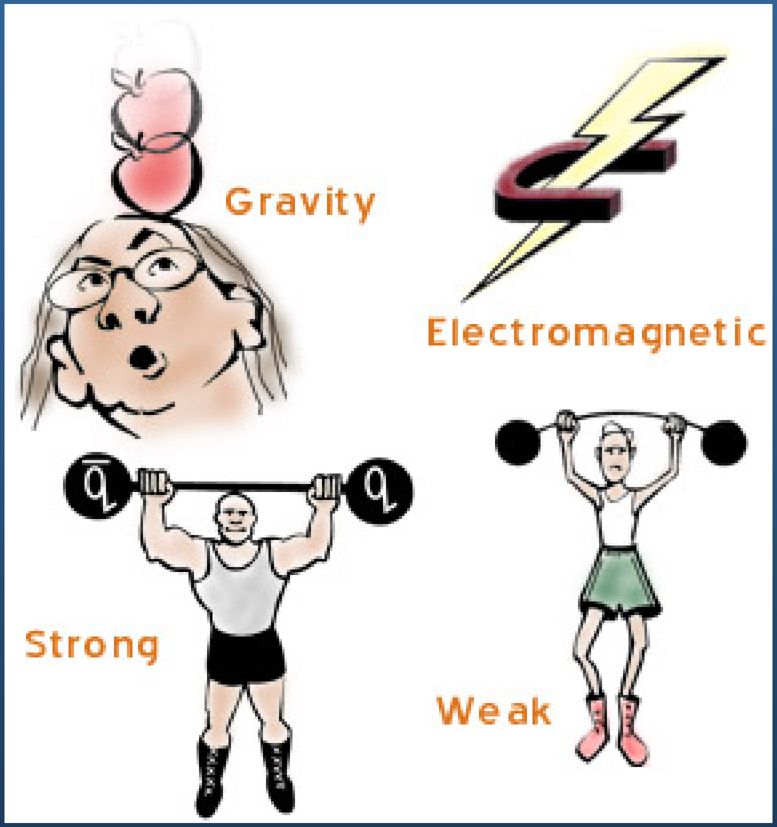
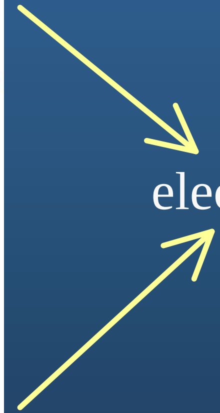
Quarks, gluons, and computers:
towards an understanding of the strong force

Colin Morningstar
Carnegie Mellon University
September 10, 2001

Four fundamental forces of nature





electricity
+ magnetism

electroweak

- Focus on the strong force in this talk

Outline

- Brief historical survey of the strong interactions
  - atom → QCD (quarks and gluons)
- Overview of lattice QCD (enter the computer)
  - new branch of particle physics
  - current state of the art
- Quark confinement
- Constituent quark model
- Gluonic excitations
  - glueballs
  - hybrid mesons
- Future work

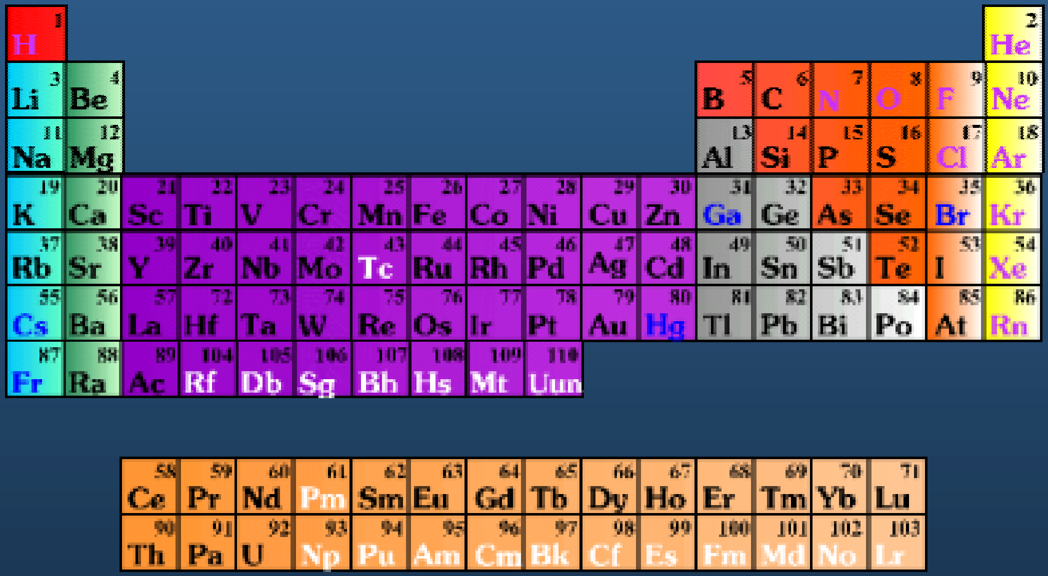
Age-old questions

- What is the world made of ?
- What holds it together ?

The thinker (1880)
by Auguste Rodin

The periodic table of the elements

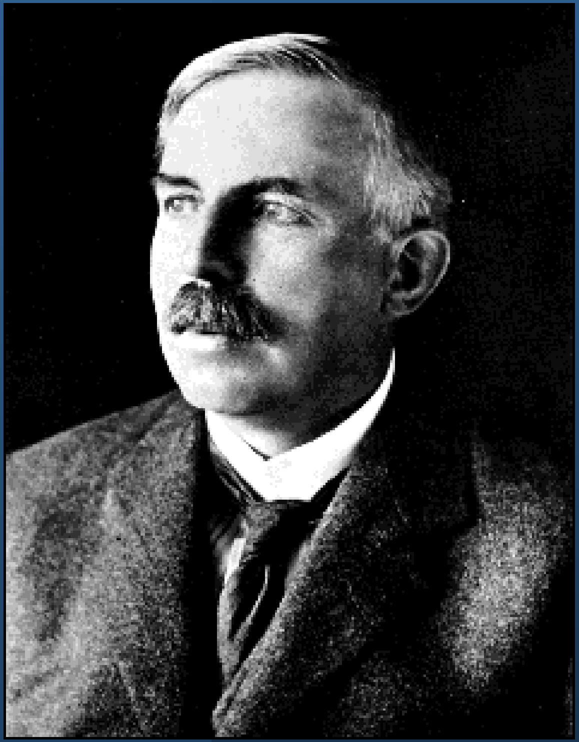
- Dimitri Mendeleev’s periodic table of the elements 1869
  - great breakthrough towards answering these questions
  - pattern recognition ⟹ substructure

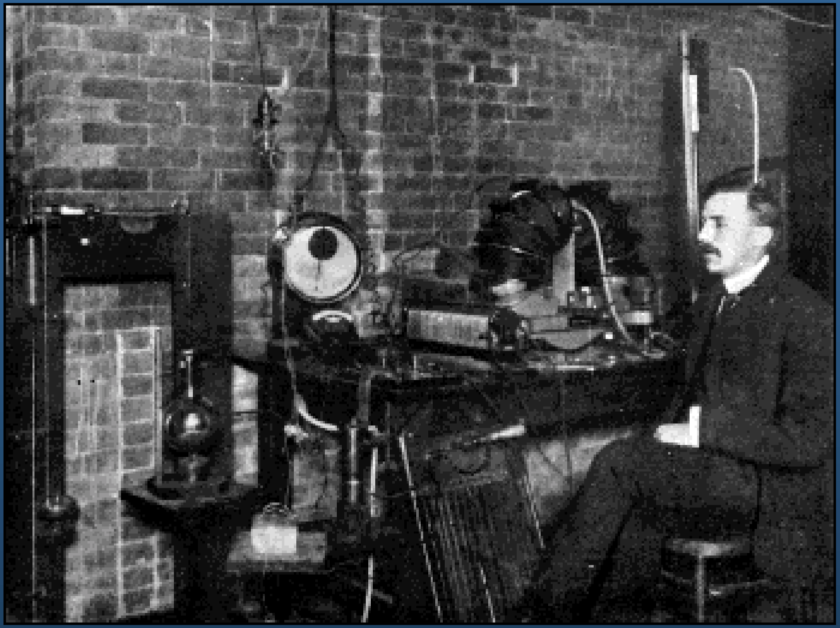


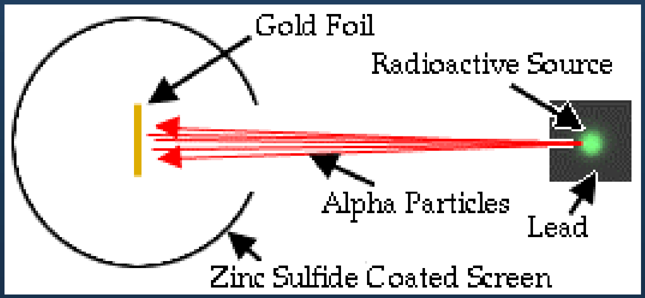
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Uun								
		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
		90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

Rutherford’s scattering experiments

- experiments of Ernest Rutherford, Geiger, and Marsden (1909)

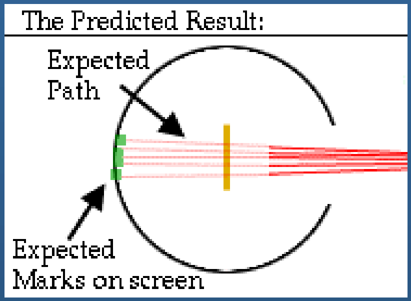




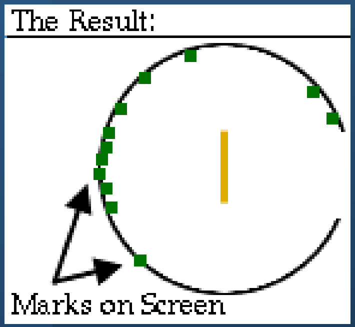


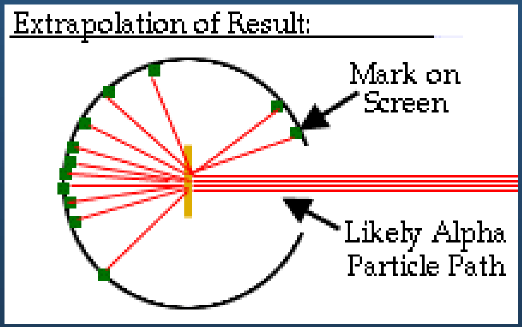
Large deflections

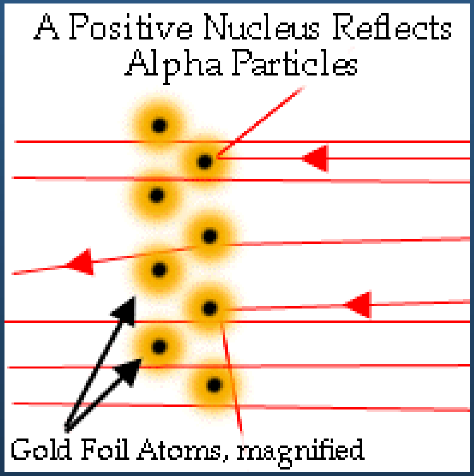
- expectation from Thomson’s “plum pudding” model of the atom



- found large deflections on rare occasions





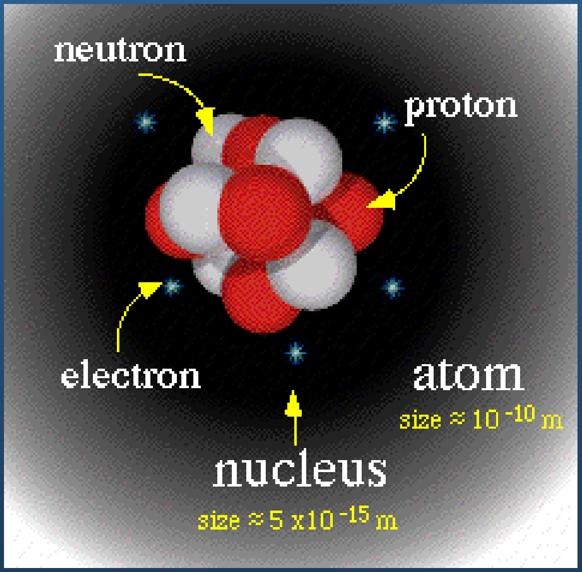


- atoms must have small but massive cores
- birth of the nucleus! (1911)

Modern theory of the atom

- regularities in Mendeleev’s periodic table
- Rutherford’s α–particle scattering experiments

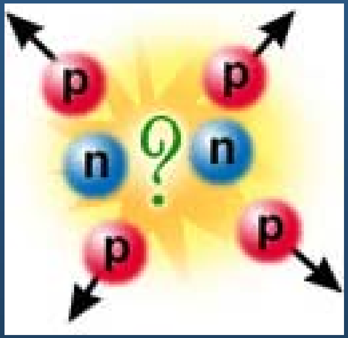
→ help fashion the modern theory of the atom



- atom = massive nucleus (protons and neutrons) surrounded by cloud of light electrons

What holds the nucleus together?

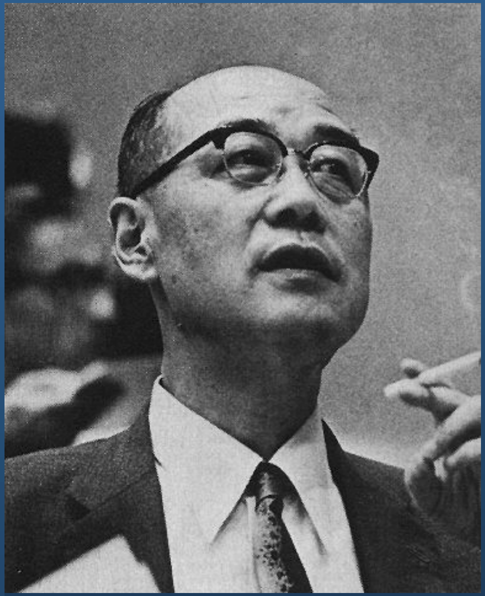
- protons: positive electric charge
- neutrons: no charge
- like charges *repel*
- what holds the nucleus together? → new force!

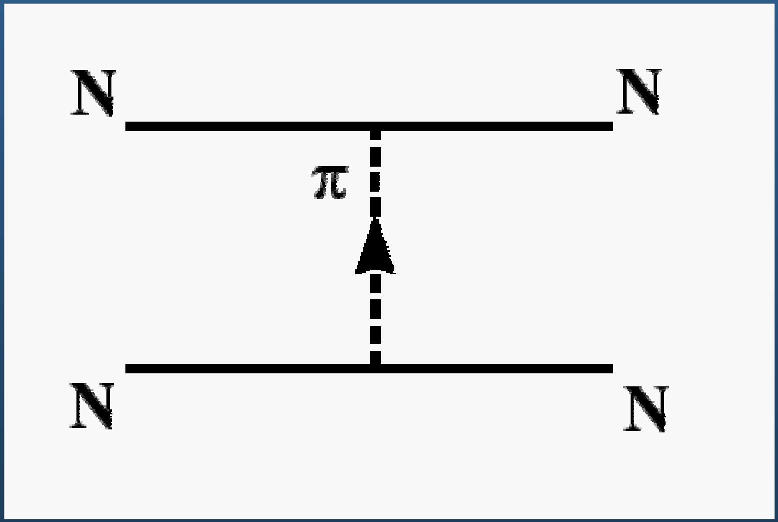


- new force must be ***strong*** to overcome electrostatic repulsion, but short-ranged

Yukawa’s strong force

- Hideki Yukawa puts forward his theory of the nuclear force (1935)
  - ❑ mediated by spinless exchange particle called the π meson
  - ❑ mass of π meson about 250 times that of the electron



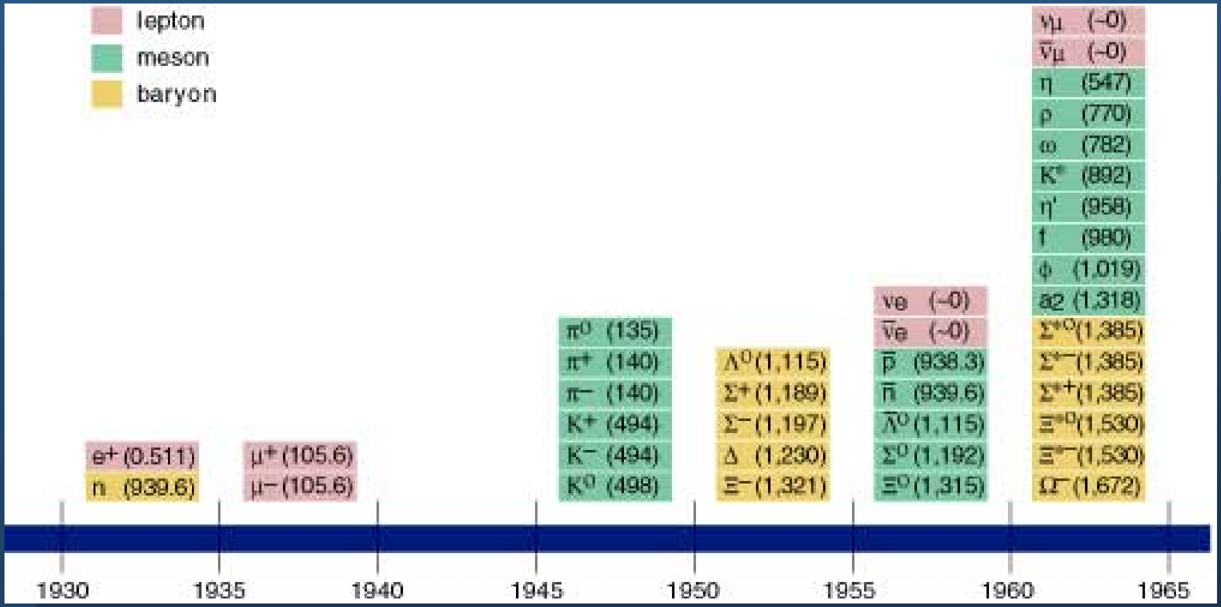


  - ❑ π meson eventually discovered in 1947
    (Lattes, Muirhead, Occhialini, Powell)

Particle zoo

- experiments reveal proton and neutron not alone
  - protons and neutrons lightest particles in a large spectrum of strongly-interacting fermions called *baryons*
  - pions lightest member of equally numerous sequence of strongly-interacting bosons called *mesons*

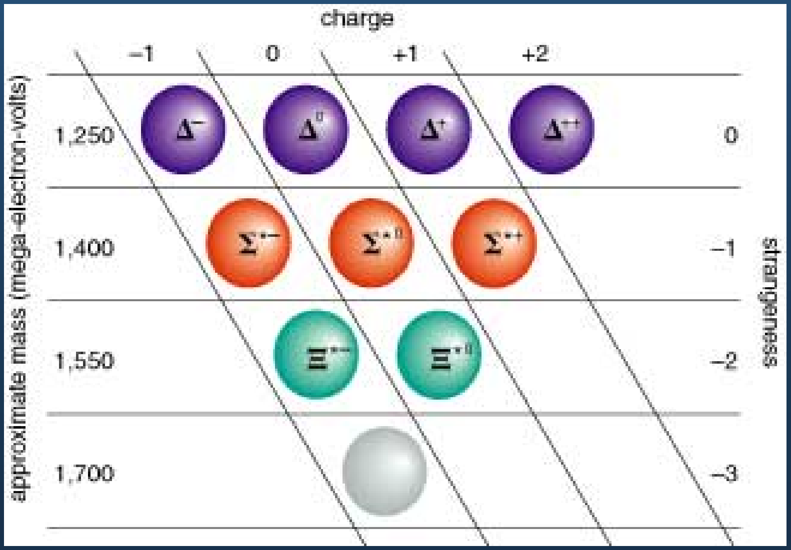
Particles discovered between 1898 and 1964



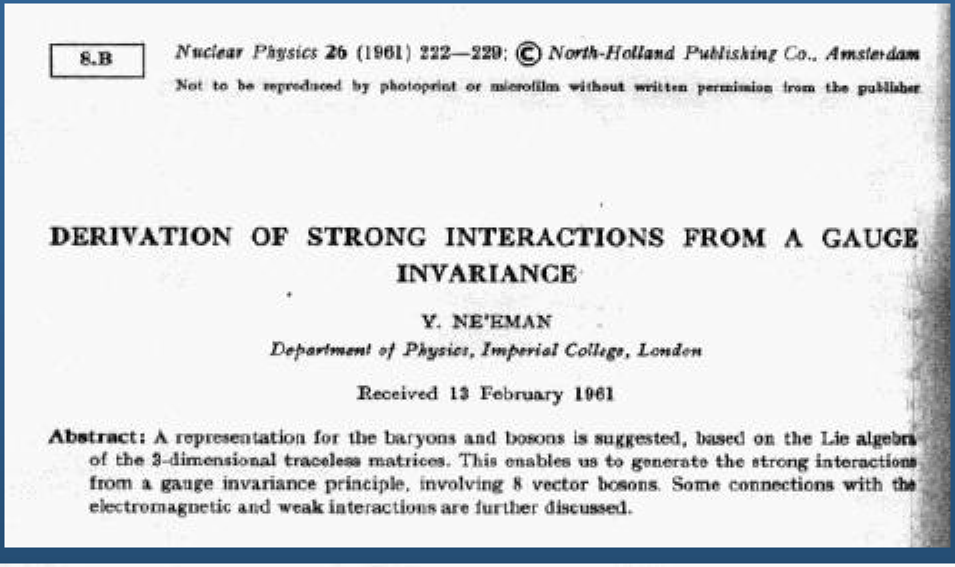
many more…

Eightfold way

- **pattern** recognition in the observed hadron spectrum 1961
  (reminiscent of Mendeleev's periodic table)
  - ❑ Murray Gell-Mann → Buddhist eightfold path to enlightenment
  - ❑ Yuval Ne'eman
- conference at CERN in 1962
  - ❑ Gell-Mann, Ne'eman learn of discovery of two new baryons Ξ^{*-}, Ξ^{*0}
  - ❑ recognize these nearly complete a group of ten related particles
  - ❑ G-M predicts new particle
  - ❑ Ω^- found BNL, CERN 1964



Two key papers on hadron symmetries



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DERIVATION OF STRONG INTERACTIONS FROM A GAUGE INVARIANCE

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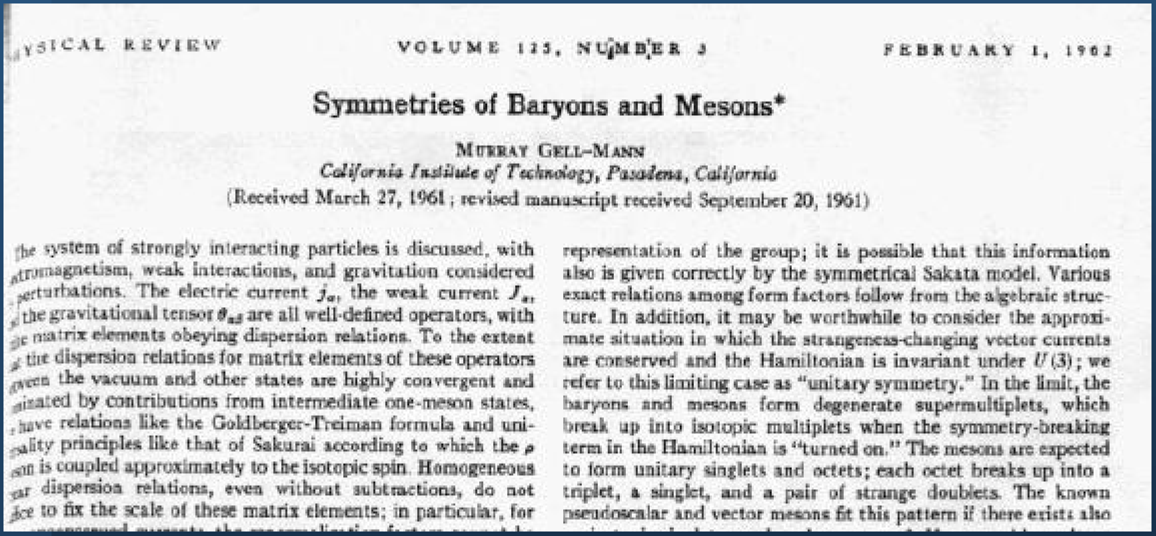
Received 13 February 1961

Abstract: A representation for the baryons and bosons is suggested, based on the Lie algebra of the 3-dimensional traceless matrices. This enables us to generate the strong interactions from a gauge invariance principle, involving 8 vector bosons. Some connections with the electromagnetic and weak interactions are further discussed.



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Symmetries of Baryons and Mesons*

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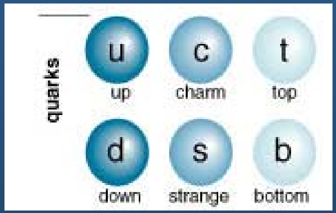
(Received March 27, 1961; revised manuscript received September 20, 1961)

The system of strongly interacting particles is discussed, with tromagnetism, weak interactions, and gravitation considered perturbations. The electric current j_α, the weak current J_α, the gravitational tensor $\theta_{\alpha\beta}$ are all well-defined operators, with matrix elements obeying dispersion relations. To the extent the dispersion relations for matrix elements of these operators ween the vacuum and other states are highly convergent and inated by contributions from intermediate one-meson states, have relations like the Goldberger-Treiman formula and uni-ality principles like that of Sakurai according to which the ρ on is coupled approximately to the isotopic spin. Homogeneous ar dispersion relations, even without subtractions, do not ice to fix the scale of these matrix elements; in particular, for representation of the group; it is possible that this information also is given correctly by the symmetrical Sakata model. Various exact relations among form factors follow from the algebraic structure. In addition, it may be worthwhile to consider the approximate situation in which the strangeness-changing vector currents are conserved and the Hamiltonian is invariant under $U(3)$; we refer to this limiting case as "unitary symmetry." In the limit, the baryons and mesons form degenerate supermultiplets, which break up into isotopic multiplets when the symmetry-breaking term in the Hamiltonian is "turned on." The mesons are expected to form unitary singlets and octets; each octet breaks up into a triplet, a singlet, and a pair of strange doublets. The known pseudoscalar and vector mesons fit this pattern if there exists also

Quark hypothesis

- pattern suggests substructure
- Murray Gell-Mann → **quarks**
  - James Joyce, *Finnegan's Wake*
- George Zweig → aces
  - unpublished (CERN preprint)

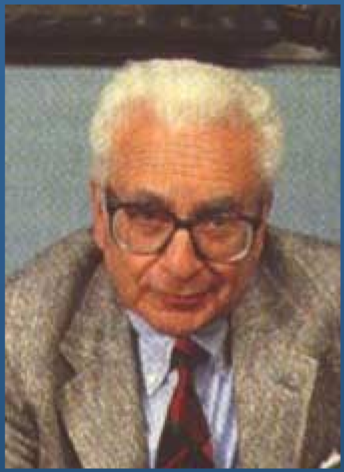


+2/3*e*

-1/3*e*

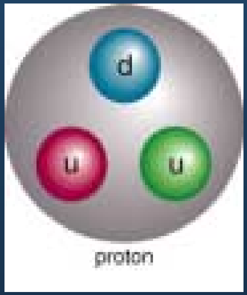
- quarks:
  - <u>fractional</u> electric charge!
  - spin 1/2
  - come in **flavors** (up, down, strange)

    three more flavors since found
- baryons = three quarks
- mesons = quark-antiquark pair

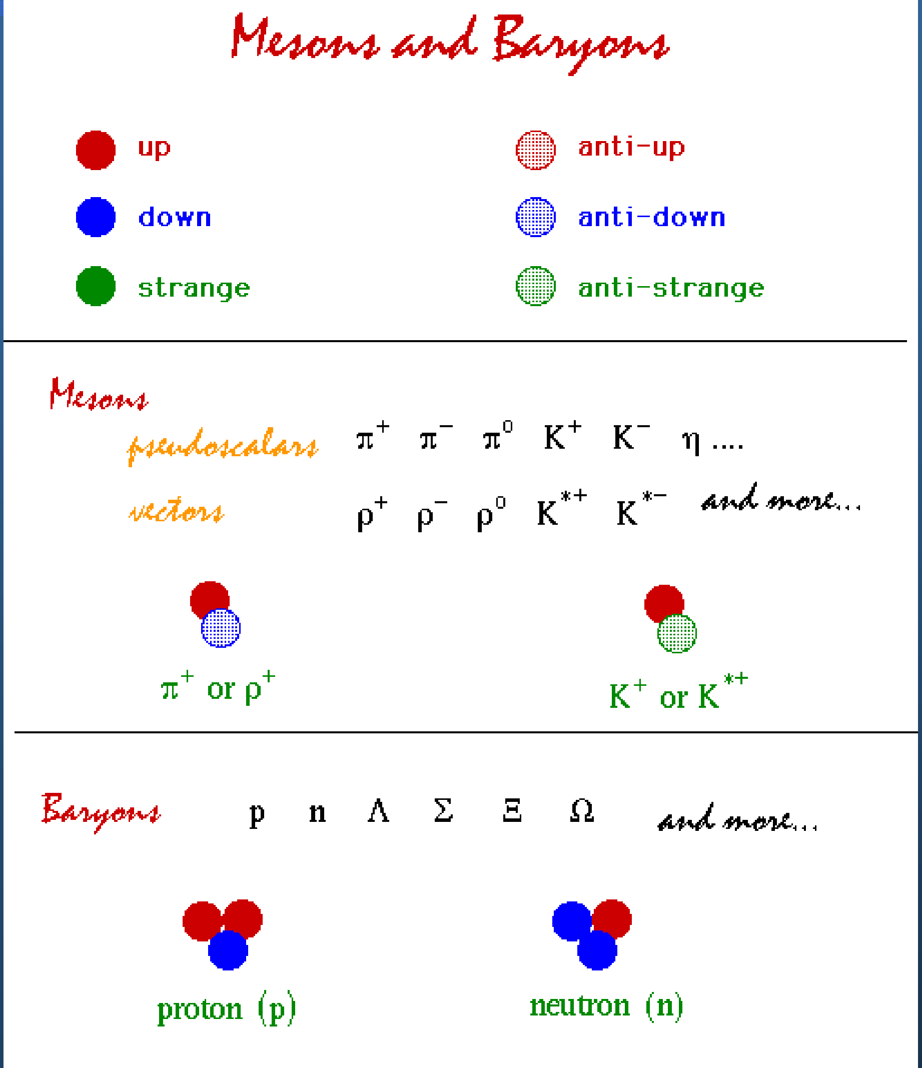


Gell-Mann

Zweig



Mesons and baryons



Mesons and Baryons

● up	▦ anti-up
● down	▦ anti-down
● strange	▦ anti-strange

Mesons

pseudoscalars π^+ π^- π^0 K^+ K^- η ....

vectors ρ^+ ρ^- ρ^0 K^{*+} K^{*-} and more...

π^+ or ρ^+

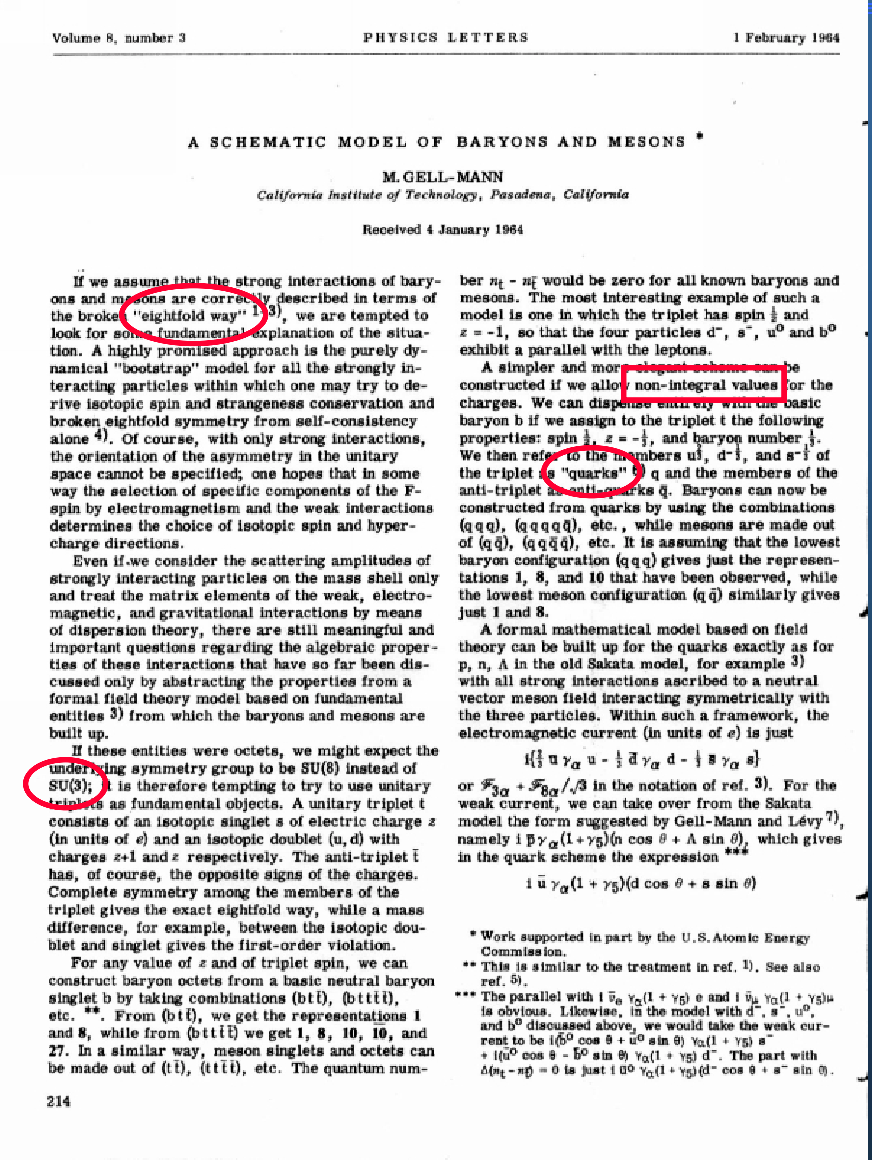
K^+ or K^{*+}

Baryons p n Λ Σ Ξ Ω and more...

proton (p)

neutron (n)

Gell-Mann’s paper



Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

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Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means of dispersion theory, there are still meaningful and important questions regarding the algebraic properties of these interactions that have so far been discussed only by abstracting the properties from a formal field theory model based on fundamental entities [3] from which the baryons and mesons are built up.

If these entities were octets, we might expect the underlying symmetry group to be SU(8) instead of SU(3); it is therefore tempting to try to use unitary triplets as fundamental objects. A unitary triplet t consists of an isotopic singlet s of electric charge z (in units of e) and an isotopic doublet (u, d) with charges z+1 and z respectively. The anti-triplet $\bar{t}$ has, of course, the opposite signs of the charges. Complete symmetry among the members of the triplet gives the exact eightfold way, while a mass difference, for example, between the isotopic doublet and singlet gives the first-order violation.

For any value of z and of triplet spin, we can construct baryon octets from a basic neutral baryon singlet b by taking combinations $(b t \bar{t})$, $(b t t \bar{t} \bar{t})$, etc. **. From $(b t \bar{t})$, we get the representations **1** and **8**, while from $(b t t \bar{t} \bar{t})$ we get **1**, **8**, **10**, $\overline{\mathbf{10}}$, and **27**. In a similar way, meson singlets and octets can be made out of $(t \bar{t})$, $(t t \bar{t} \bar{t})$, etc. The quantum number $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q q q)$, $(q q q q \bar{q})$, etc., while mesons are made out of $(q \bar{q})$, $(q q \bar{q} \bar{q})$, etc. It is assuming that the lowest baryon configuration $(q q q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q \bar{q})$ similarly gives just **1** and **8**.

A formal mathematical model based on field theory can be built up for the quarks exactly as for p, n, Λ in the old Sakata model, for example [3] with all strong interactions ascribed to a neutral vector meson field interacting symmetrically with the three particles. Within such a framework, the electromagnetic current (in units of e) is just

$$i\{\tfrac{2}{3}\bar{u}\gamma_\alpha u - \tfrac{1}{3}\bar{d}\gamma_\alpha d - \tfrac{1}{3}\bar{s}\gamma_\alpha s\}$$

or $\mathscr{F}_{3\alpha} + \mathscr{F}_{8\alpha}/\sqrt{3}$ in the notation of ref. [3]. For the weak current, we can take over from the Sakata model the form suggested by Gell-Mann and Lévy [7], namely $i\bar{p}\gamma_\alpha(1+\gamma_5)(n\cos\theta + \Lambda\sin\theta)$, which gives in the quark scheme the expression ***

$$i\bar{u}\gamma_\alpha(1+\gamma_5)(d\cos\theta + s\sin\theta)$$

---

* Work supported in part by the U.S. Atomic Energy Commission.

** This is similar to the treatment in ref. [1]. See also ref. [5].

*** The parallel with $i\bar{\nu}_e\gamma_\alpha(1+\gamma_5)e$ and $i\bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\mu$ is obvious. Likewise, in the model with d^-, s^-, u^0, and b^0 discussed above, we would take the weak current to be $i(\bar{b}^0\cos\theta + \bar{u}^0\sin\theta)\gamma_\alpha(1+\gamma_5)s^- + i(\bar{u}^0\cos\theta - \bar{b}^0\sin\theta)\gamma_\alpha(1+\gamma_5)d^-$. The part with $\Delta(n_t - n_{\bar{t}}) = 0$ is just $i\bar{u}^0\gamma_\alpha(1+\gamma_5)(d^-\cos\theta + s^-\sin\theta)$.

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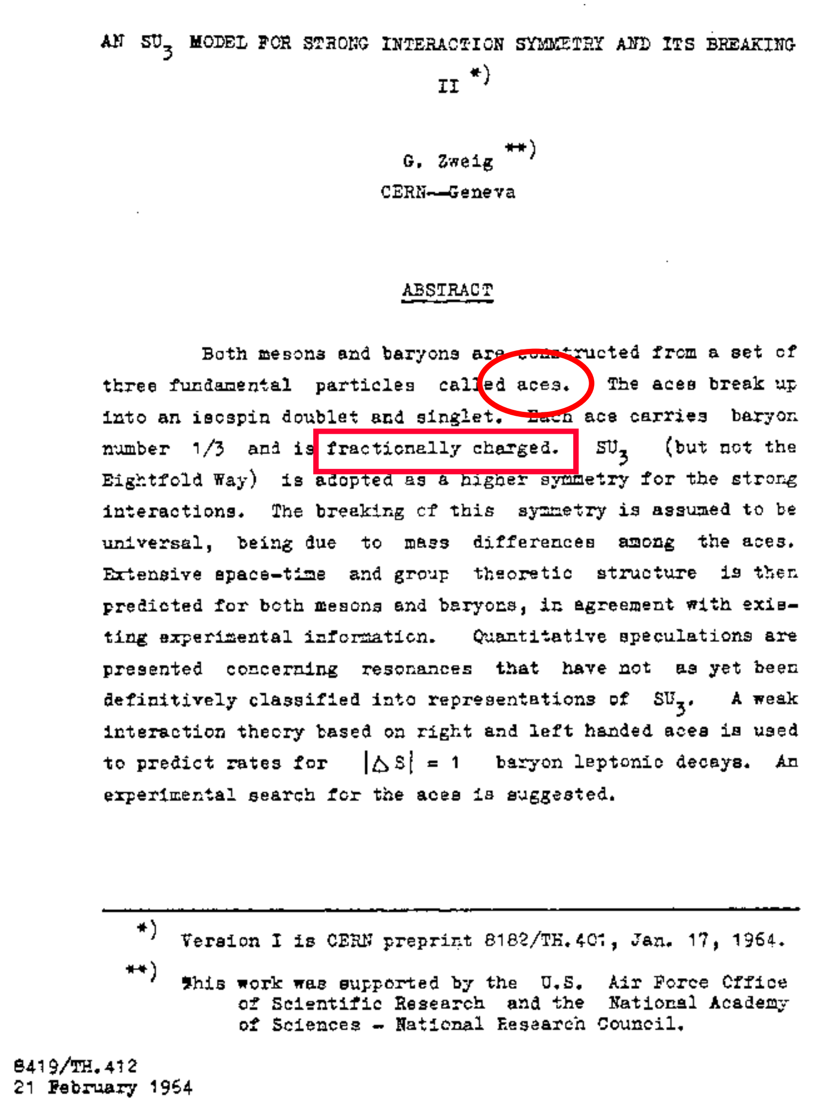
- quark hypothesis brought order to the particle zoo
- fractional electric charge never observed
- Gell-Mann initially considered quarks as purely mathematical objects
- but on last page of paper:

“It is fun to speculate about the way quarks would behave if they were physical particles of finite mass (instead of purely mathematical entities…).”

Zweig’s paper



AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig **)

CERN---Geneva

ABSTRACT

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number 1/3 and is fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. Quantitative speculations are presented concerning resonances that have not as yet been definitively classified into representations of SU_3. A weak interaction theory based on right and left handed aces is used to predict rates for $|\Delta S| = 1$ baryon leptonic decays. An experimental search for the aces is suggested.

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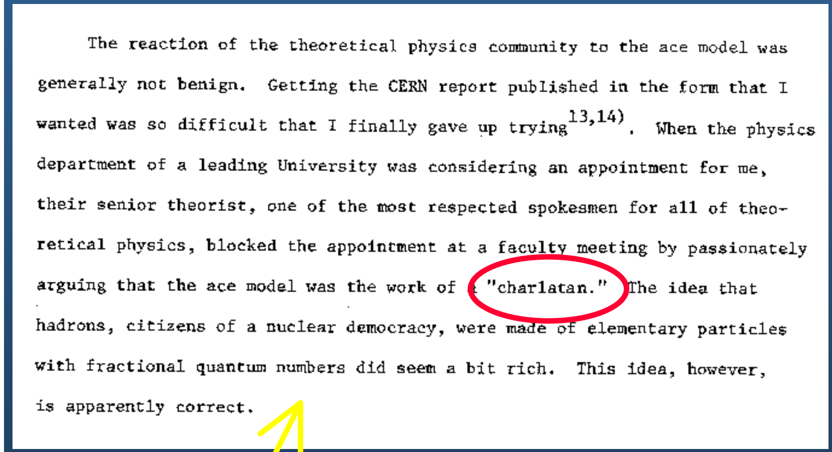
*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

**) This work was supported by the U.S. Air Force Office of Scientific Research and the National Academy of Sciences - National Research Council.

8419/TH.412
21 February 1964

- graduate student at CalTech
- CERN preprint

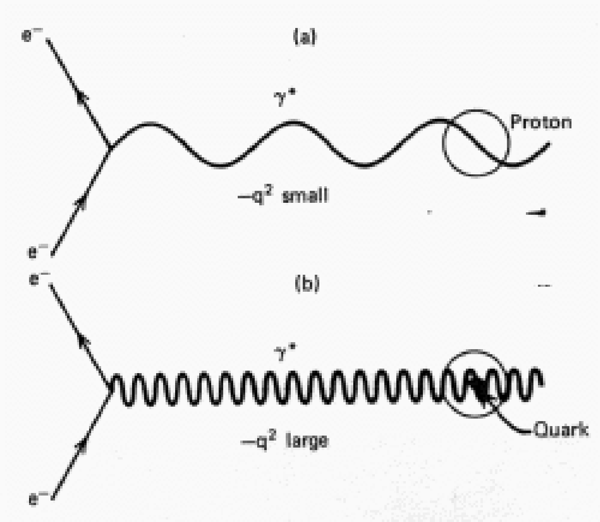


The reaction of the theoretical physics community to the ace model was generally not benign. Getting the CERN report published in the form that I wanted was so difficult that I finally gave up trying[13,14]. When the physics department of a leading University was considering an appointment for me, their senior theorist, one of the most respected spokesmen for all of theoretical physics, blocked the appointment at a faculty meeting by passionately arguing that the ace model was the work of a "charlatan." The idea that hadrons, citizens of a nuclear democracy, were made of elementary particles with fractional quantum numbers did seem a bit rich. This idea, however, is apparently correct.

- from talk at Baryons 1980 in Toronto

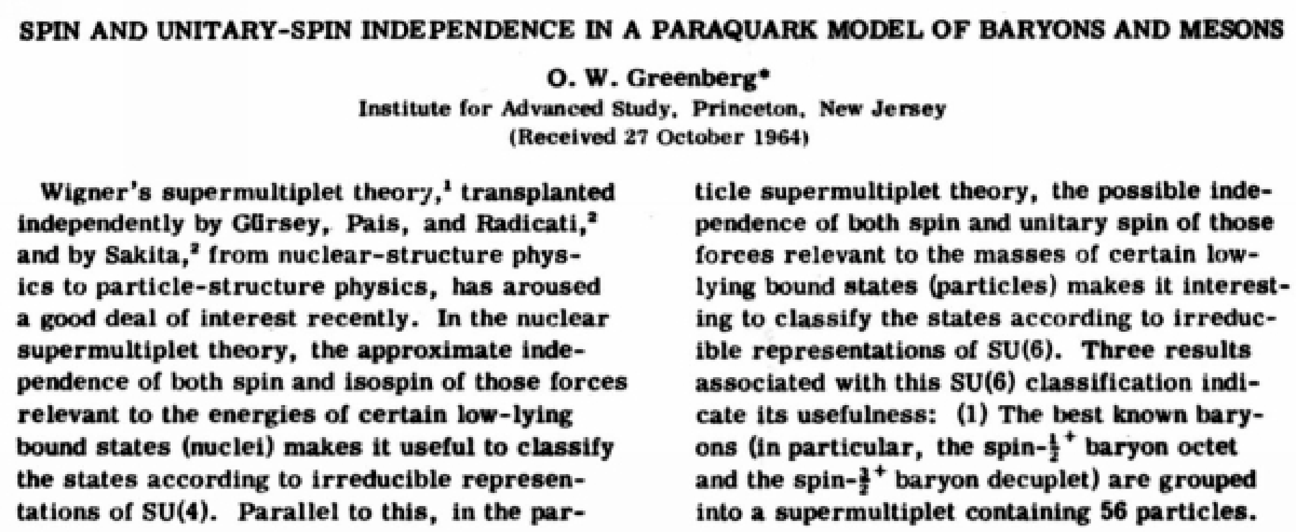
Discovery of the quark

- watershed experiments at Stanford Linear Accelerator Center (1968)
  - modern rendition of Rutherford scattering experiments
  - scattered electrons off protons
  - small-wavelength photons to resolve quarks inside the proton
- scattering consistent with quark hypothesis!
- further probing provides evidence that proton constituents have the expected fractional electric charges of quarks
- asymptotic freedom
  - interactions between quarks *weakens* at very short distances



New quantum number

- nagging problem with quark hypothesis:
  - certain baryons seemed to violate the Pauli exclusion principle!
- example: the $\Delta^{++} = uuu$ baryon with spin $J = \frac{3}{2}$
  - three identical fermion *u* quarks in symmetric state
    - → forbidden by Fermi-Dirac statistics
- resolution by Greenberg 1964 (Han and Nambu 1965)
  - quarks must have a *new quantum number*



SPIN AND UNITARY-SPIN INDEPENDENCE IN A PARAQUARK MODEL OF BARYONS AND MESONS

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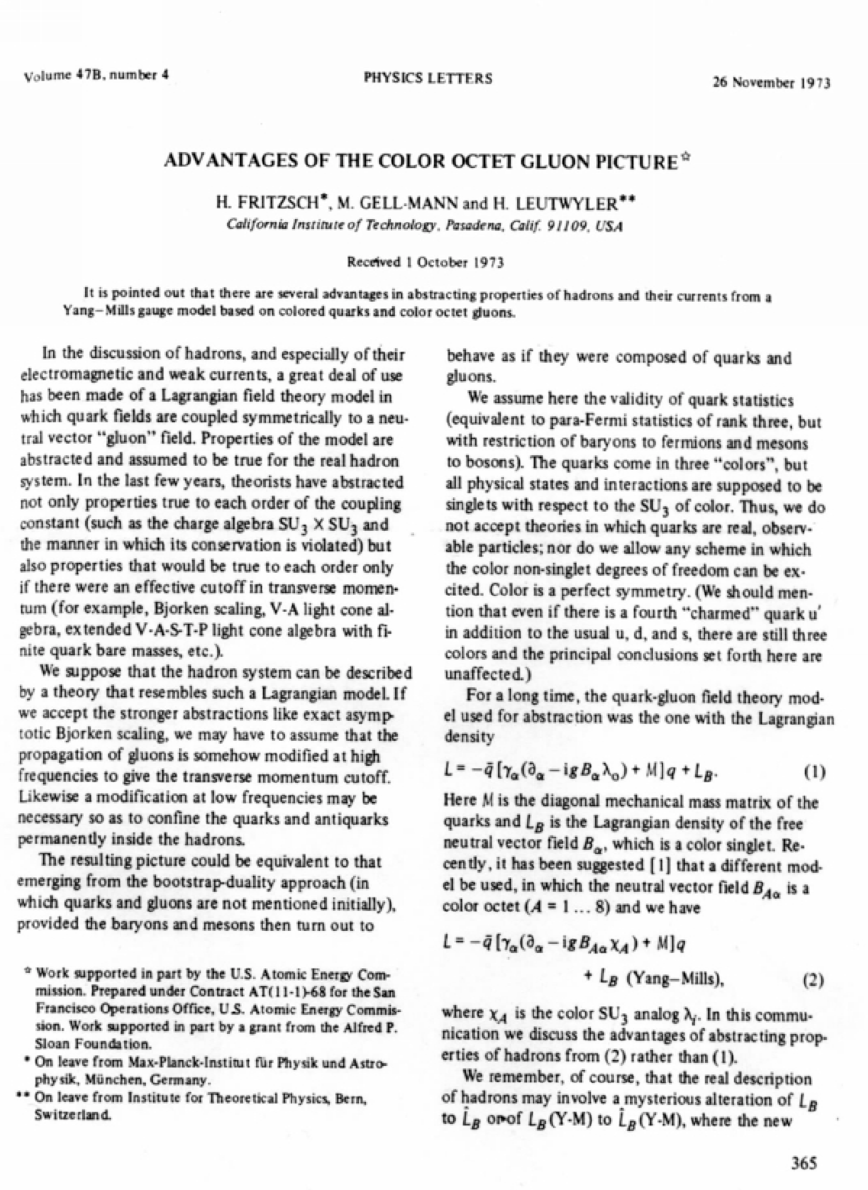
(Received 27 October 1964)

Wigner's supermultiplet theory,[1] transplanted independently by Gürsey, Pais, and Radicati,[2] and by Sakita,[3] from nuclear-structure physics to particle-structure physics, has aroused a good deal of interest recently. In the nuclear supermultiplet theory, the approximate independence of both spin and isospin of those forces relevant to the energies of certain low-lying bound states (nuclei) makes it useful to classify the states according to irreducible representations of SU(4). Parallel to this, in the particle supermultiplet theory, the possible independence of both spin and unitary spin of those forces relevant to the masses of certain low-lying bound states (particles) makes it interesting to classify the states according to irreducible representations of SU(6). Three results associated with this SU(6) classification indicate its usefulness: (1) The best known baryons (in particular, the spin-$\frac{1}{2}^+$ baryon octet and the spin-$\frac{3}{2}^+$ baryon decuplet) are grouped into a supermultiplet containing 56 particles.

PRL 13, 598

Origin of “color”

- Gell-Mann coins the term “color” for this new quantum number
- three different colors are needed: red, green, blue
- birth of *quantum chromodynamics (QCD)*
  - Fritzsch, Gell-Mann, Leutwyler 1973
  - based on an SU(3) local gauge symmetry (Yang-Mills)
  - quarks interact by the exchange of colored **gluons**



Volume 47B, number 4 PHYSICS LETTERS 26 November 1973

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE ☆

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Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

In the discussion of hadrons, and especially of their electromagnetic and weak currents, a great deal of use has been made of a Lagrangian field theory model in which quark fields are coupled symmetrically to a neutral vector “gluon” field. Properties of the model are abstracted and assumed to be true for the real hadron system. In the last few years, theorists have abstracted not only properties true to each order of the coupling constant (such as the charge algebra $SU_3 \times SU_3$ and the manner in which its conservation is violated) but also properties that would be true to each order only if there were an effective cutoff in transverse momentum (for example, Bjorken scaling, V-A light cone algebra, extended V-A-S-T-P light cone algebra with finite quark bare masses, etc.).

We suppose that the hadron system can be described by a theory that resembles such a Lagrangian model. If we accept the stronger abstractions like exact asymptotic Bjorken scaling, we may have to assume that the propagation of gluons is somehow modified at high frequencies to give the transverse momentum cutoff. Likewise a modification at low frequencies may be necessary so as to confine the quarks and antiquarks permanently inside the hadrons.

The resulting picture could be equivalent to that emerging from the bootstrap-duality approach (in which quarks and gluons are not mentioned initially), provided the baryons and mesons then turn out to behave as if they were composed of quarks and gluons.

We assume here the validity of quark statistics (equivalent to para-Fermi statistics of rank three, but with restriction of baryons to fermions and mesons to bosons). The quarks come in three “colors”, but all physical states and interactions are supposed to be singlets with respect to the SU_3 of color. Thus, we do not accept theories in which quarks are real, observable particles; nor do we allow any scheme in which the color non-singlet degrees of freedom can be excited. Color is a perfect symmetry. (We should mention that even if there is a fourth “charmed” quark u′ in addition to the usual u, d, and s, there are still three colors and the principal conclusions set forth here are unaffected.)

For a long time, the quark-gluon field theory model used for abstraction was the one with the Lagrangian density

$$L = -\bar{q}\,[\gamma_\alpha(\partial_\alpha - igB_\alpha\lambda_0) + M]\,q + L_B. \qquad (1)$$

Here M is the diagonal mechanical mass matrix of the quarks and L_B is the Lagrangian density of the free neutral vector field B_α, which is a color singlet. Recently, it has been suggested [1] that a different model be used, in which the neutral vector field $B_{A\alpha}$ is a color octet ($A = 1 \ldots 8$) and we have

$$L = -\bar{q}\,[\gamma_\alpha(\partial_\alpha - igB_{A\alpha}\chi_A) + M]\,q + L_B \ \text{(Yang–Mills)}, \qquad (2)$$

where χ_A is the color SU_3 analog λ_i. In this communication we discuss the advantages of abstracting properties of hadrons from (2) rather than (1).

We remember, of course, that the real description of hadrons may involve a mysterious alteration of L_B to $\dot{L}_B$ or of L_B(Y-M) to $\dot{L}_B$(Y-M), where the new

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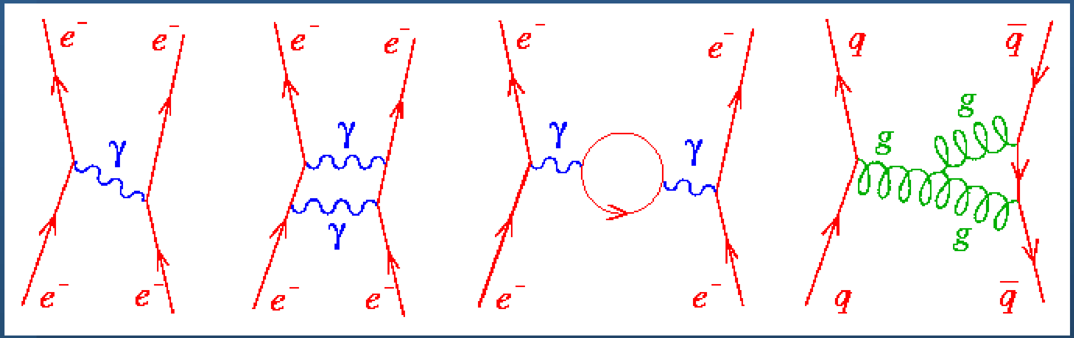
☆ Work supported in part by the U.S. Atomic Energy Commission. Prepared under Contract AT(11-1)-68 for the San Francisco Operations Office, U.S. Atomic Energy Commission. Work supported in part by a grant from the Alfred P. Sloan Foundation.

* On leave from Max-Planck-Institut für Physik und Astrophysik, München, Germany.

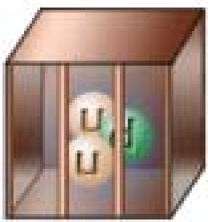
** On leave from Institute for Theoretical Physics, Bern, Switzerland.

Quantum chromodynamics

- resemblance to quantum electrodynamics
  - quarks (3 colors) ←→ electrons
  - gluons (8 color varieties) ←→ photons (massless spin-1)
- major difference: gluon has color charge (photon neutral)



- seemingly simple difference
  - physical content of QCD completely *unlike* that of QED
- physical states (hadrons) *colorless*
  - color confinement



Charmonium

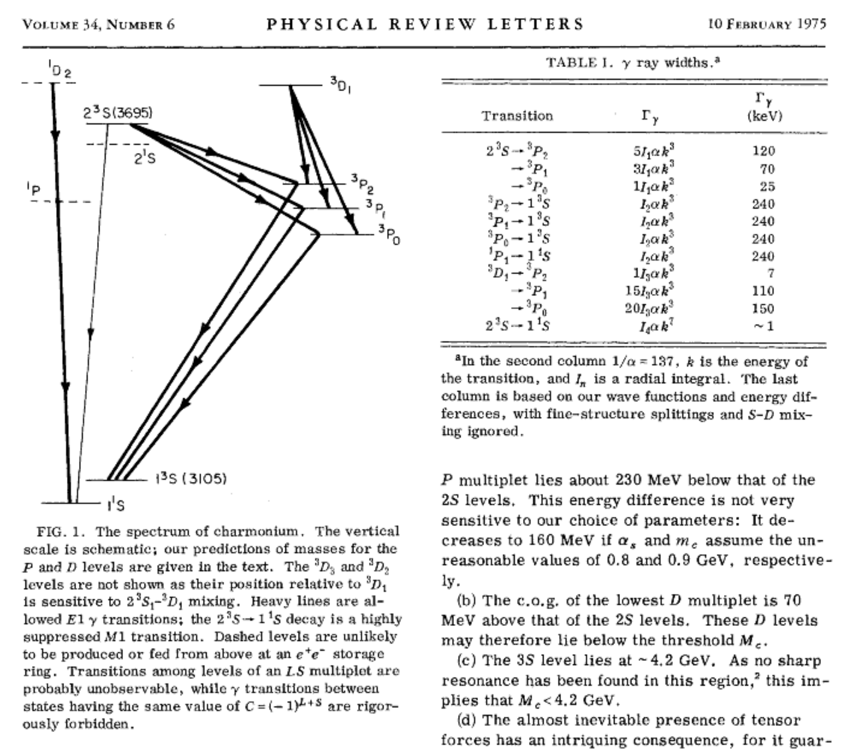
- discovery of *J*/Ψ in November 1974 (BNL, SLAC)
- interpretation as bound state of new flavor of quark called *charm*

- charm quark rather heavy
  - nonrelativistic analysis
- Cornell potential
  - Coulomb + linear

short-range
asymptotic freedom

long-range
confinement

  - reproduces spectrum



VOLUME 34, NUMBER 6 PHYSICAL REVIEW LETTERS 10 FEBRUARY 1975

FIG. 1. The spectrum of charmonium. The vertical scale is schematic; our predictions of masses for the *P* and *D* levels are given in the text. The $^3D_3$ and $^3D_2$ levels are not shown as their position relative to $^3D_1$ is sensitive to $2\,^3S_1$–$^3D_1$ mixing. Heavy lines are allowed $E1$ γ transitions; the $2\,^3S \to 1\,^1S$ decay is a highly suppressed $M1$ transition. Dashed levels are unlikely to be produced or fed from above at an e^+e^- storage ring. Transitions among levels of an *LS* multiplet are probably unobservable, while γ transitions between states having the same value of $C=(-1)^{L+S}$ are rigorously forbidden.

TABLE I. γ ray widths.[a]

Transition	Γ_γ	Γ_γ (keV)
$2\,^3S \to {}^3P_2$	$5I_1\alpha k^3$	120
$\to {}^3P_1$	$3I_1\alpha k^3$	70
$\to {}^3P_0$	$1I_1\alpha k^3$	25
${}^3P_2 \to 1\,^3S$	$I_2\alpha k^3$	240
${}^3P_1 \to 1\,^3S$	$I_2\alpha k^3$	240
${}^3P_0 \to 1\,^3S$	$I_2\alpha k^3$	240
${}^1P_1 \to 1\,^1S$	$I_2\alpha k^3$	240
${}^3D_1 \to {}^3P_2$	$1I_3\alpha k^3$	7
$\to {}^3P_1$	$15I_3\alpha k^3$	110
$\to {}^3P_0$	$20I_3\alpha k^3$	150
$2\,^3S \to 1\,^1S$	$I_4\alpha k^7$	~1

[a] In the second column $1/\alpha = 137$, k is the energy of the transition, and I_n is a radial integral. The last column is based on our wave functions and energy differences, with fine-structure splittings and *S*-*D* mixing ignored.

P multiplet lies about 230 MeV below that of the 2*S* levels. This energy difference is not very sensitive to our choice of parameters: It decreases to 160 MeV if α_s and m_c assume the unreasonable values of 0.8 and 0.9 GeV, respectively.

(b) The c.o.g. of the lowest *D* multiplet is 70 MeV above that of the 2*S* levels. These *D* levels may therefore lie below the threshold M_c.

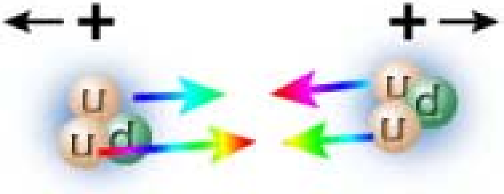
(c) The 3*S* level lies at ~4.2 GeV. As no sharp resonance has been found in this region,[2] this implies that $M_c < 4.2$ GeV.

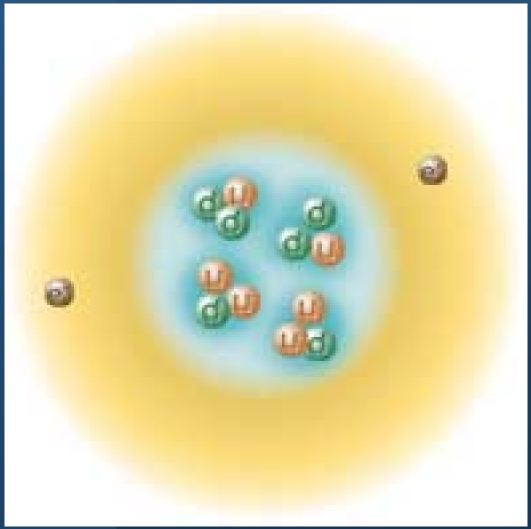
(d) The almost inevitable presence of tensor forces has an intriguing consequence, for it guarantees some mixing between the rather nearby

Eichten *et al.*, Phys. Rev. D**34**, 369 (1975)

Residual strong force

- true strong force = gluon exchange between quarks
- binding of protons + neutrons by pion exchange in nucleus
  - from feeble vestige of strong force (van der Waals-like)





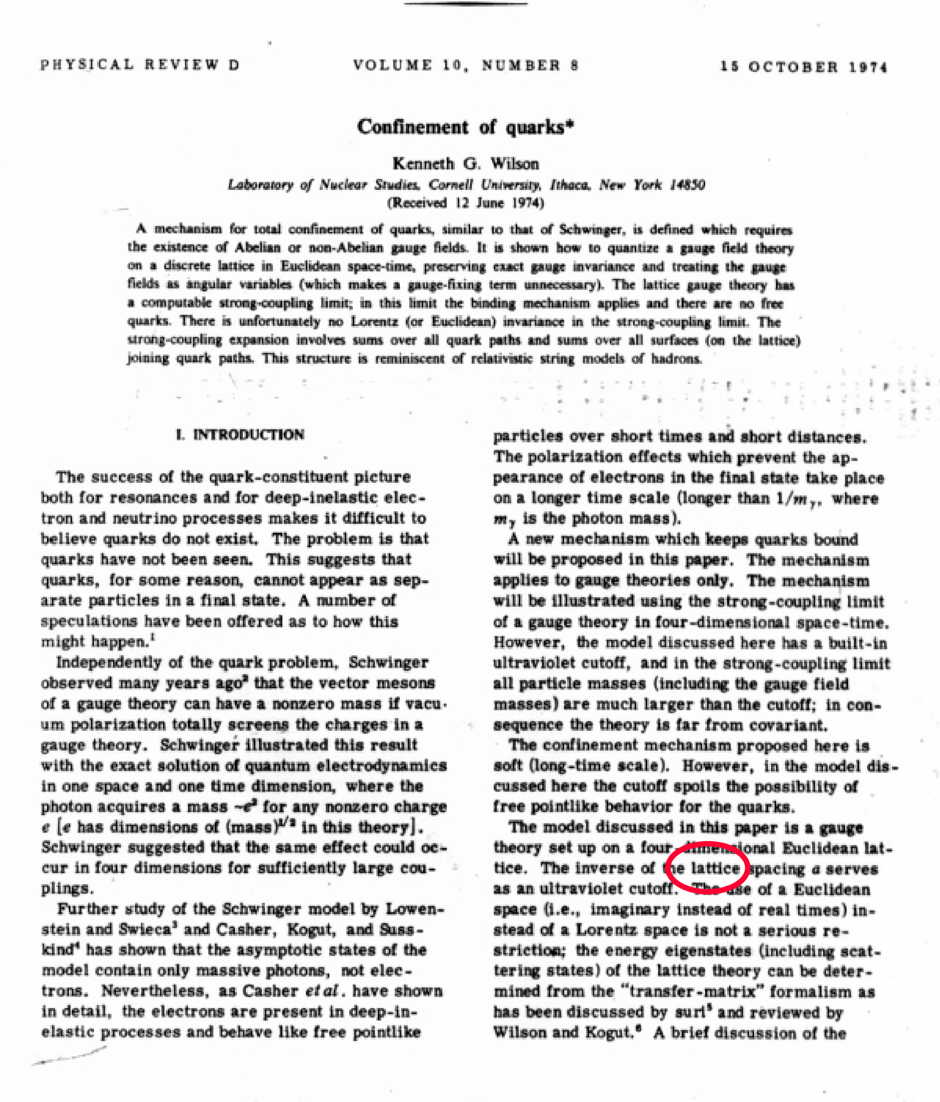
current view of the atom

How feeble is this residual strong force?

Calculation problems

- calculational tools of QED (small coupling expansions)
  - work well for deep inelastic scattering in QCD
    - asymptotic freedom
  - *utterly fail* for hadron formation
    - bound state problem $\propto e^{-1/g^2}$
- Ken Wilson (1974) suggests novel approach
  - formulate QCD using a discrete space-time lattice
  - lattice acts as ultraviolet regulator
  - Wilson formulation preserves exact local gauge invariance
- advantage of this approach
  - facilitates computer simulations of quarks and gluons
  - theorists now free of the shackles of small-coupling expansions!

Birth of Lattice QCD



PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Confinement of quarks*

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(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

I. INTRODUCTION

The success of the quark-constituent picture both for resonances and for deep-inelastic electron and neutrino processes makes it difficult to believe quarks do not exist. The problem is that quarks have not been seen. This suggests that quarks, for some reason, cannot appear as separate particles in a final state. A number of speculations have been offered as to how this might happen.[1]

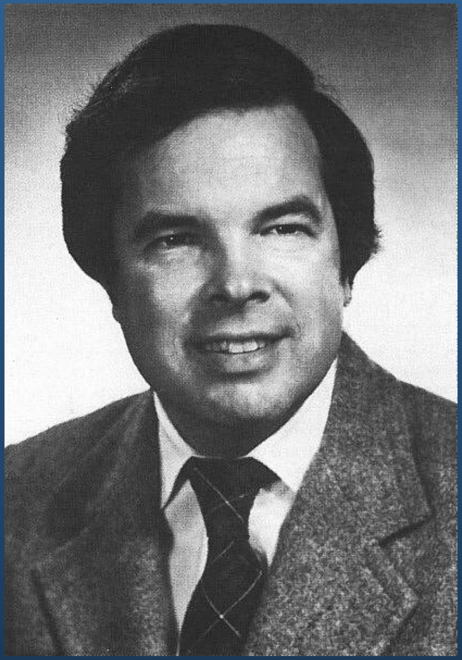
Independently of the quark problem, Schwinger observed many years ago[2] that the vector mesons of a gauge theory can have a nonzero mass if vacuum polarization totally screens the charges in a gauge theory. Schwinger illustrated this result with the exact solution of quantum electrodynamics in one space and one time dimension, where the photon acquires a mass $\sim e^2$ for any nonzero charge e [e has dimensions of $(\text{mass})^{1/2}$ in this theory]. Schwinger suggested that the same effect could occur in four dimensions for sufficiently large couplings.

Further study of the Schwinger model by Lowenstein and Swieca[3] and Casher, Kogut, and Susskind[4] has shown that the asymptotic states of the model contain only massive photons, not electrons. Nevertheless, as Casher *et al.* have shown in detail, the electrons are present in deep-inelastic processes and behave like free pointlike particles over short times and short distances. The polarization effects which prevent the appearance of electrons in the final state take place on a longer time scale (longer than $1/m_\gamma$, where m_γ is the photon mass).

A new mechanism which keeps quarks bound will be proposed in this paper. The mechanism applies to gauge theories only. The mechanism will be illustrated using the strong-coupling limit of a gauge theory in four-dimensional space-time. However, the model discussed here has a built-in ultraviolet cutoff, and in the strong-coupling limit all particle masses (including the gauge field masses) are much larger than the cutoff; in consequence the theory is far from covariant.

The confinement mechanism proposed here is soft (long-time scale). However, in the model discussed here the cutoff spoils the possibility of free pointlike behavior for the quarks.

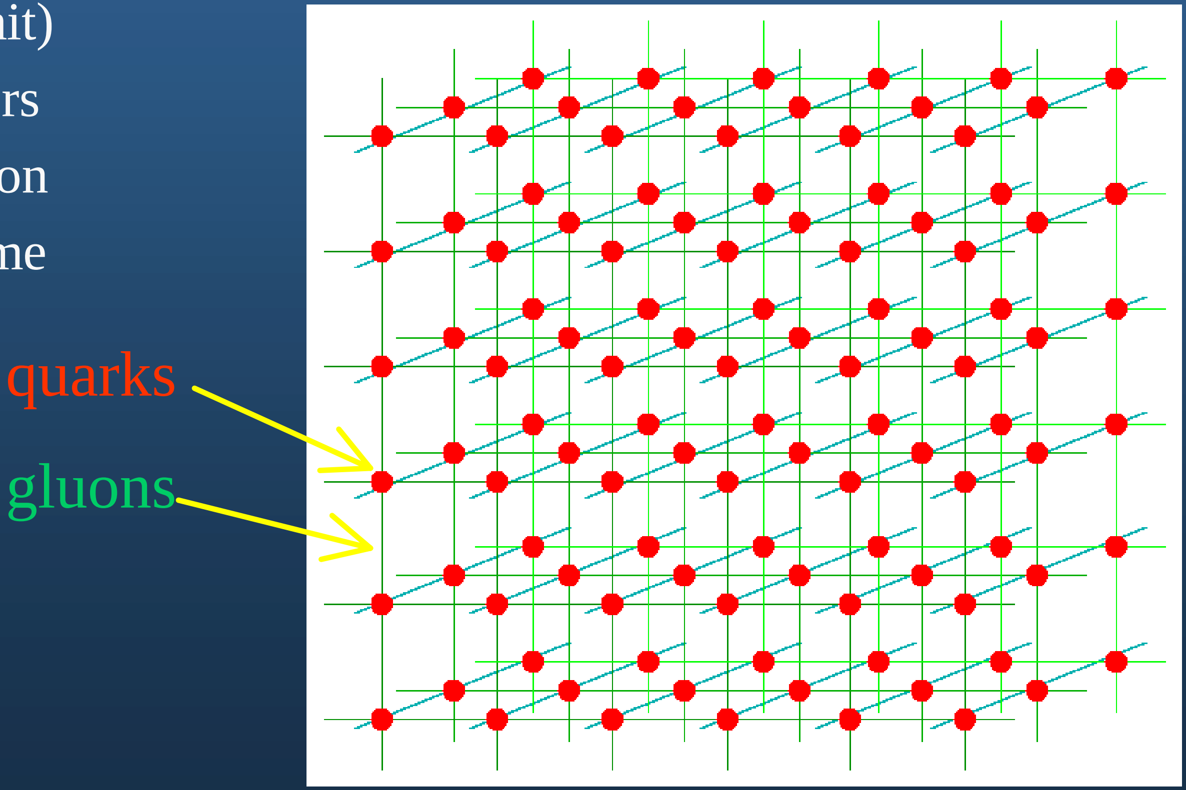
The model discussed in this paper is a gauge theory set up on a four-dimensional Euclidean lattice. The inverse of the lattice spacing a serves as an ultraviolet cutoff. The use of a Euclidean space (i.e., imaginary instead of real times) instead of a Lorentz space is not a serious restriction; the energy eigenstates (including scattering states) of the lattice theory can be determined from the "transfer-matrix" formalism as has been discussed by suri[5] and reviewed by Wilson and Kogut.[6] A brief discussion of the



Ken Wilson

Lattice regularization

- hypercubic space-time lattice
- quarks reside on sites, gluons reside on links between sites
- lattice excludes short wavelengths from theory (regulator)
- regulator removed using standard renormalization procedures (continuum limit)
- systematic errors
  - ❑ discretization
  - ❑ finite volume



Monte Carlo methods

- vacuum expectation value in terms of path integrals

$$\langle \Phi(t)\Phi^*(0) \rangle = \frac{\int D\Phi\ \Phi(t)\Phi^*(0)\ e^{-S[\Phi]}}{\int D\Phi\ e^{-S[\Phi]}}$$

- S[Φ] is the Euclidean space action, $\Phi^*(t)$ creates state of interest
- evaluation of path integrals:
  - Markov-chain Monte Carlo methods
    - Metropolis
    - heatbath
    - overrelaxation
    - hybrid methods
  - no expansions in a small parameter
  - statistical errors
- first principles approach

New branch of particle physics

- Wilson’s paper marks beginning of new branch of particle physics
  - *lattice QCD* or lattice gauge theory
- field has grown considerably during past 25 years
- large grand-challenge collaborations with dedicated supercomputer resources in several countries
  - USA, Japan, Germany, Italy, UK, Australia,…
- annual International Symposium on Lattice Field Theory
  - typically over 300 participants
- e-print archive at xxx.lanl.gov: hep-lat

- I am a member of the *Lattice Hadron Physics collaboration*
  - focus on hadron physics relevant to experimental program at the *Thomas Jefferson National Accelerator Facility* (Newport News)
  - acquiring large Pentium-based clusters at Jlab and MIT with funds obtained through DOE’s SciDAC Initiative

Role of lattice QCD

- useful for "brute force" calculations of strong interaction observables
  - spectra, decay constants f_B, structure functions
  - quantities needed to go beyond the standard model
- useful tool to help understand QCD
  - testing mechanisms of confinement
  - answering why the naïve quark model works as well as it does
  - do certain field configurations dominate path integral?
- develop better tools for nonperturbative aspects of gauge theories
  - repercussions beyond QCD
- lattice simulations have told us much about QCD
  - but still much more to learn!

Technology advancements

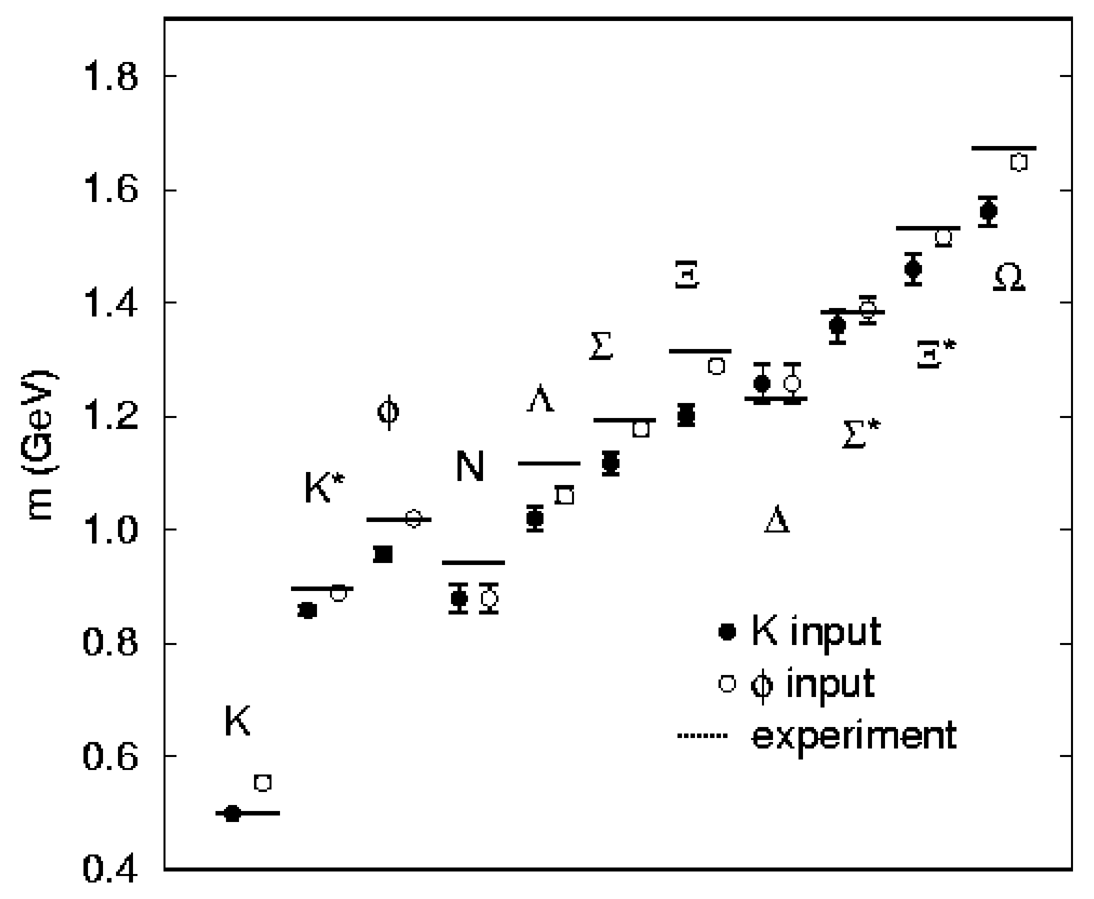
- significant advances in simulation techniques during 1990’s
    - improved actions (quantum corrections to couplings, tadpoles)
    - excited states using variational techniques with correlator matrices
    - anisotropic lattices
    - chiral fermions (domain wall, overlap)
- computers → vastly improved performance, lower costs
- accurate calculations of many quantities for the first time
    - Yang-Mills glueball spectrum
    - quenched light hadron spectrum
    - quenched f_B, *etc.*
- full incorporation of virtual quark-antiquark pairs still a problem
    - very active subject of research

Current state of the art

Stable single particle masses $N_f = 0$	A
Stable single particle masses $N_f > 0$	B-
Two particle energies	C-
EM Current matrix elements $N_f = 0$	A-
EM Current matrix elements $N_f > 0$	C+
Unstable single particle masses	D
Decay strengths, resonance widths	R

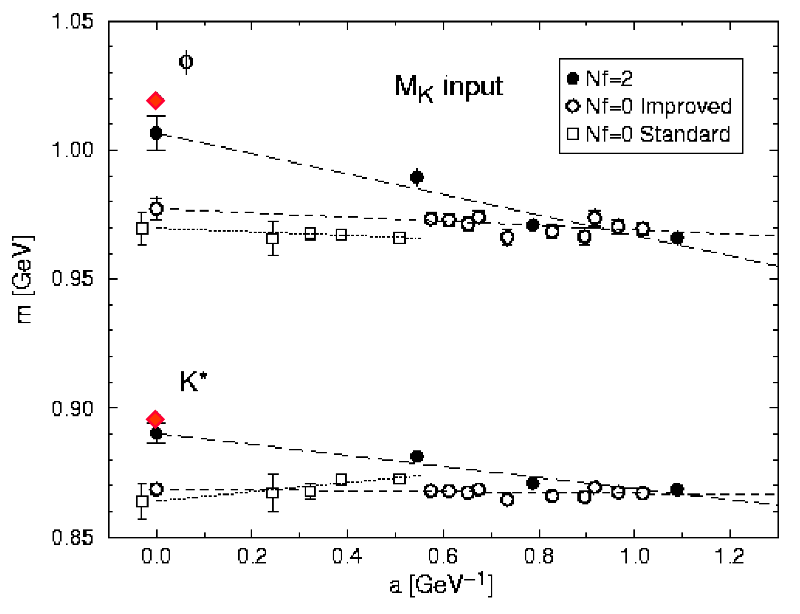
Light hadron spectrum (quenched)

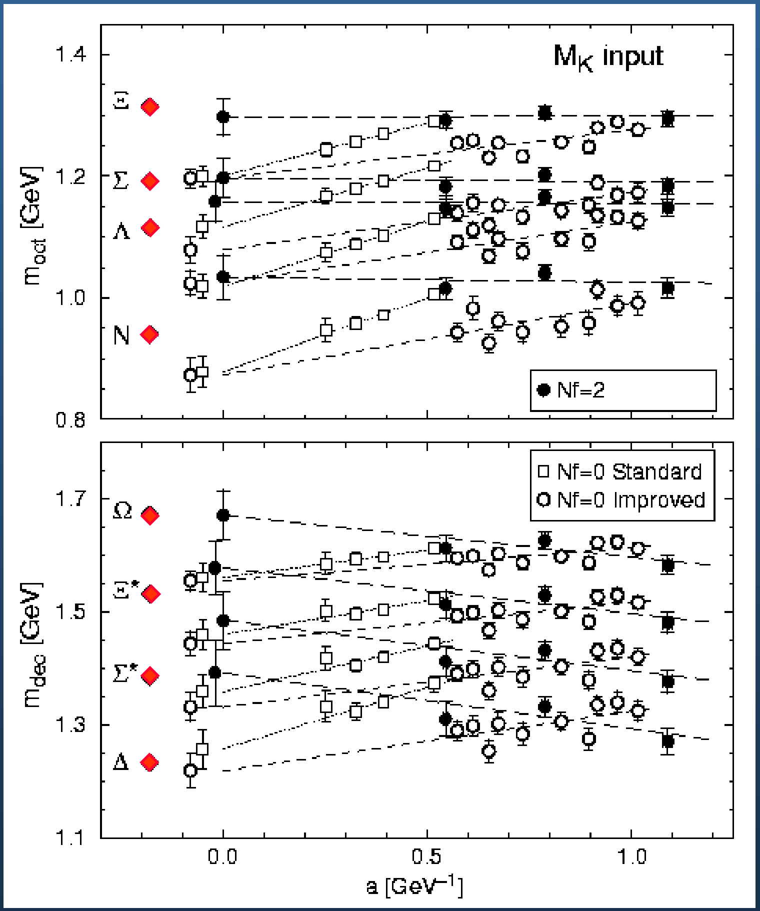
- CP-PACS collaboration (Japan) Phys Rev Lett **84**, 238 (2000)



Light hadron spectrum ($N_f = 2$)

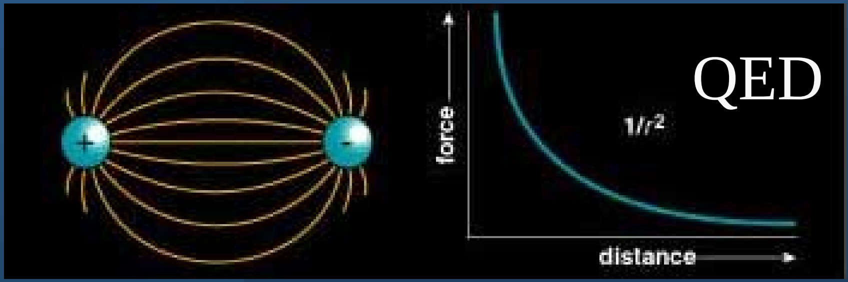
- CP-PACS collaboration hep-lat/0010078
- Iwasaki gauge, SWTI fermion
- baryons → finite volume effects
- experimental values ◆

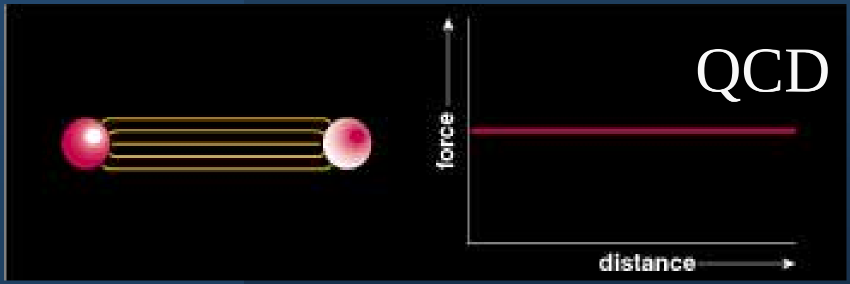




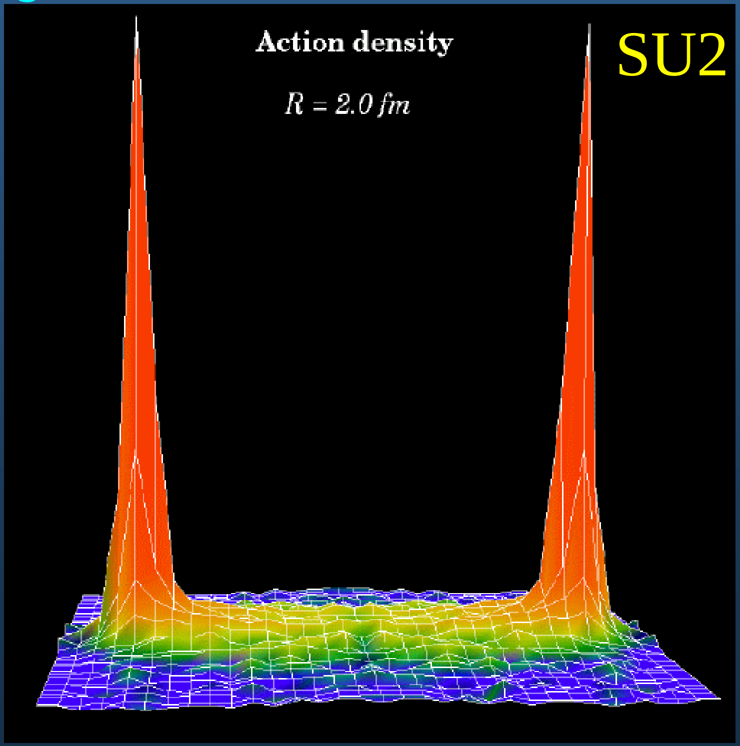
Key feature of QCD

- fundamental degrees of freedom (quarks and gluons) cannot be observed in isolation
- attractive force between quark-antiquark is *constant* with separation
- suggests that gluon field forms a *string*-like object between quark-antiquark



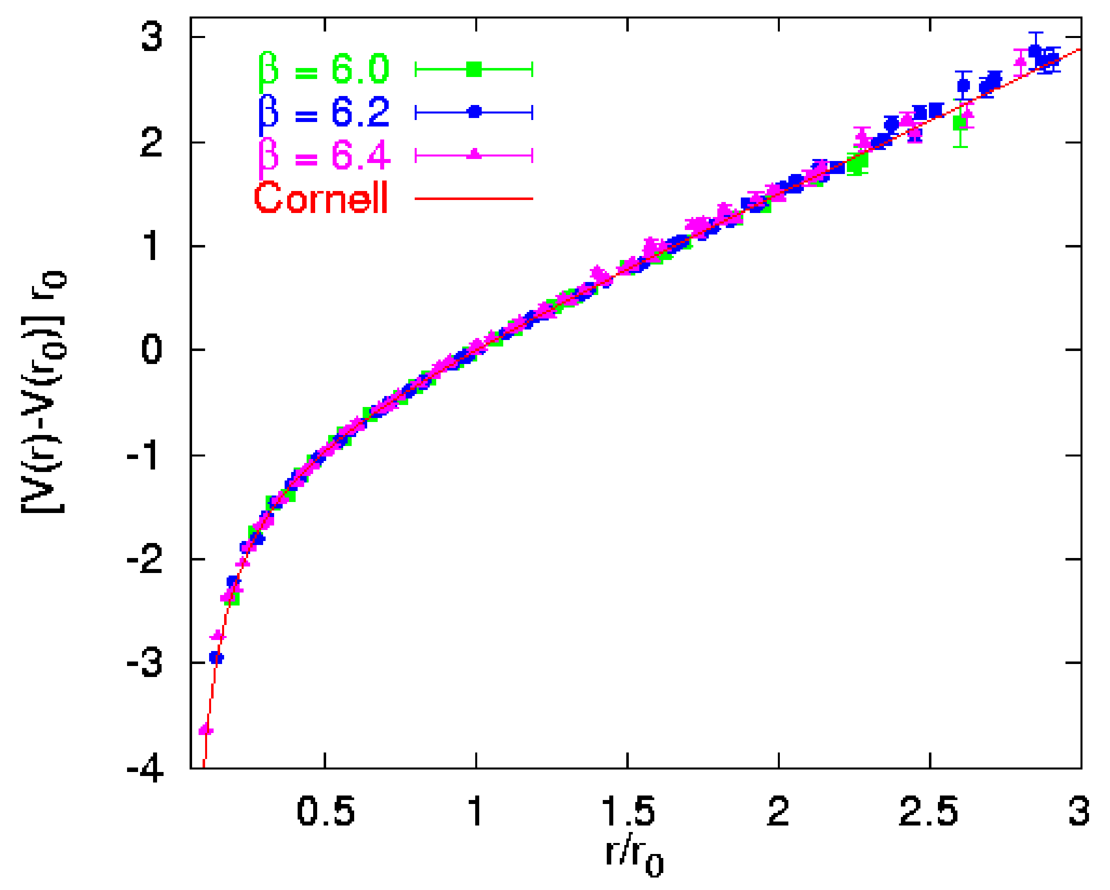


Bali *et al.*



Static quark potential

- lattice simulations confirm linearly rising potential from gluon exchange

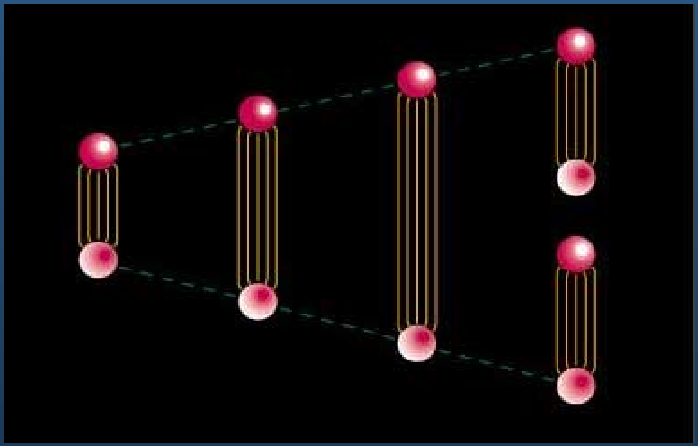


Bali *et al.*

$r_0 = 0.5\ \text{fm}$

Quark confinement

- quarks can never be isolated
- linearly rising potential
  - separation of quark from antiquark takes an infinite amount of energy
  - gluon flux breaks, new quark-antiquark pair produced



Constituent quark model

- much of our understanding of hadron formation comes from the *constituent quark model*
  - motivated by QCD
  - valence quarks interacting via Coulomb + linear potential
  - gluons: source of the potential, dynamics ignored
- mesons: only certain J^{PC} allowed:
  - $0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, \ldots$ forbidden

$$P = (-1)^{L+1} \quad L = 0,1,2,\ldots$$

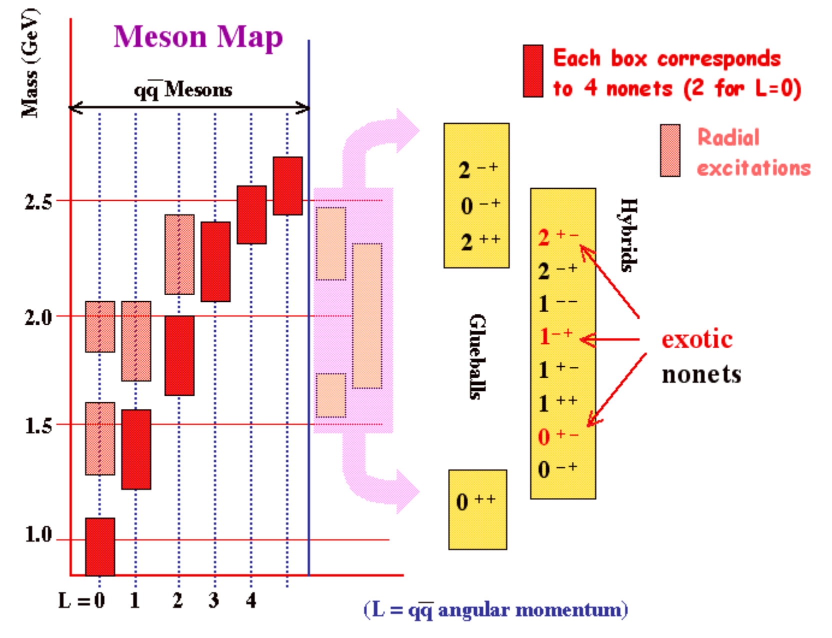
$$C = (-1)^{L+S} \quad S = 0,1$$

- *most* of observed low-lying hadron spectrum described reasonably well by quark model
  - agreement is amazing given the crudeness of the model
- experimental results now need input beyond the quark model
  - over-abundance of states
  - forbidden 1^{-+} states

Gluonic excitations

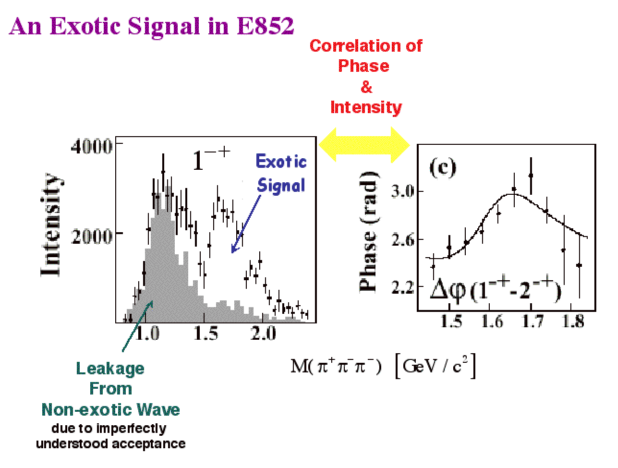
- QCD suggests existence of states in which *gluon* field is excited
  - glueballs (*excited glue*)
  - hybrid mesons ($q\bar{q}$ + *excited glue*)
  - hybrid baryons (qqq + *excited glue*)
- such states not understood
  - quark model fails
  - perturbative methods fail
- lack of understanding

  makes identification difficult!
- clues to confinement

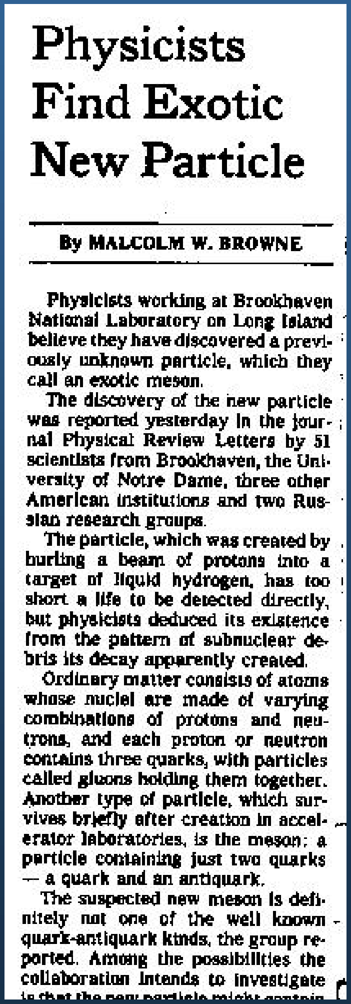


Experimental candidates

- scalar glueball: $f_0(1500)$ or $f_J(1710)$
  - mixing
- tensor glueball: $\xi(2230)$ or $f_2(1980)$
- 1^{-+} hybrid mesons (E852 BNL 1997)
  - 1.4 GeV (controversial)
  - 1.6 GeV (lattice predicts 1.9 GeV)
- higher $c\bar{c}$ and $b\bar{b}$ states
- hybrid baryon $P_{11}(1710)$
- others



New York Times, Sept. 2, 1997



Physicists Find Exotic New Particle

By MALCOLM W. BROWNE

Physicists working at Brookhaven National Laboratory on Long Island believe they have discovered a previously unknown particle, which they call an exotic meson.

The discovery of the new particle was reported yesterday in the journal Physical Review Letters by 51 scientists from Brookhaven, the University of Notre Dame, three other American institutions and two Russian research groups.

The particle, which was created by hurling a beam of protons into a target of liquid hydrogen, has too short a life to be detected directly, but physicists deduced its existence from the pattern of subnuclear debris its decay apparently created.

Ordinary matter consists of atoms whose nuclei are made of varying combinations of protons and neutrons, and each proton or neutron contains three quarks, with particles called gluons holding them together. Another type of particle, which survives briefly after creation in accelerator laboratories, is the meson: a particle containing just two quarks — a quark and an antiquark.

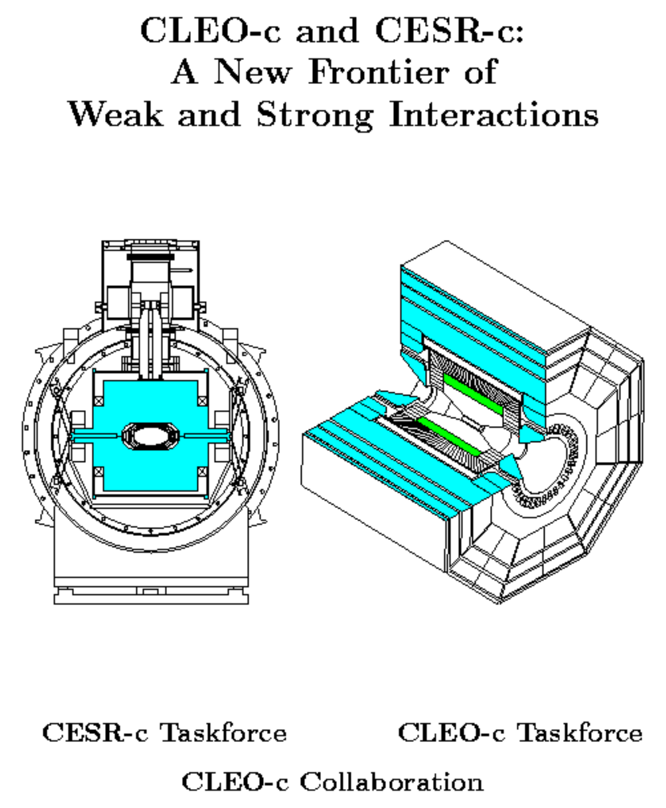
The suspected new meson is definitely not one of the well known quark-antiquark kinds, the group reported. Among the possibilities the collaboration intends to investigate

Future experiments

- experimental focus on such states intensifying

glueballs

hybrid mesons

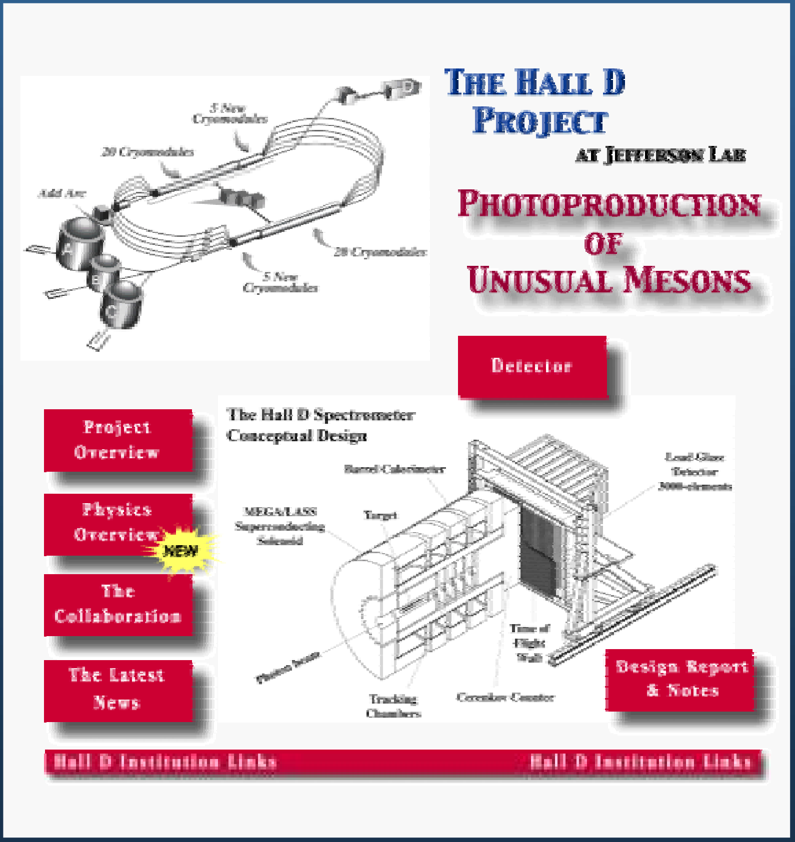


CLEO-c and CESR-c:
A New Frontier of
Weak and Strong Interactions

CESR-c Taskforce

CLEO-c Taskforce

CLEO-c Collaboration



THE HALL D
PROJECT
AT JEFFERSON LAB

PHOTOPRODUCTION
OF
UNUSUAL MESONS

Detector

Project Overview

Physics Overview

NEW

The Collaboration

The Latest News

The Hall D Spectrometer
Conceptual Design

Design Report & Notes

Hall D Institution Links

Hall D Institution Links

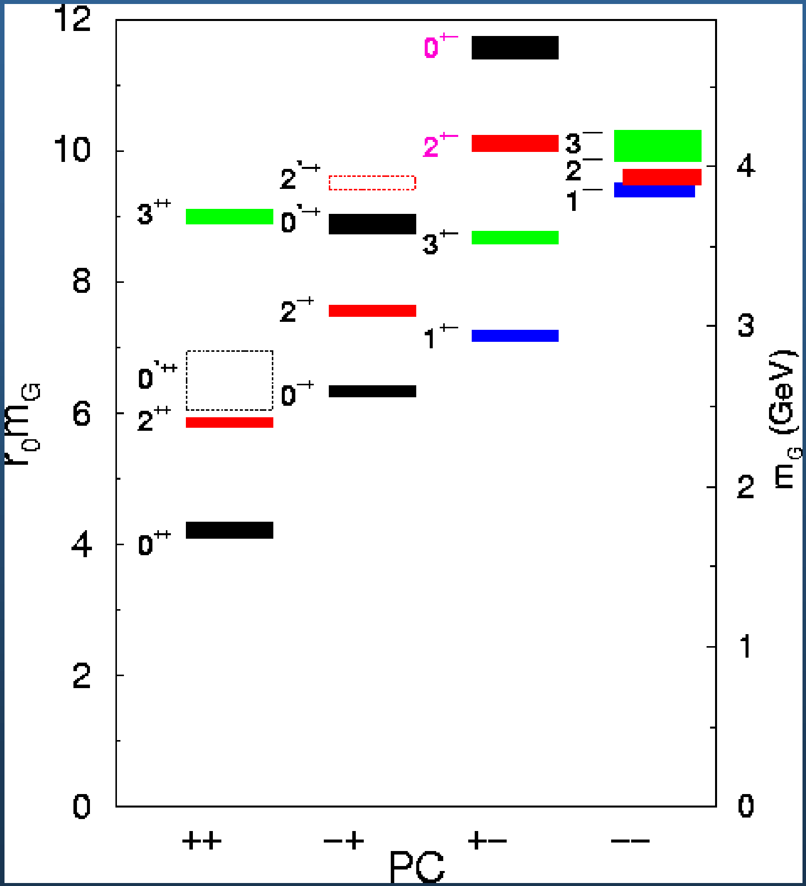
Lattice takes aim

glueball masses	$N_f = 0$	C.M.,Peardon	99	
	$N_f > 0$	SESAM	00	
glueball decays	$N_f = 0$	IBM (GF11)	96	infancy
glueball-$q\bar{q}$ mixing	$N_f = 0$	IBM (GF11)	99	
excitations of $Q\bar{Q}$ potential	$N_f = 0$	Juge, Kuti, C.M.	99	
light 1^{-+} meson mass	$N_f = 0$	UKQCD	97	2.0(2) GeV
	$N_f = 0$	MILC	98	2.1(1) GeV
	$N_f > 0$	Lacock, Schilling	98	1.9(2) GeV
$c\bar{c}$ 1^{-+} meson mass (above $1S$)	$N_f = 0$	MILC	98	1.22(15) GeV
	$N_f = 0$	CP-PACS	98	1.32(2) GeV
	$N_f = 0$	Juge,Kuti,C.M.	99	1.19 GeV
$b\bar{b}$ 1^{-+} meson mass (above $1S$)	$N_f = 0$	UKQCD	98	1.68(10) GeV
	$N_f = 0$	CP-PACS	98	1.542(8) GeV
	$N_f = 0$	Juge,Kuti,C.M.	99	1.49(2)(5) GeV

Yang-Mills SU(3) Glueball Spectrum

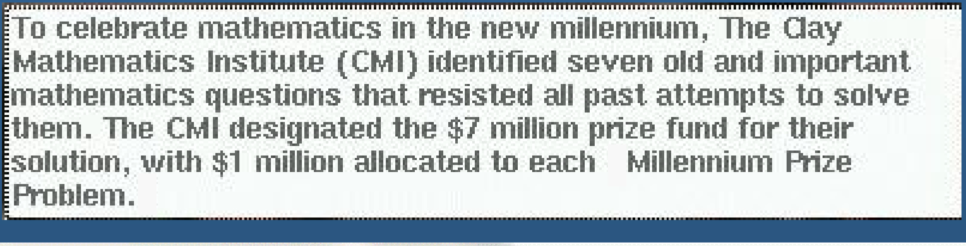
- gluons can bind to form glueballs
    - e.m. analogue: massive globules of pure light!
- technology advancements permit first glimpse of glueball spectrum
- states labeled by J^{PC} and scale set using $r_0^{-1} = 410(20)$ MeV
- probe of confinement
- “experimental” results in simpler world (no quarks) to help build phenomenological models
- first step towards realistic glueball study

C. Morningstar and M. Peardon,
Phys. Rev. D 60, 034509 (1999)

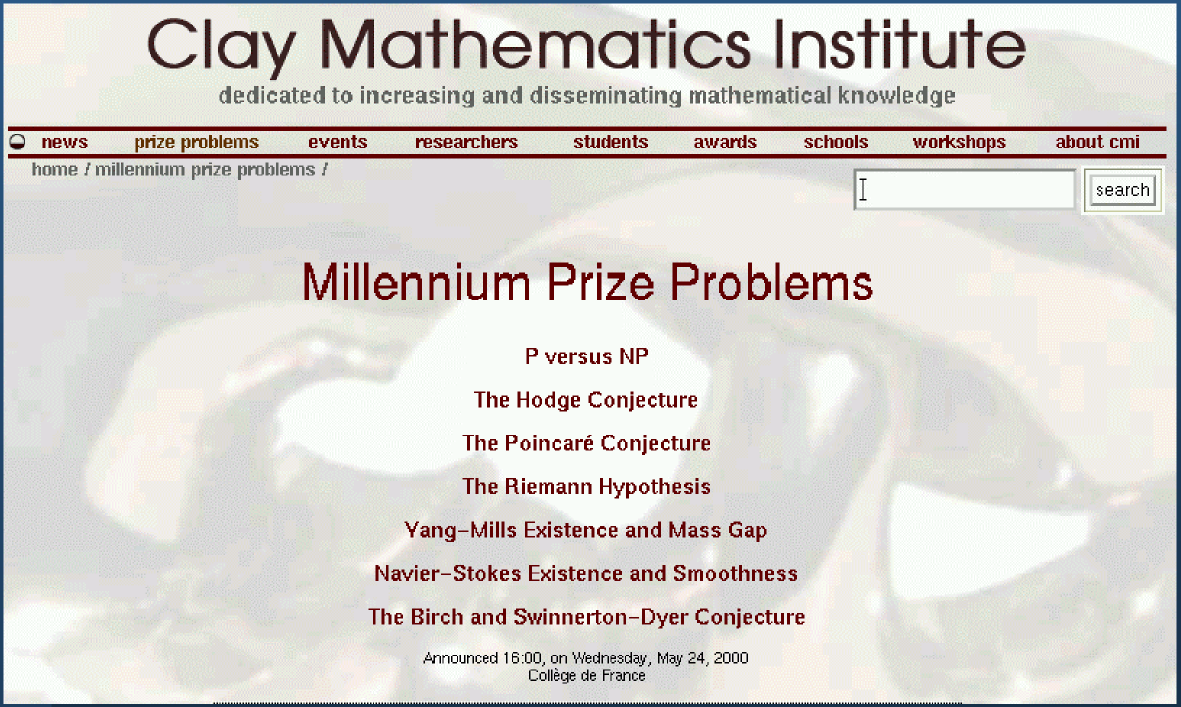


Clay Millenium Prize

- understanding this glueball spectrum worth **$1 million**
- www.claymath.org



To celebrate mathematics in the new millennium, The Clay Mathematics Institute (CMI) identified seven old and important mathematics questions that resisted all past attempts to solve them. The CMI designated the $7 million prize fund for their solution, with $1 million allocated to each Millennium Prize Problem.



Clay Mathematics Institute

dedicated to increasing and disseminating mathematical knowledge

Millennium Prize Problems

P versus NP

The Hodge Conjecture

The Poincaré Conjecture

The Riemann Hypothesis

Yang–Mills Existence and Mass Gap

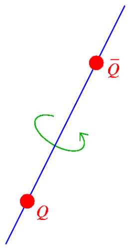
Navier–Stokes Existence and Smoothness

The Birch and Swinnerton–Dyer Conjecture

Announced 16:00, on Wednesday, May 24, 2000
Collège de France

Excitations of static quark potential

- gluon field in presence of static quark-antiquark pair can be *excited*
- classification of states: (notation from molecular physics)
  - magnitude of glue spin projected onto molecular axis



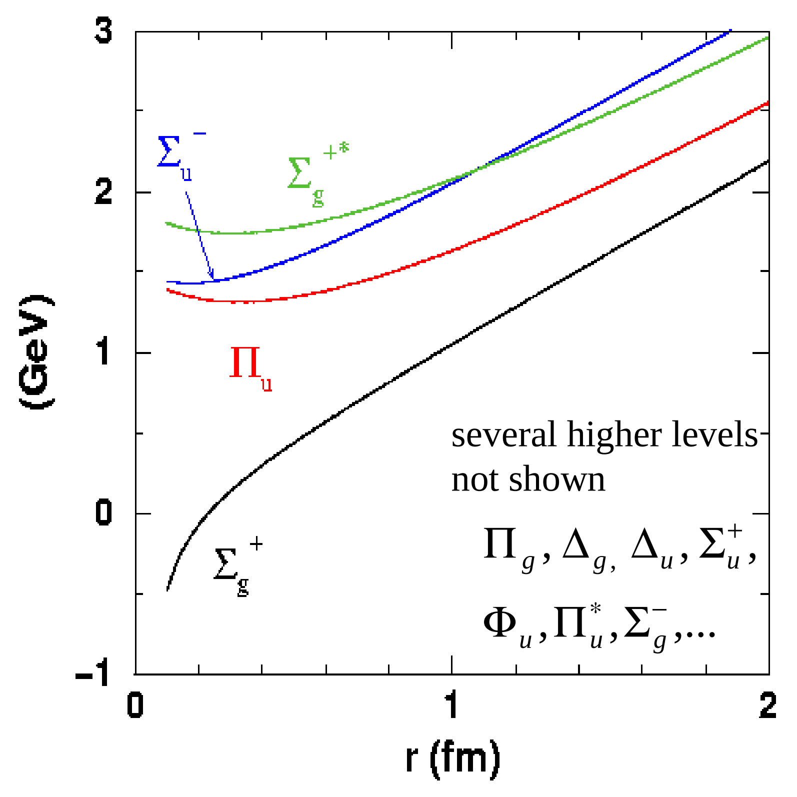
$$\Lambda = 0,1,2,\ldots$$
$$= \Sigma, \Pi, \Delta, \ldots$$

  - charge conjugation + parity about midpoint

$$\eta = g \text{ (even)}$$
$$= u \text{ (odd)}$$

  - chirality (reflections in plane containing axis) Σ^+, Σ^-

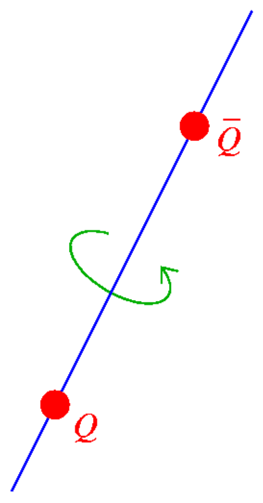
$\Pi, \Delta, \ldots$ doubly degenerate (Λ doubling)



Juge, Kuti, Morningstar

Heavy-quark hybrid mesons

- more amenable to theoretical treatment than light-quark hybrids
- possible treatment like diatomic molecule (Born-Oppenheimer)
  - ❑ slow heavy quarks ←→ nuclei
  - ❑ fast gluon field ←→ electrons
    (and light quarks)



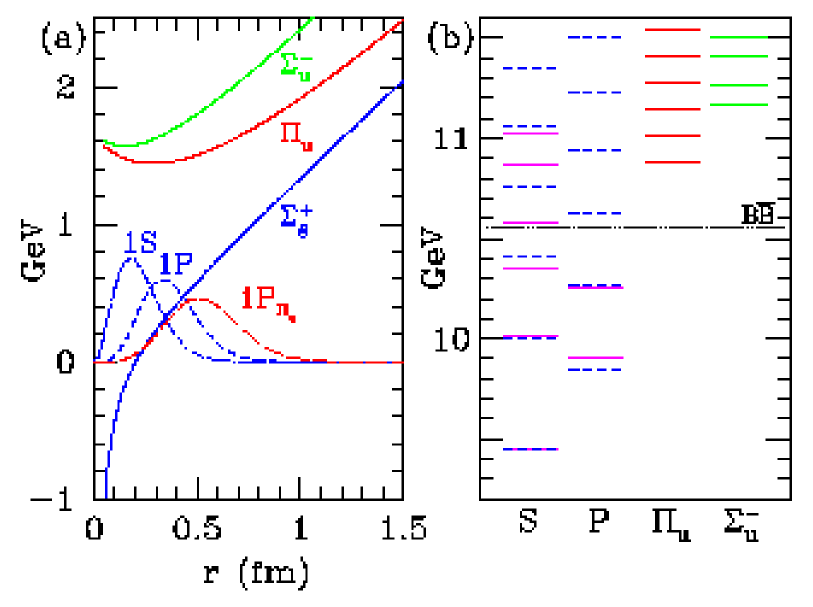
- gluons provide adiabatic potentials $V_{Q\bar{Q}}(r)$
  - ❑ gluons fully relativistic, interacting
  - ❑ potentials computing in lattice simulations
- nonrelativistic quark motion described in *leading order* by solving Schrodinger equation for each $V_{Q\bar{Q}}(r)$

$$\left\{\frac{p^2}{2\mu}+V_{Q\bar{Q}}(r)\right\}\psi_{Q\bar{Q}}(r)=E\psi_{Q\bar{Q}}(r)$$

- conventional mesons from Σ_g^+ ; hybrids from $\Pi_u, \Sigma_u^-, \ldots$

Leading Born-Oppenheimer

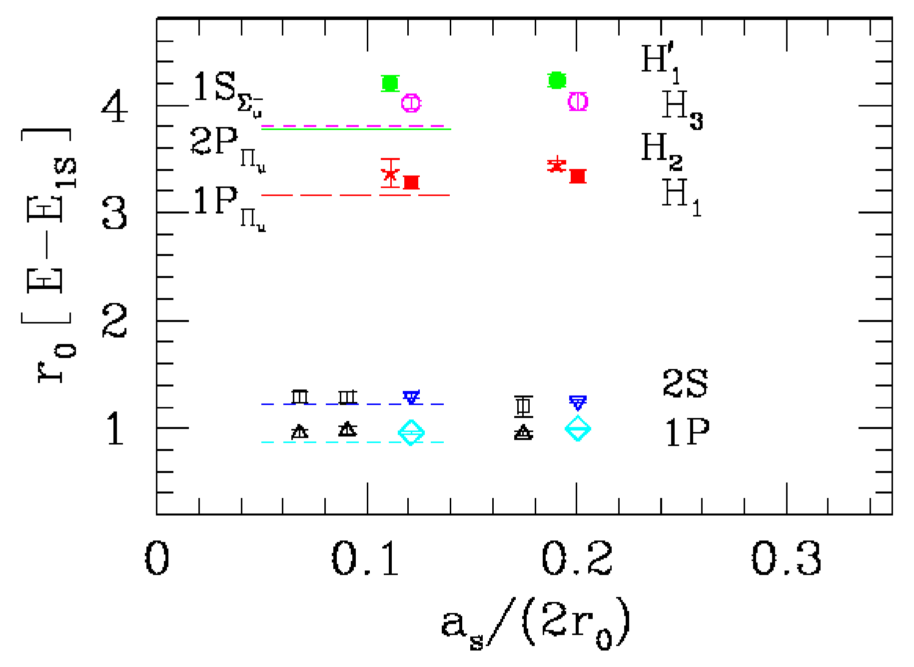
- results obtained (in absence of light quark loops)
- good agreement with experiment below $B\bar{B}$ threshold
- plethora of hybrid states predicted (caution! quark loops)
- but is a Born-Oppenheimer treatment valid?



Juge, Kuti, Morningstar, Phys Rev Lett **82**, 4400 (1999)

Testing LBO

- test LBO by comparison of spectrum with NRQCD simulations
  - include retardation effects, but no quark spin, no light quarks
  - allow possible mixings between adiabatic potentials
- dramatic evidence of validity of LBO
  - level splittings agree to 10% for 2 conventional mesons, 4 hybrids



$$H_1, H_1' = 1^{--}, 0^{-+}, 1^{-+}, 2^{-+}$$

$$H_2 = 1^{++}, 0^{+-}, 1^{+-}, 2^{+-}$$

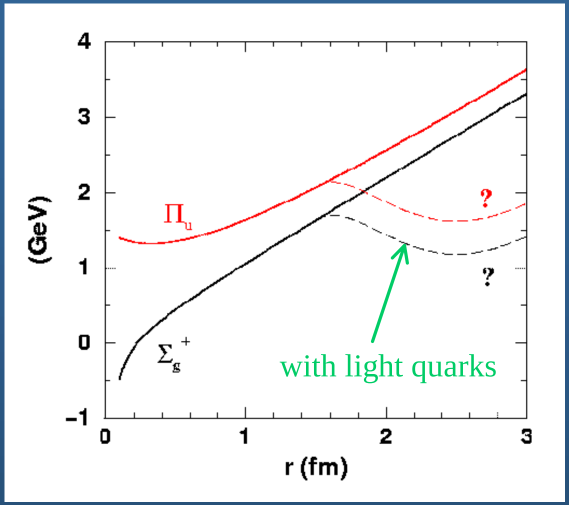
$$H_3 = 0^{++}, 1^{+-}$$

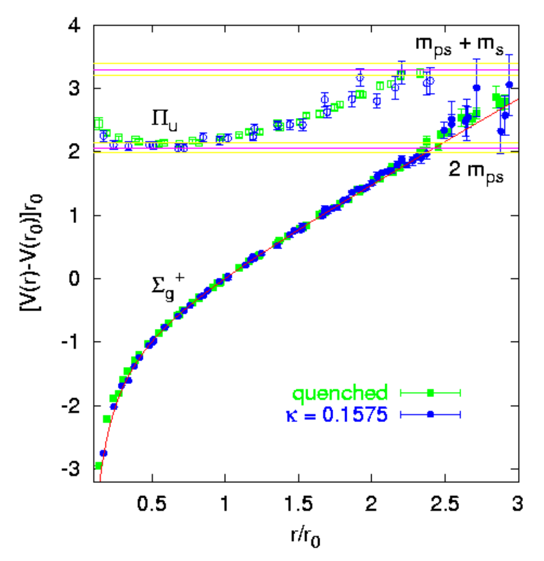
Compelling physical picture

- Born-Oppenheimer provides simple physical picture for heavy-quark conventional and hybrid meson states
  - partial explanation of quark model success (light quarks?)
  - allows incorporation of gluon dynamics (beyond quark model)
- does this BO picture survive inclusion of
  - quark spin?
  - light-quark effects?
- quark spin: two recent studies *suggest* BO picture survives
  - Drummond *et al.* Phys.Lett.B478, 151 (2000)
    - looked at 4 hybrids degenerate in LBO
    - found significant shifts from $\sigma \cdot B / M$ but used bag model to interpret results as not arising from surface mixing effects
  - Toussaint *et al.* Phys.Rev.D64, 074505 (2001)
    - found very small probability admixture of hybrid in Υ from $\sigma \cdot B / M$
  - more conclusive tests needed

Light quark spoiler?

- spoil B.O.? → unknown
- light quarks change $V_{Q\bar{Q}}(r)$
  - small corrections at small r
    - fixes low-lying spectrum
  - large changes for r>1 fm
    → fission into $(Q\bar{q})(\bar{Q}q)$
- states with diameters over 1 fm
  - most likely *cannot exist* as observable resonances
- dense spectrum of states from pure glue potentials will not be realized
  - survival of a few states conceivable given results from Bali *et al.*
- discrepancy with experiment above $B\bar{B}$
  - most likely due to light quark effects



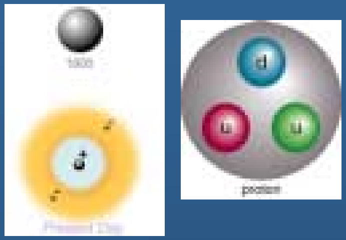


Future work

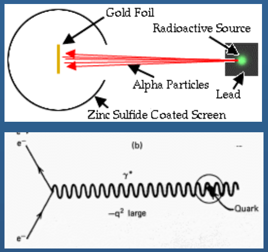
- heavy-quark hybrid mesons
  - ❑ revisit quark spin effects
  - ❑ $V_{Q\overline{Q}}(r)$ in presence of virtual light quarks (string breaking)
- light quark hybrid mass calculations
- glueball mixing with scalar quarkonium
- tests of confinements scenarios with glueball spectrum
- “baryon” potentials (gluons in presence of static *qqq*)
- *N** spectrum
- $qq\overline{qq}$ states (study of the a_0(980), the light 1^{-+} exotic)
- flux tube profiles (energy, angular momentum)
- strings in other representations (adjoint,…)
- torelon spectrum
- much, much more….

Conclusion

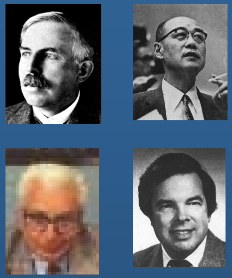
- quest to understand the strong force is a classic story



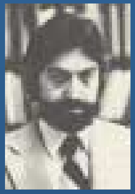
great conceptual leaps



great experiments



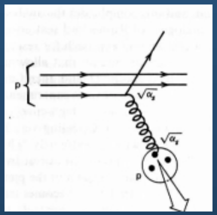
heroes



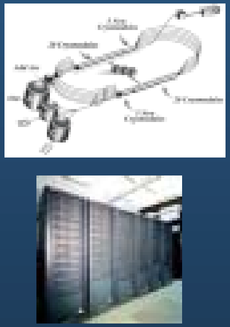
the horribly wronged

incredible violence

confinement



freedom (asymptotic)



amazing technology

large sums of money

comic relief

- lattice simulations of quarks and gluons will continue to play an important role in this quest