80-211: HOMEWORK 8

- You may work with other students from class, but everyone should write up solutions independently.
- Homework is due by the end of class time, either as a hard copy or email attachment.
- Remember that no late homework is accepted, but you have two drops.

1. (10 points) Give a formal proof of this sequent:
   \[ \vdash \exists x (x = a). \]

2. (16 points) Let \( \varphi \) be the wff
   \[ \exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land P(x, y) \land P(y, z)), \]
   let \( \psi \) be the wff
   \[ \forall x \exists y \forall z (P(x, z) \rightarrow (P(y, x) \land P(z, y))), \]
   and let \( \theta \) be the wff
   \[ \exists x \exists y \neg P(x, y). \]
   Construct an interpretation \( \mathcal{M} \) whose universe is \( U = \{1, 2, 3, 4, 5\} \), and which is such that
   \[ \mathcal{M} \models \varphi \land \psi \land \theta. \]

3. (28 points) Exercise 4 (Velleman p. 186)
4. (16 points) Exercise 11 (Velleman p. 187)
5. (30 points) Exercise 14 (Velleman p. 187)

Date: Due on Wednesday, April 14.