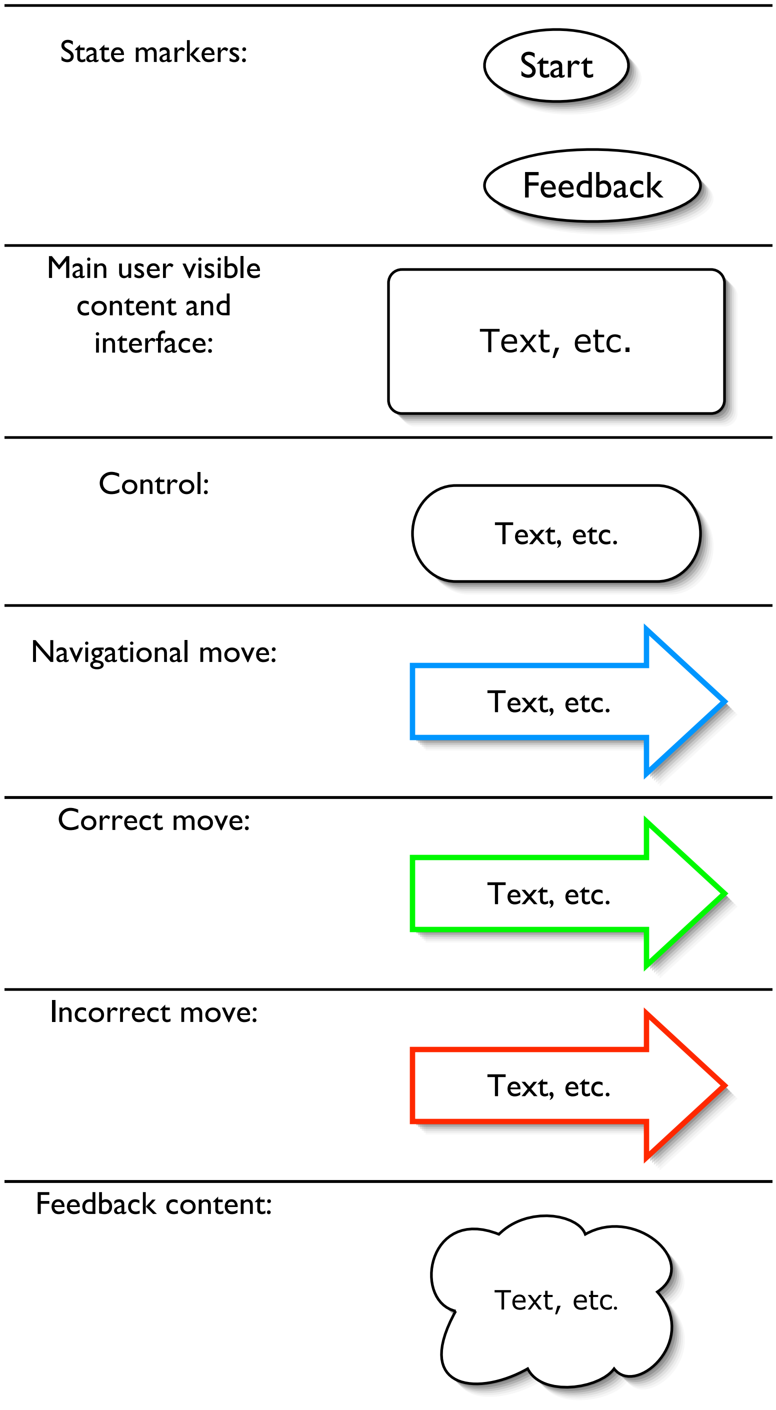
Legend:

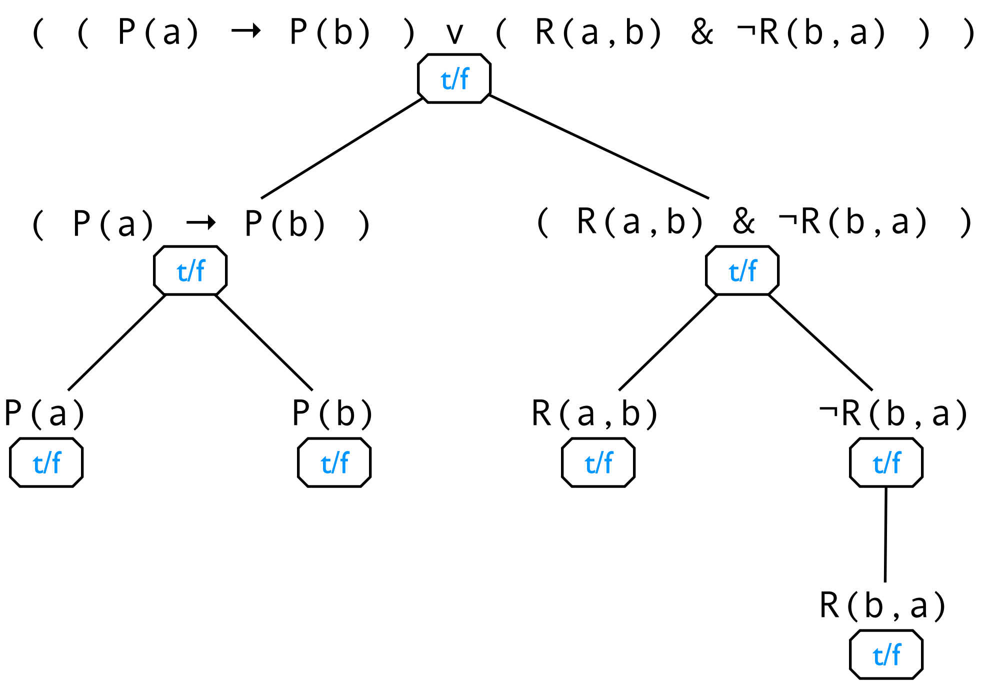

State markers:
Start
Feedback
Main user visible content and interface:
Text, etc.
Control:
Text, etc.
Navigational move:
Text, etc.
Correct move:
Text, etc.
Incorrect move:
Text, etc.
Feedback content:
Text, etc.

Start

Determine whether the given interpretation makes the formula below true or false.

We've provided you with the completed parse tree for the formula. All you need to do is "chase truth up the tree", marking each node as either true or false. Once you've correctly marked each node, you're done.

$$\mathfrak{I}(a) = \boldsymbol{a}$$
$$\mathfrak{I}(b) = \boldsymbol{b}$$
$$\mathfrak{I}(P) = \{ \langle \boldsymbol{a} \rangle , \langle \boldsymbol{c} \rangle \}$$
$$\mathfrak{I}(R) = \{ \langle \boldsymbol{a}, \boldsymbol{a} \rangle , \langle \boldsymbol{a}, \boldsymbol{b} \rangle , \langle \boldsymbol{a}, \boldsymbol{c} \rangle \}$$



Hint

Items marked as t/f are the comboboxes. The formulae should be visible even after the node has been classified a either true or false, so the formula at the node should not be included in the combobox itself.

Feedback

P(a)

t

That's right.

f

The atomic formula P(a) will be false on an interpretation $\mathfrak{I}$ just in case $\mathfrak{I}$(a) is not in the extension of the predicate letter, $\mathfrak{I}$(P). Check the interpretation again carefully.

P(b)

f

That's right.

t

The atomic formula P(b) will be true on an interpretation $\mathfrak{I}$ just in case $\mathfrak{I}$(b) is in the extension of the predicate letter, $\mathfrak{I}$(P). Check the interpretation again carefully.

(P(a) → P(b))

f

That's right.

t

A conditional is false if its antecedent is true and its consequent is false.

$R(a,b)$

t

That's right.

f

The atomic formula $R(a,b)$ will be false on an interpretation $\mathfrak{I}$ just in case $\langle \mathfrak{I}(a), \mathfrak{I}(b) \rangle$ is not in the extension of the predicate letter, $\mathfrak{I}(R)$. Check the interpretation again carefully.

$R(b,a)$

f

That's right.

t

The atomic formula $R(a,b)$ will be true on an interpretation $\mathfrak{I}$ just in case $\langle \mathfrak{I}(b), \mathfrak{I}(a) \rangle$ is in the extension of the predicate letter, $\mathfrak{I}(R)$. Check the interpretation again carefully.

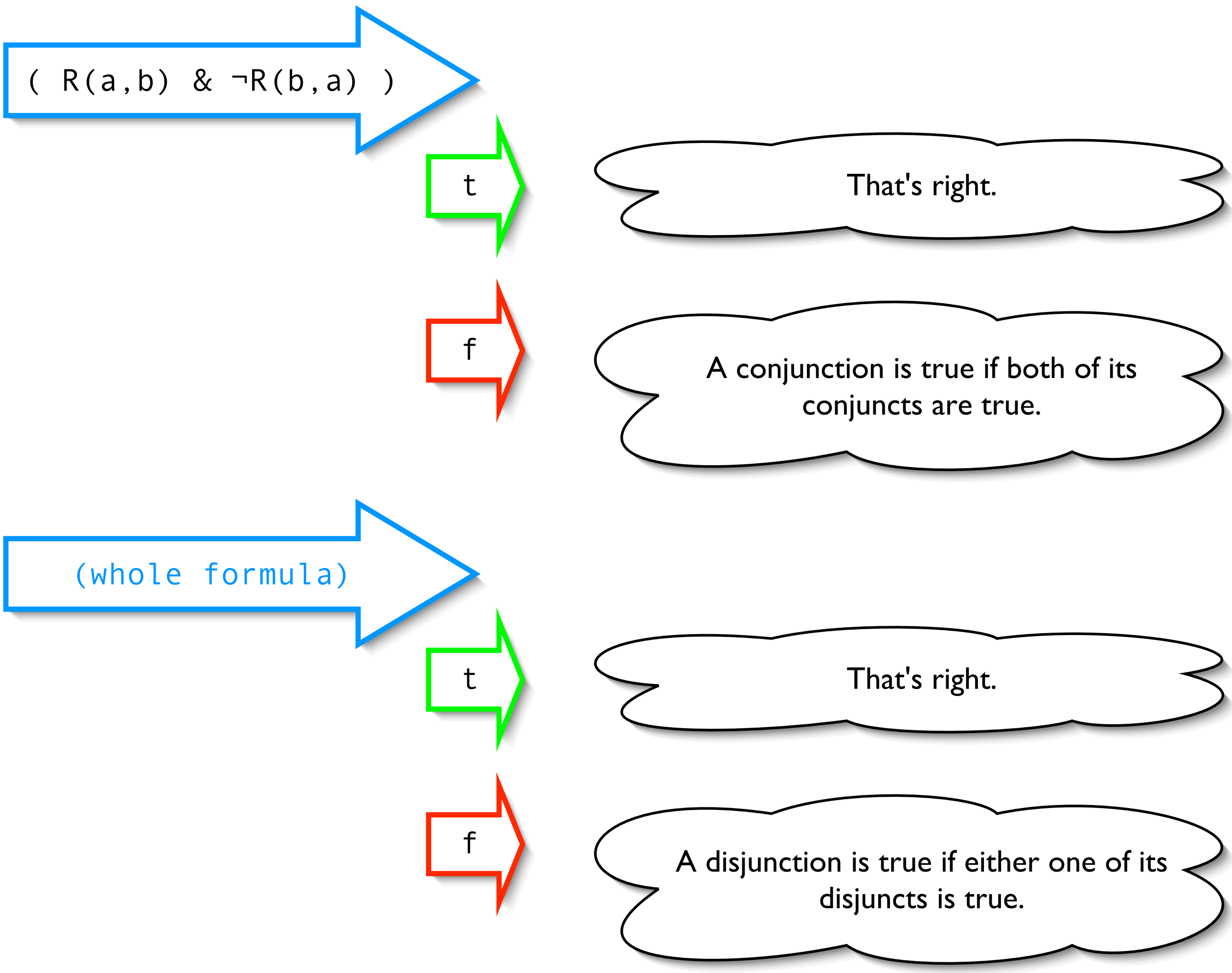
$\neg R(b,a)$

t

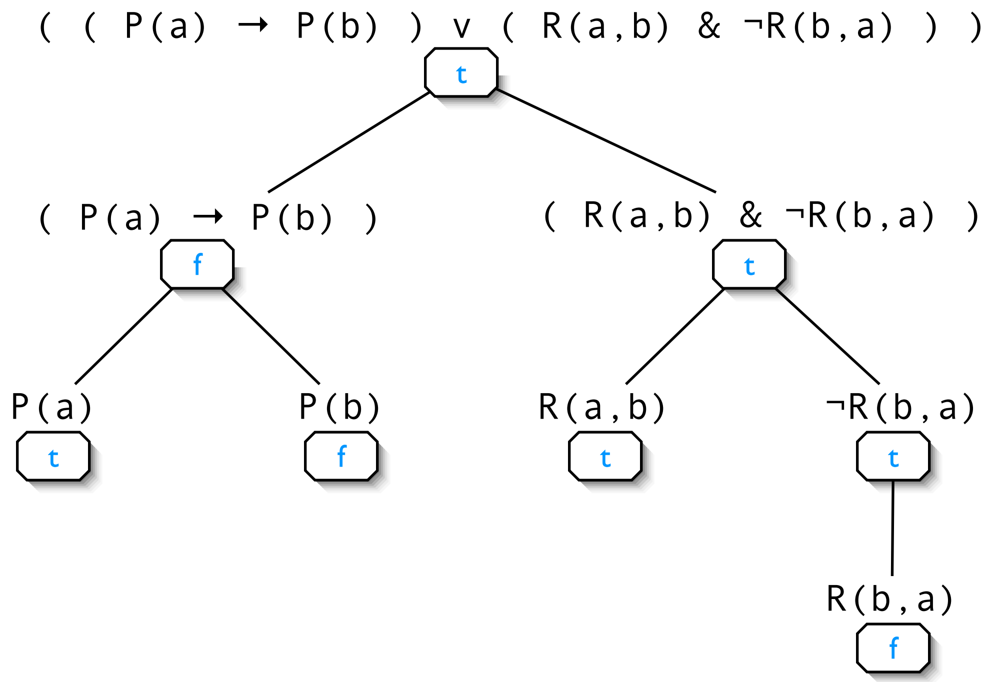
That's right.

f

A negation is true if its immediate subformula is false.


(R(a,b) & ¬R(b,a))
t
That's right.
f
A conjunction is true if both of its conjuncts are true.
(whole formula)
t
That's right.
f
A disjunction is true if either one of its disjuncts is true.

Solution:



Hint

Each hint should contain the following at the bottom, after specific hint content:

Click here to view the characteristic truth-tables for the connectives.

The link should be to the following file:

interpretationshint.gif

The hint to be displayed at a given stage is that for the first incomplete node, from bottom to top and left to right, i.e., the first in the following order not answered correctly when the hint is requested.

R(b,a)

An atomic formula is true on an interpretation just in case the ordered tuple of the interpretations of the terms in the formula is a member of the interpretation of the predicate.

The formula R(b,a) will be true on an interpretation $\mathfrak{I}$ if $\langle \mathfrak{I}(\mathrm{b}), \mathfrak{I}(\mathrm{a}) \rangle$ is in $\mathfrak{I}(\mathrm{R})$.

Since $\mathfrak{I}(\mathrm{b})$ is ***b***, $\mathfrak{I}(\mathrm{a})$ is ***a***, and $\langle$***b***,***a***$\rangle$ is not in $\mathfrak{I}(\mathrm{R})$, R(b,a) is false on the given interpretation.

P(a)

An atomic formula is true on an interpretation just in case the ordered tuple of the interpretations of the terms in the formula is a member of the interpretation of the predicate.

The formula P(a) will be true on an interpretation $\mathfrak{I}$ if $\langle \mathfrak{I}(\mathrm{a}) \rangle$ is in $\mathfrak{I}(\mathrm{P})$.

Since $\mathfrak{I}(\mathrm{a})$ is ***a***, and $\langle$***a***$\rangle$ is in $\mathfrak{I}(\mathrm{P})$, P(a) is true on the given interpretation.

P(b)

An atomic formula is true on an interpretation just in case the ordered tuple of the interpretations of the terms in the formula is a member of the interpretation of the predicate.

The formula P(b) will be true on an interpretation $\mathfrak{I}$ if $\langle \mathfrak{I}(b) \rangle$ is in $\mathfrak{I}(P)$.

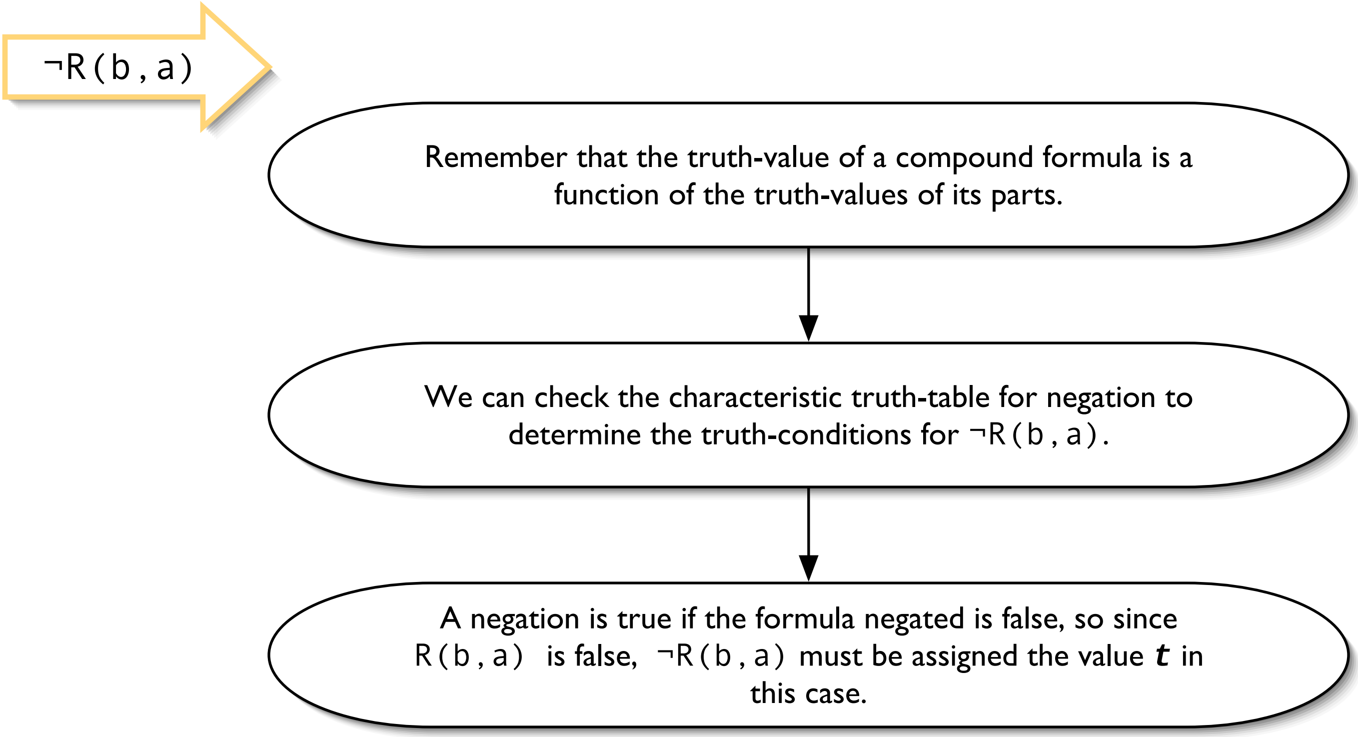
Since $\mathfrak{I}(b)$ is ***b***, and $\langle \boldsymbol{b} \rangle$ is not in $\mathfrak{I}(P)$, P(b) is false on the given interpretation.

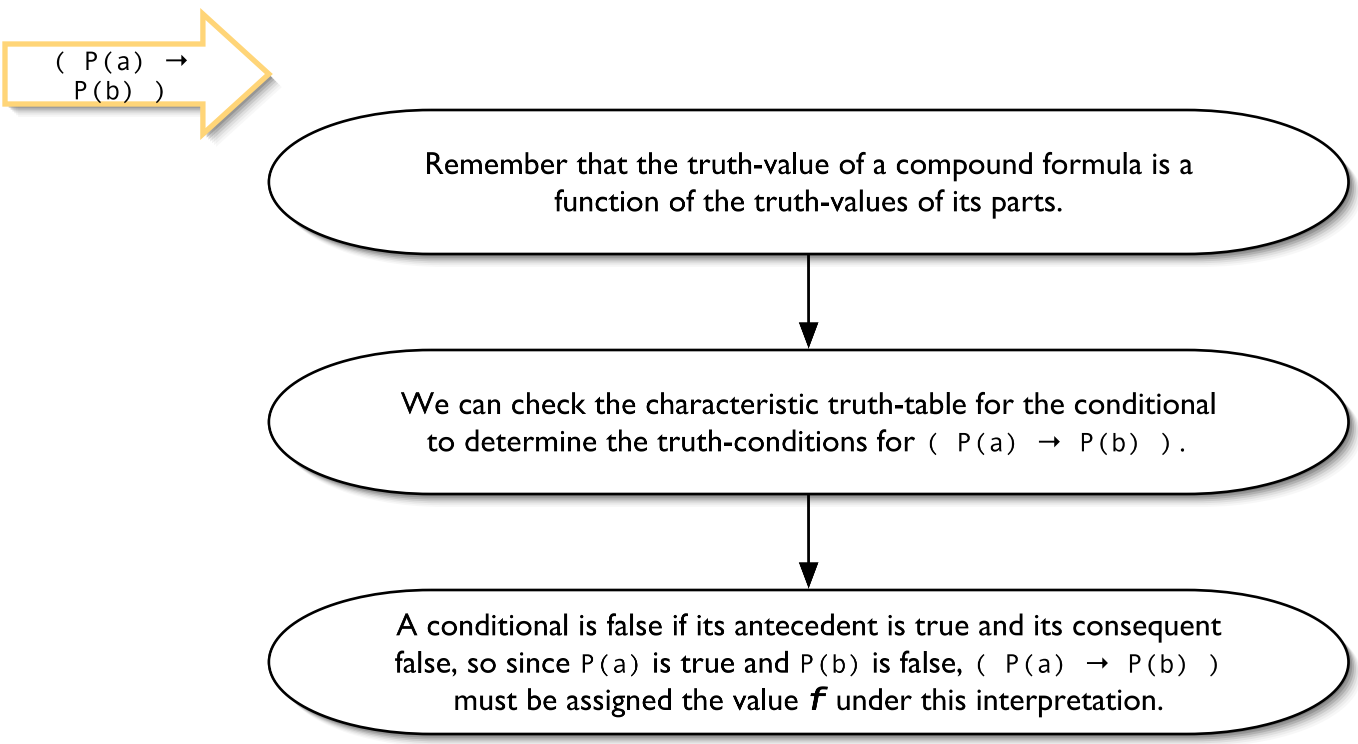
R(a,b)

An atomic formula is true on an interpretation just in case the ordered tuple of the interpretations of the terms in the formula is a member of the interpretation of the predicate.

The formula R(a,b) will be true on an interpretation $\mathfrak{I}$ if $\langle \mathfrak{I}(a), \mathfrak{I}(b) \rangle$ is in $\mathfrak{I}(R)$.

Since $\mathfrak{I}(a)$ is ***a***, $\mathfrak{I}(b)$ is ***b***, and $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$ is in $\mathfrak{I}(R)$, R(b,a) is true on the given interpretation.


¬R(b,a)
Remember that the truth-value of a compound formula is a function of the truth-values of its parts.
We can check the characteristic truth-table for negation to determine the truth-conditions for ¬R(b,a).
A negation is true if the formula negated is false, so since R(b,a) is false, ¬R(b,a) must be assigned the value ***t*** in this case.


(P(a) → P(b))
Remember that the truth-value of a compound formula is a function of the truth-values of its parts.
We can check the characteristic truth-table for the conditional to determine the truth-conditions for (P(a) → P(b)).
A conditional is false if its antecedent is true and its consequent false, so since P(a) is true and P(b) is false, (P(a) → P(b)) must be assigned the value ***f*** under this interpretation.

(R(a,b) & ¬R(b,a))

Remember that the truth-value of a compound formula is a function of the truth-values of its parts.

We can check the characteristic truth-table for conjunction to determine the truth-conditions for (R(a,b) & ¬R(b,a)).

A conjunction is true when both conjuncts are true, so under this interpretation, (R(a,b) & ¬R(b,a)) should be assigned the value ***t***.

(top node)

Remember that the truth-value of a compound formula is a function of the truth-values of its parts.

We can check the characteristic truth-table for disjunction to determine the truth-conditions for ((P(a) → P(b)) v (R(a,b) & ¬R(b,a))).

Since a disjunction is true if either of its disjuncts is true, we know that this formula should be assigned the value ***t*** on this intrepretation.