c) contingency

A wff that evaluates as true under some truth-assignments to its propositional variables, and false under others.

(d) contradiction

A contradiction any wff obtained by conjoining some wff with its negation.

(e) valid sequent

A sequent for which: any truth-assignment that makes all of its assumptions true, also makes its conclusion true.

(f) corresponding conditional

Given a sequent $A_1, A_2, \ldots, A_n \vdash B$, its corresponding conditional is the wff

\[ (A_1 \rightarrow (A_2 \rightarrow (\cdots (A_n \rightarrow B) \cdots))). \]
2. Answer the following true/false questions (circle the right answer). (6 points each)

(a) A valid sequent must be provable in our formal proof system.

   \[ \begin{array}{cl}
   & \text{T} \quad \text{F} \\
   \end{array} \]

   This is what the metatheorem about the completeness of our formal system tells us.

(b) A sequent, all of whose assumptions are inconsistencies, could possibly be valid.

   \[ \begin{array}{cl}
   & \text{T} \quad \text{F} \\
   \end{array} \]

   In fact, any sequent that has at least one inconsistent assumption will necessarily be valid. The reason? There are no truth-assignments that make all the assumptions true, and so the sequent is valid in a vacuous way.

(c) A valid sequent could possibly have an inconsistent conclusion.

   \[ \begin{array}{cl}
   & \text{T} \quad \text{F} \\
   \end{array} \]

   Per the explanation above, if the sequent has some inconsistent assumption, then despite an inconsistent conclusion, it will still be valid.

(d) An invalid sequent could possibly have an inconsistent conclusion.

   \[ \begin{array}{cl}
   & \text{T} \quad \text{F} \\
   \end{array} \]
3. Draw parse trees for the following formulas. (5 points each)

Nobody missed this, so I won’t draw the parse trees.

(a) \(-(A \rightarrow B) \& (\neg P \lor Q)\)

(b) \(-(P \lor (Q \rightarrow \neg R))\)

(c) \(((V \& W) \rightarrow \neg (V \& W)) \lor (V \& W)\)
4. Prove the law of the excluded middle: \( \vdash P \lor \neg P \). You may use primitive rules only. (20 points)

See Lemmon p. 52 if you’re uncomfortable with this proof.
5. Decide whether each of the following is tautologous, contingent or inconsistent. All I need is your answer. (Hint: if you are unsure, the next question and our overall point distribution should at least tell you how many of these are tautologies.) (4 points each)

(a) $\neg(\neg P \lor Q) \land (Q \rightarrow P)$

inconsistent

(b) $(R \rightarrow S) \rightarrow ((P \lor R) \rightarrow (P \lor S))$

tautology

(c) $(P \land Q) \rightarrow \neg(P \rightarrow \neg Q)$

tautology

(d) $((P \land Q) \rightarrow R) \rightarrow ((P \rightarrow R) \land (Q \rightarrow R))$

contingent
6. For each tautology from the previous question, show that it is a theorem. That is, show that we can give a formal proof of it that rests on no assumptions. (10 points each)

\[ \vdash (R \rightarrow S) \rightarrow ((P \lor R) \rightarrow (P \lor S)) \]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R → S</td>
<td>P ∨ R</td>
<td>P</td>
<td>P ∨ S</td>
<td>R</td>
<td>S</td>
<td>P ∨ S</td>
<td>P ∨ R → (P ∨ S)</td>
<td>(P ∨ R) → (P ∨ S)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>P</td>
<td>A</td>
<td></td>
<td>3</td>
<td></td>
<td>1,5 MPP</td>
<td></td>
<td></td>
<td>2,8 CP</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>P</td>
<td>A</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td>6 ∨I</td>
<td></td>
<td></td>
<td>1,9 CP</td>
</tr>
<tr>
<td>4</td>
<td>P ∨ S</td>
<td>3 ∨I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>R</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \vdash (P \& Q) \rightarrow \neg(P \rightarrow Q) \]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P &amp; Q</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td>5,4 &amp;I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>P → Q</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2,3 MPP</td>
<td>2,6 RAA</td>
</tr>
<tr>
<td>1</td>
<td>P</td>
<td>1 &amp;E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td>(4)</td>
<td>¬Q</td>
<td>1 &amp;E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(5)</td>
<td>Q</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td>(6)</td>
<td>Q &amp; ¬Q</td>
<td>1 &amp;E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(7)</td>
<td>¬(P → Q)</td>
<td>2,6 RAA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>(P &amp; Q) → ¬(P → Q)</td>
<td>1,7 CP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>