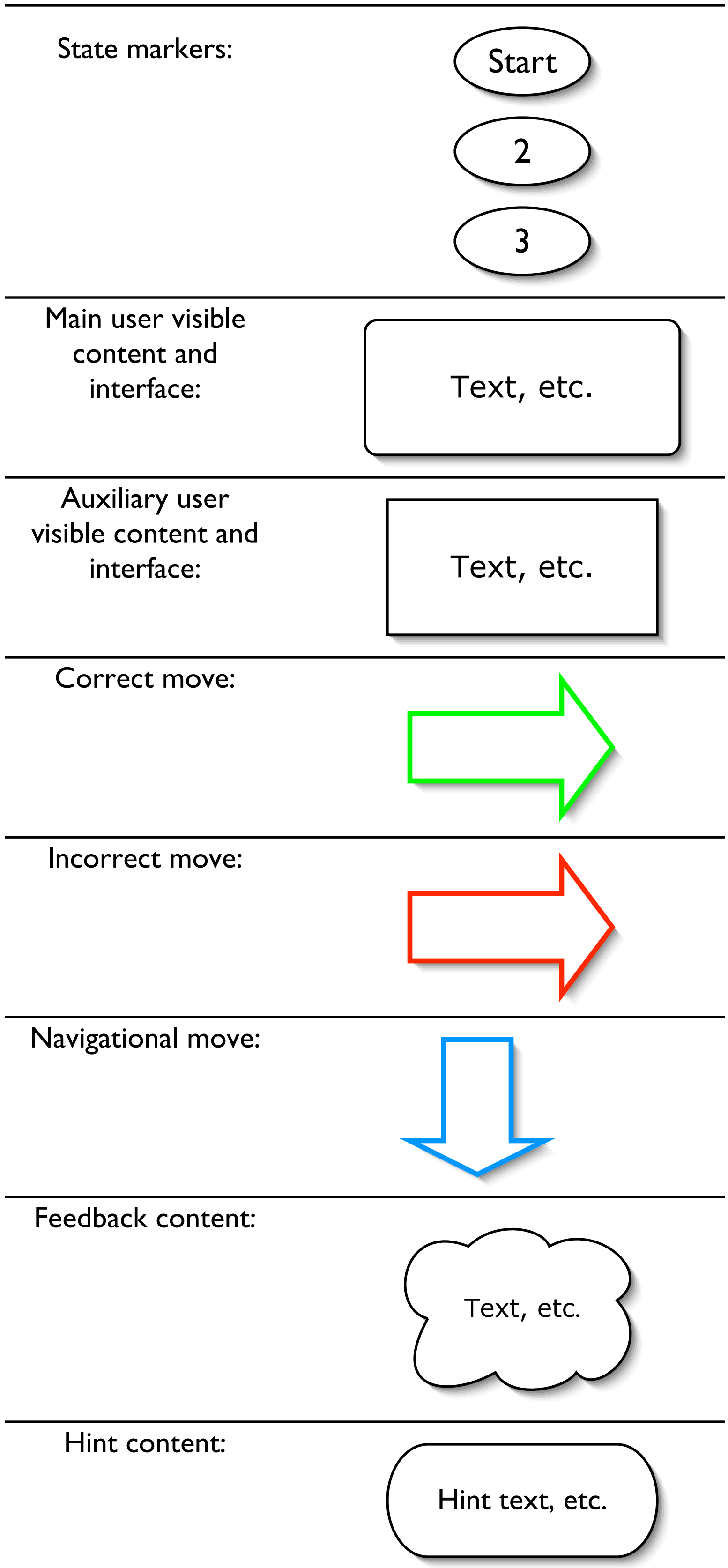
Legend:


State markers:
Start
2
3
Main user visible content and interface:
Text, etc.
Auxiliary user visible content and interface:
Text, etc.
Correct move:
Incorrect move:
Navigational move:
Feedback content:
Text, etc.
Hint content:
Hint text, etc.

Start

Complete the following derivation by filling in the missing formulae. To fill in the formula on a given line, just click anywhere on that line.

1.	$P \lor \neg S$	Premise
2.	$\neg P \lor Q$	Premise
3.	?	Cut: 1, 2
4.	?	Def. →I: 3
5.	?	Trans: 4
6.	?	Def. →E: 5
7.	?	DeM: 6

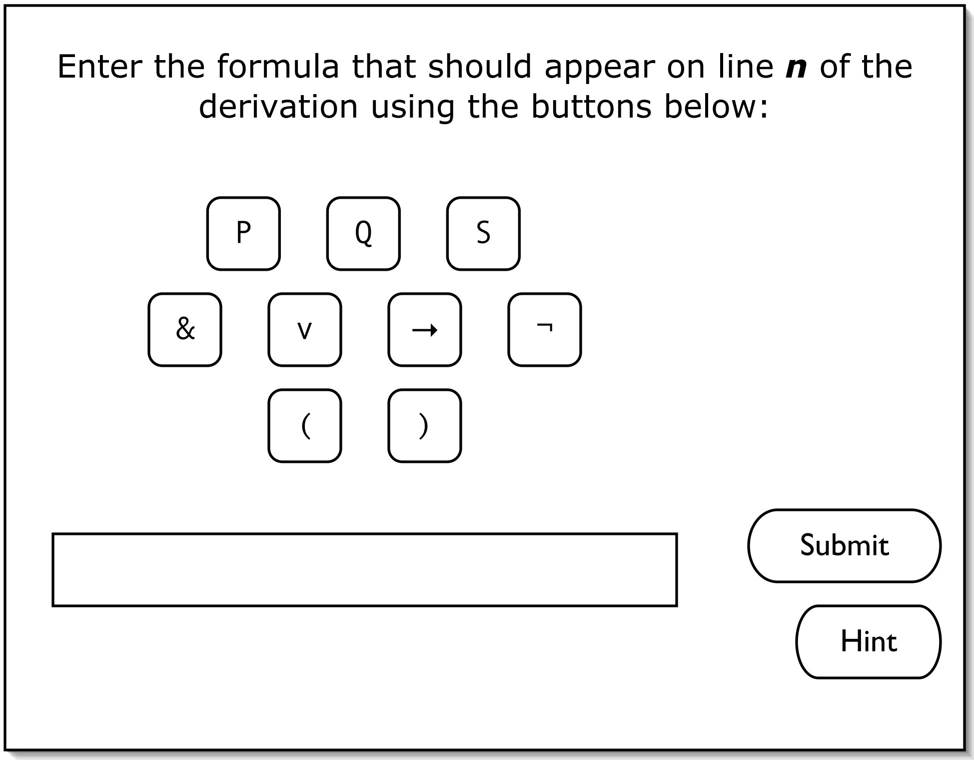
3 4 5 6 7

2 3 4 5 6

Completed derivation:

1.	$P \lor \neg S$	Premise
2.	$\neg P \lor Q$	Premise
3.	$\neg S \lor Q$	Cut: 1, 2
4.	$S \rightarrow Q$	Def. →I: 3
5.	$\neg Q \rightarrow \neg S$	Trans: 4
6.	$\neg\neg Q \lor \neg S$	Def. →E: 5
7.	$\neg(Q \,\&\, S)$	DeM: 6

Interface for entering formulae:



I've included the ideal version of the interface, here, which contains all and only those symbols actually appearing in the exercise.

2

¬S v Q

That's right.

Q v ¬S
¬Q v S
S v ¬Q

Not quite. The left-hand disjunct in the formula derived using Cut is the same as the right-hand disjunct of the formula on the first line cited, while the right-hand disjunct corresponds to the right-hand disjunct on the second line cited.

anything else

That's not right. Recall that `Cut` is the following rule:

```
a. φ v ψ
b. ¬φ v ψ
.
.
.
d. ψ v ρ     Cut: a, b
```

3

$S \rightarrow Q$

That's right.

$P \rightarrow Q$

That formula could be derived from line 2 by `Def.→I`, but the line cited is 3, not 2.

(see list below)

$P \rightarrow P$
$P \rightarrow S$
$Q \rightarrow P$
$Q \rightarrow Q$
$Q \rightarrow S$
$S \rightarrow P$
$S \rightarrow S$

Not quite. You are correct that the formula should be a conditional, but you haven't got the antecedent and consequent quite right. Recall that a conditional is true if its antecedent is false or its consequent is true, double-check the line cited and consider the truth-table for $\neg S \vee Q$ with that in mind.

anything else

Any line justified by means of `Def.→I` is going to be a conditional. Double check the line cited and consider the truth-table for the formula on that line with this in mind.

4

$\neg Q \rightarrow \neg S$

That's right.

$\neg S \rightarrow \neg Q$

Not quite. Transposition doesn't just negate the antecedent and consequent, it also exchanges them.

anything else

That's not right. Recall that `Trans` is the following rule:

```
a. φ → ψ
.
.
.
c. ¬ψ → ¬φ    Trans: a
```

5

¬¬Q v ¬S

That's right.

¬S v ¬Q

That formula could be derived from line 4 by `Def.→I`, but the line cited is 5, not 4.

(see list below)

¬¬S v ¬Q
¬¬S v ¬¬Q
¬Q v ¬S
¬Q v ¬¬S
¬¬Q v ¬¬S

Not quite. You are correct that the formula should be a disjunction, but you haven't got the disjuncts quite right. Recall that a conditional is true if its antecedent is false or its consequent is true, and consider ¬Q → ¬S with that in mind.

anything else

Any line justified by means of `Def.→E` is going to be a disjunction. Double check the line cited and consider the truth-table for the formula on that line with this in mind.

6

¬(¬Q & S)

That's right.

too many/few negations, no other problems

You're on the right track, but check your negations carefully.

atomic subformulae other than Q and/or S

The formula derived using DeMorgan's cannot include atomic subformulae not in the line to which the rule was applied.

anything else

Recall that DeMorgan's laws demonstrate the relationship between a disjunction of negations and a negated conjunction, and also between a conjunction of negations and a negated disjunction.

Hints

Each hint should contain the following, after specific hint content:

Click here to view the rule Cut, here to view the DeM rules, here to view the rule Trans., and here to view the rules for the definition of the conditional.

The links should be to the following files, as indicated by both order and colour:

missingformulae2hintCut.gif
missingformulae2hintDeM.gif
missingformulae2hintTrans.gif
missingformulae2hintDefCond.gif

2

Remember that applying Cut to two disjunctions has the effect of "cutting out" a tautology of the form $\varphi \lor \neg\varphi$, leaving the choice between the remaining disjuncts.

The formulae on lines 1 and 2, to which Cut is being applied, are $P \lor \neg S$, and $\neg P \lor Q$, so if P and $\neg P$ are cut out, that leaves $\neg S$ and Q.

The formula to enter is the disjunction of $\neg S$ and Q, in that order: $\neg S \lor Q$.

3

Remember the characteristic truth-table for the conditional: a conditional is true just in case either its antecedent is false or its consequent is true.

The disjunction $\neg S \lor Q$ is true just in case either S is false or Q is true.

The formula to enter is the conditional $S \rightarrow Q$.

4

Remember that `Trans` exchanges and then negates the antecedent and consequent of the conditional to which it is applied.

The conditional of line 4 is $S \rightarrow Q$, so exchanging antecedent and consequent would result in the conditional $Q \rightarrow S$, negating S results in $\neg S$, and negating Q results in $\neg Q$. Just put all these pieces together to get the final result.

The formula to enter is the conditional $\neg Q \rightarrow \neg S$.

5

Remember the characteristic truth-table for the conditional: a conditional is true just in case either its antecedent is false or its consequent is true.

The conditional $\neg Q \rightarrow \neg S$ is true just in case either $\neg\neg Q$ is true or $\neg S$ is true.

The formula to enter is the disjunction $\neg\neg Q \lor \neg S$.

6

Remember the DeMorgan's transforms a disjunction of negations into a negated conjunction.

The formula to enter is $\neg(\neg Q \,\&\, S)$.