Customized Data Plans for Mobile Users: Feasibility and Benefits of Data Trading

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***Abstract*— The growing volume of mobile data traffic has led many Internet service providers (ISPs) to cap the monthly data usage of their users and to charge overage fees, when the data caps are exceeded. Yet data caps imperfectly capture the reality of heterogeneous data usage over a month—even the same user may have varied requirements from month to month. In response, some ISPs are providing alternative avenues for users to customize data plans to their needs. In this paper, we examine a secondary data market, as for example created by China Mobile Hong Kong, in which users can buy and sell leftover data caps from one another. While similar to an auction in that users submit bids to buy and sell data, it differs from traditional double auctions in that the ISP serves as the middleman between buyers and sellers. Such a market faces two questions. First, can users learn each others' trading behavior well enough for the market to function, and second, do ISPs have a financial incentive to offer such a market? Different users' abilities to trade data depend on others, thus forcing users to not only optimize the amounts of data they bid, but also to learn and adjust for other users' trading behavior. We derive users' optimal behavior and propose an algorithm for ISPs to match buyers and sellers. We compare the optimal matchings for different ISP objectives and derive conditions under which the secondary market increases ISP revenue: while the ISP loses revenue from overage fees, it can assess administration fees and profit from the differences between the buyer and seller prices. Finally, we use one year of usage data from 100 U.S. mobile users to simulate the market dynamics and to illustrate that sustainable conditions for a revenue increase for the ISP can hold in practice.**

***Index Terms*— Smart data pricing, double auction, mobile data trading.**

I. Introduction

TWO of the primary challenges facing Internet service providers (ISPs) today are the growing volume and diversity of mobile data traffic [2]. In response, ISPs have attempted to simultaneously customize mobile data access to allow for different user or application behavior and find ways to limit user demand to available network capacity [3]–[5]. While much of this work focuses on technical modifications to network operations, e.g., automatically adapting application usage to network congestion [6], in this paper we focus on a higher-level concern: is there a market for data usage?

Though different types of pricing do not determine application data demand, ISPs can influence users' data consumption patterns and incentivize users to behave in ways that reduce network congestion [7], [8]. Pricing is also an important component of perceived user experience: users who expect better network service are willing to pay more for data usage. Thus, different pricing plans can complement other lower-layer or automated mechanisms for customizing and limiting data usage. In order to do so, these data plans should be able to accommodate different user behavior, e.g., users consume different amounts of data over a month, and the same user's data consumption can change from month to month [9], [10]. There is then high variability in users' willingness to limit their data usage and potentially prevent excessive network congestion. Most ISPs today have attempted to limit excessive data usage by charging users a fee for a maximum amount of data quota within a month, i.e., a monthly data cap [4], with additional steep overage fees or throttling for data usage over the cap [11]. Yet this rigid pricing does not take into account usage heterogeneity. We consider an emerging alternative, *traded data plans*, in this paper.

A. Traded Data Plans

The discrepancy between heterogeneous data usage and fixed data caps has been somewhat mitigated by shared data plans [12], [13]. Such plans allow data caps to be shared across multiple users and devices; thus, users with a high data demand can reduce the likelihood of exceeding their data caps by sharing a cap with other users who have less data demand. Yet most users share their data caps only with their immediate family members.

Understandably, the majority of the users may not be willing to give away their leftover data caps to strangers, but there is a possibility that they might *sell* their leftover data. Users with a large data demand could then purchase additional data from other users, thus avoiding ISPs' high overage fees and customizing their own data caps to their needs in each month. Interestingly, ISPs can have an instrumental role in this secondary market, both to enforce the traded data caps in

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users' bills (e.g., ensuring that buyers are not charged overage fees for their purchased data), and to help buyers and sellers locate one another (e.g., through an exchange platform). China Mobile Hong Kong (CMHK) introduced such a secondary market at the end of 2013 [14]. CMHK's 2cm data exchange platform allows users to submit bids to buy and sell data, with CMHK acting as a middleman to match buyers and sellers, and as a bookkeeping facilitator, to ensure that the sellers' trading revenue and buyers' purchased data are reflected on their monthly bills.

Traded data plans have been studied in [1] and [5], with [15] particularly emphasizing user trading behavior, but much research remains to be done. Our work considers several important research questions: *how do users choose the bids to submit, and how does an ISP match buyers to sellers?* More fundamentally, *are traded data plans a profitable way for ISPs to accommodate user heterogeneity? Are users adaptive enough to trade data in this secondary market?*

Intuitively, we would expect ISPs to lose revenue with the secondary market: instead of purchasing overage data from the ISP, users can potentially buy data directly from other users at lower prices. However, the ISP's role as a middleman between the buyers and sellers allows it to extract revenue from buyer-seller transactions.[1] In this work, *we derive the optimal economic behavior of the buyers, sellers, and ISP, as well as propose algorithms to learn data trading behavior. We show that all three parties can benefit from the option of a secondary market, and validate our analysis with simulations over a one-year dataset of 100 users' monthly usage from a U.S. ISP.*

B. Related Work

Auction solutions have been proposed to form a self-organizing market that incentivizes users to participate in data trading. Most previously studied data auctions aim to mitigate network congestion [16], [17], while recent works propose variants of data auctions by leveraging user behavior. The trading mechanism proposed in [18] allows users to sell their data by becoming hotspots for others, but user mobility can jeopardize the viability of such a market. With the ISP's involvement, the data transactions in our work can be done remotely. Considering a similar trading model, the author in [19] discusses users' decisions on their usage at different times of the billing cycle. We argue that users will adjust their decisions over the billing cycle depending on their abilities to buy and sell data.

Conventional two-sided matching is applied in the literature mostly for allocating resources to users. For instance, secondary users are matched to desirable spectrum from primary users in cognitive radio networks [20], [21]. In a general wireless setting, matching enables assignments between operators and small cells as well as between small cells and users [22], [23]. However, these resources are only held on a temporary basis, so the user incentives are different from those in data trading. Moreover, most two-sided auction works do not consider the incentives of an auction middleman.

[1]There may be long-term branding and marketing benefits, beyond the monetary benefit, for an ISP to offer a secondary market. We do not consider these long-term effects in this paper, and instead focus on user and ISP behavior within a month.

C. Modeling User and ISP Behavior

We suppose that each seller (resp. buyer) can submit a bid to the secondary market consisting of the volume of data that the user wishes to sell (or buy) and the unit price to accept (or pay). The ISP then matches buyers and sellers to each other. While the ISP determines the amount of data that users can buy or sell, buyers always pay their bid prices for any data bought, and similarly sellers always receive their bid prices (any differences between the amounts paid and received go to the ISP). Thus, users have little incentive to lie about the prices that they are willing to accept (sellers) or pay (buyers).

1) Choosing Optimal Bids (Section II): When choosing how much data to bid, users must account for its effect on their usage in the rest of the month, which also depends on their unknown future usage preferences. For instance, buyers may use more data if they can buy data in the secondary market. However, users might not be able to trade their entire bid amount; thus, if they benefit more from trading a very small amount of data rather than an amount near the optimum, they may bid a smaller amount of data. *We show that it is optimal for users to assume they can trade their entire bid and derive the amount of data to bid as a function of the bid price, accounting for its effect on future usage.*

The prices that users bid affect whether their bids can be fully matched: for instance, some buyers may not pay the high price set by a seller. However, users do not know a priori how much of their bids can be matched, as they do not have information about the ISP's matching algorithm or the other users' bids. They can, however, learn from their previous trading experience and adjust their bid prices accordingly. We first examine ISPs' matching policies before proposing an algorithm for users to dynamically increase their likelihood of being matched in the secondary market.

2) Matching Buyers and Sellers (Section III): The ISP matches users so as to optimize its revenue, including volume-based administration fees and "bid" revenue, or the price difference between buyers who pay higher price and sellers who accept lower prices. Since buyers will buy more data in the secondary market due to its low prices as compared to ISP overage fees, the ISP can collect substantial administration fees, which can exceed its primary market revenue and compensate for a loss of overage revenue in the secondary data market. Moreover, buyers' purchased data comes from sellers' existing data caps, thus leading to less overall traffic. *We compare the users matched when the ISP optimizes its different types of revenue and derive conditions under which the ISP gains revenue as compared to the primary market.*

3) Market Dynamics (Section IV): As users participate in more matchings, they can more reliably estimate the amount of data they can buy or sell at given prices. This process forms a feedback loop between users and the ISP as users learn more about other users' bids and the ISP's matching algorithm. *We propose an algorithm for users to adjust their expectations of being matched and change their bid prices accordingly.*

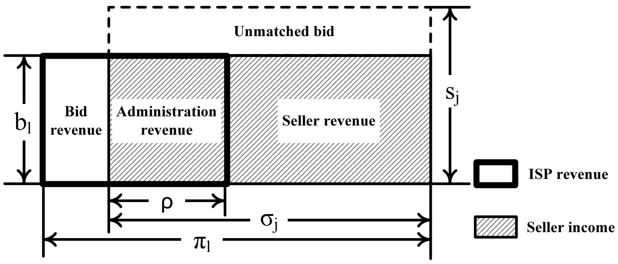


Fig. 1. Buyer-seller matching with their bids and ISP revenue.

We simulate the day-to-day market interactions over a one-year dataset of monthly usage for 100 U.S. ISP customers in Section V. We show that the buyers, sellers, and ISP can mutually benefit from the secondary market. We conclude the paper in Section VI. All proofs can be found in the Appendix.

II. User Trading Behavior

The secondary market consists of L buyers who purchase data from other users and J sellers who sell their leftover data. In this section, we discuss how sellers (Section II-A) and buyers (Section II-B) choose their bids to maximize their utilities,[2] and then consider how users choose whether to become a buyer or seller in Section II-C. Since users can choose whether or not to participate in the secondary market, they can benefit from having the option of participating. We now introduce notation and behaviorial considerations common to both buyers and sellers.

Since different users can purchase different data caps from their ISPs [11], we denote a buyer l and seller j's data caps before trading as d_l^b and d_j^s respectively. Each buyer and seller has a maximum amount of leftover data, denoted as o_l^b and o_j^s; thus, each user consumes at least $d_l^b - o_l^b$ (buyers) or $d_j^s - o_j^s$ (sellers) amount of data. For instance, users will likely have some predictable usage over a month, e.g., for habitual web browsing and checking email. Note that this leftover data must be less than the data cap: $o_l^b \leq d_l^b$ and $o_j^s \leq d_j^s$.

We define a buyer l's bid by an amount of data b_l and a price π_l that she is willing to pay. Similarly, each seller j bids a price σ_j for an amount of data s_j. Figure 1 shows how buyers' bids are matched to sellers' bids and how the ISP receives revenue in the secondary market. In this example, the buyer purchases her entire bid. The seller's income is split between the *administration revenue* paid to the ISP and the revenue kept by the seller. The ISP receives *bid revenue* from the difference between buyer and seller prices. The bid prices are lower bounded by an administration fee ρ per unit data sold that the ISP imposes on the sellers, as in CMHK's traded data plan [14]. Although the administration fee could also be charged to the buyers, it would give them less incentive to join the secondary market; however, sellers can always deduct this fee from their income and still be guaranteed a positive profit if $\pi_l \geq \rho$.[3] Thus, sellers will not accept a buyer l's price if $\pi_l < \rho$. The prices are upper-bounded by the ISP's overage fee p per unit data: buyers prefer to buy data from the ISP at price p rather than accept seller j's price if $\sigma_j > p$.

[2]The utility maximization may be performed by third-party agents working on behalf of buyers and sellers.

[3]Our model can easily be adapted to include both buyer and seller administration fees; since the qualitative results will not change, we assume seller-only fees for simplicity of presentation.

Without knowing other users' bids and the ISP's matching algorithm, each user decides the optimal amount of data to trade by maximizing his or her utility, given a price to accept or pay. Absent the cost or revenue from trading data, users gain utility from consuming data. We use the standard α-fair utility functions with $\alpha \in [0, 1)$ to model the usage utility from consuming c amount of data [24], [25]:

$$V(c) = \frac{\theta c^{1-\alpha}}{1-\alpha}, \tag{1}$$

where $\theta > 0$ is a normalization constant representing users' relative utility from their data consumption and payment to sellers (resp. income from buyers). A higher θ also scales up the marginal return of usage utility, encouraging users to consume more data. The strict concavity of this α-fair utility function captures the diminishing utility increase for heavy data consumption: as users consume more data, they receive less satisfaction from each additional unit of data consumed. As α increases, users' demands are more sensitive to the increase in usage utility.

A. Sellers' Optimal Bids

Since sellers can submit bids before the end of the month, they do not know their exact eventual monthly usage. Thus, we suppose that each seller j's realized usage c_j^s for the month is a random variable with distribution f. This distribution depends not only on the amount of data sold s_j, but also on the user's maximum leftover data o_j^s and data cap before trading d_j^s.

Figure 2 shows that the jth seller consumes at least $d_j^s - o_j^s$ amount of data, i.e., his minimum usage, and at most $d_j^s - s_j$ amount of data, i.e., the data cap after selling data ($s_j \leq o_j^s$). The jth seller's expected usage utility from selling s_j data is then $\int_{d_j^s-o_j^s}^{d_j^s-s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s$, and his revenue equals $(\sigma_j - \rho)s_j$, so the expected utility of the seller when selling s_j data is given by:

$$E(U_j^s \mid s_j) = \int_{d_j^s-o_j^s}^{d_j^s-s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s + (\sigma_j - \rho)s_j. \tag{2}$$

We note that (2) is always increasing in the price σ_j. *Thus, sellers always bid higher prices, subject to their ability to be matched to buyers (cf. Section IV).* Given σ_j and the distribution f, the seller chooses $s_j^\star(\sigma_j) \in [0, o_j^s]$ so as to maximize the utility (2). Though it is possible that the seller will not be able to sell all of his data, it is still optimal for the seller to bid the utility-maximizing $s_j^\star$ as long as $E(U_j^s \mid s_j)$ is concave: If $E(U_j^s \mid s_j)$ is concave, then $E(U_j^s \mid s_j)$ increases in s_j for $s_j \in [0, s_j^\star]$. Thus, the seller always increases his utility by bidding the maximum amount of data up to the optimal amount. Though we formulate this utility maximization problem in terms of a general distribution f, to provide analytical insights, we show below some illustrative distributions for which $E(U_j^s \mid s_j)$ is concave.

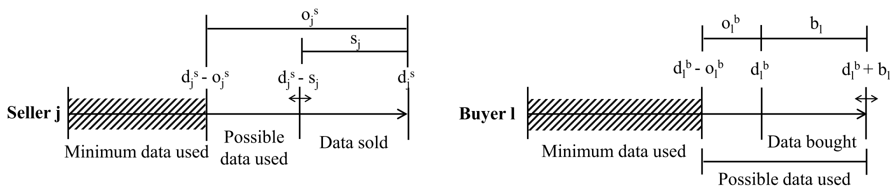


Fig. 2. Relationships between the data caps (d_j^s and d_l^b), leftover data (o_j^s and o_l^b), and data sold/bought (s_j and b_l).

1) Example Distributions: We first consider the extreme case of a delta distribution, where a user's realized usage for the month only has one possibility: for instance, some sellers may only use the minimum data (i.e., f is a delta distribution centered at $d_j^s - o_j^s$), while others may use up their entire data caps in the month, (i.e., f is the delta distribution centered at $d_j^s - s_j$). This distribution can model a seller whose usage is consistent from month to month, e.g., someone who mainly uses cellular data while commuting to and from work, and otherwise uses WiFi. In the former case, $E(U_j^s \mid s_j)$ is linear in s_j and the seller bids $s_j^\star = o_j^s$ amount of data. In the latter case, the utility function in (2) can be written as:

$$E_\delta(U_j^s \mid s_j) = V_j^s(d_j^s - s_j) + (\sigma_j - \rho)s_j. \tag{3}$$

Thus, we compute the optimal bid as $s_j^\star = \max\left\{0, \min\left\{o_j^s, d_j^s - \left((\sigma_j - \rho)/\theta_j^s\right)^{-1/\alpha_j^s}\right\}\right\}$.

In most cases, the seller's usage will fall somewhere between these two extremes. We thus follow [12] in supposing that it follows a uniform distribution $f(c_j^s) = (o_j^s - s_j)^{-1}$ between $d_j^s - o_j^s$ and $d_j^s - s_j$. For instance, a user who consumes more data when traveling may have unpredictable data usage based on his or her travel plans in a given month. In this case, we first show that $E(U_j^s \mid s_j)$ is a concave function.

Proposition 1: The utility function of the jth seller $E(U_j^s \mid s_j)$ in (2) is concave in s_j if $f(c_j^s)$ is a uniform distribution. Then, the optimal bid $s_j^\star$ satisfies

$$(o_j^s - s_j^\star)(\sigma_j - \rho) = V_j^s(d_j^s - s_j^\star) - \int_{d_j^s - o_j^s}^{d_j^s - s_j^\star} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s. \tag{4}$$

We now observe that $s_j^\star$ is increasing in σ_j, as we could intuitively expect:

Corollary 1: The optimal amount sold $s_j^\star(\sigma_j)$ for each seller j increases as σ_j increases if $E(U_j^s \mid s_j)$ is concave.

To solve for $s_j^\star$ satisfying (4), we use the nonlinear Perron-Frobenius theory for Algorithm 1. We refer the reader to [26], [27] for more details of the nonlinear Perron-Frobenius theory and its applications in wireless networks.

Lemma 1: Algorithm 1 converges geometrically fast to the fixed point $s_j^\star$ in (4) from any initial point $s_j(0)$ if $s_j^\star \leq d_j^s - \left(\theta_j^s(1 + \alpha_j^s o_j^s / d_j^s)/(2(\sigma_j - \rho))\right)^{1/\alpha_j^s}$.

Since the right-hand side of Lemma 1's condition decreases in the utility scaling factor θ_j^s, we expect it to be satisfied for relatively small θ_j^s. For such θ_j^s, the user has relatively low utility from usage, as we would expect from a seller. We formalize this intuition in Section II-C.

Algorithm 1 Sellers' Utility Maximization

Initialize $\mathbf{s}(0) \in (\mathbf{0}, \mathbf{o}^s)$.

1) The jth seller updates the data caps to be sold:

$$s_j(k+1) = o_j^s - \frac{1}{\sigma_j - \rho} V_j^s(d_j^s - s_j(k)) + \frac{1}{\sigma_j - \rho} \int_{d_j^s - o_j^s}^{d_j^s - s_j(k)} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s.$$

2) Normalize $s_j(k+1)$:

$$s_j(k+1) \leftarrow \min\left\{ s_j(k+1), d_j^s - \left(\frac{\theta_j^s\left(d_j^s + \alpha_j^s o_j^s\right)}{2d_j^s(\sigma - \rho)} \right)^{\frac{1}{\alpha_j^s}} \right\}.$$

B. Buyers' Optimal Bids

Like the sellers, buyers do not exactly know their future usage. Thus, we take the buyer's monthly usage c_l^b to be a random variable with distribution $f(c_l^b)$ between the minimum usage $d_l^b - o_l^b$ and data cap after trading $d_l^b + b_l$ (Figure 2). Hence, the expected data usage utility of the lth buyer purchasing b_l amount of data is given by $\int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b$. Each buyer l's cost of purchasing b_l amount of data is $b_l \pi_l$, so the expected utility of the lth buyer is

$$E(U_l^b \mid b_l) = \int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b - b_l \pi_l. \tag{5}$$

Since (5) is decreasing in π_l, *buyers wish to bid at lower prices, subject to their ability to be matched to sellers (Section IV).* As with the seller, the buyer will always bid her utility-maximizing $b_l^\star$ if $E(U_l^b \mid b_l)$ is concave.

1) Example Distributions: As for sellers in Section II-A, some buyers will use only their minimum usage $d_l^b - o_l^b$; these buyers will therefore not purchase any data in the market. Other buyers will use up their entire data caps, i.e., their distributions f will be the delta distribution centered at $d_l^b + b_l$. The utility function under this delta distribution is given by

$$E_\delta(U_l^b \mid b_l) = V_l^b(d_l^b + b_l) - \pi_l b_l, \tag{6}$$

yielding the optimal data bid $b_l^\star(\pi_l) = \max\left\{ \left(\pi_l/\theta_l^b\right)^{-1/\alpha_l^b} - d_l^b, 0\right\}$.

In most cases, however, the buyer's usage will lie between the two extremes; as for sellers, this distribution models users with unpredictable data usage over the month. We thus consider f to be the uniform distribution $f(c_l^b) = 1/(o_l^b + b_l)$. We first show that the utility in (5) is concave:

Proposition 2: The utility function of the lth buyer $E(U_l^b \mid b_l)$ in (5) is concave in b_l if $f(c_l^b)$ is a uniform distribution. Then, the optimal bid $b_l^\star$ satisfies:

$$(o_l^b + b_l^\star)\pi_l = V_l^b(d_l^b + b_l^\star) - \int_{d_l^b - o_l^s}^{d_l^b + b_l^\star} V_l^b(c_l^b) f(c_l^b)\mathrm{d}c_l^b. \quad (7)$$

We also note that $b_l^\star$ is a decreasing function of the price, as we would intuitively expect:

Corollary 2: The optimal bid $b_l^\star(\pi_l)$ for each buyer l decreases as π_l increases if $E(U_l^b \mid b_l)$ is concave.

We again use the nonlinear Perron-Frobenius theory in [26] to solve for $b_l^\star$ in Algorithm 2.

Algorithm 2 Buyers' Utility Maximization

Initialize $\mathbf{b}(0) \in \mathbb{R}_+^L$.

1) The lth buyer updates the amount of data to be purchased:

$$b_l(k+1) = \frac{1}{\pi_l} V_l^b(d_l^b + b_l(k)) - \frac{1}{\pi_l}\int_{d_l^b - o_l^b}^{d_l^b + b_l(k)} V_l^b(c_l^b) f(c_l^b)\mathrm{d}c_l^b - o_l^b.$$

2) Normalize $b_l(k+1)$:

$$b_l(k+1) \leftarrow \min\left\{ b_l(k+1), \left(\frac{\theta_l^b(d_l^b + \alpha_l^b o_l^b)}{2 d_l^b \pi_l}\right)^{\frac{1}{\alpha_l^b}} - d_l^b \right\}.$$

Lemma 2: Algorithm 2 converges geometrically fast to the fixed point $b_l^\star$ in (7) from any initial point $b_l(0)$ if $b_l^\star \le \left(\theta_l^b(1 + \alpha_l^b o_l^b / d_l^b)/(2\pi_l)\right)^{1/\alpha_l^b} - d_l^b$.

We thus observe that the algorithm converges for buyers with high utility scaling factors θ_l^b. We show in the next section that buyers will likely satisfy this condition.

C. Selling or Buying Data

Users choose to become a buyer or seller based on the utilities they can achieve from buying or selling data. Thus, if

$$E(U_j^s \mid s_j^\star(p)) \ge E(U_l^b \mid b_l^\star(\rho)), \quad (8)$$

the user becomes a seller: the user's maximum utility from selling data (assuming all data is sold at the maximum price) must be higher than the maximum utility from purchasing data (assuming all data is bought at the minimum price).[4] If (8) is reversed, the user becomes a buyer instead.

To illustrate this decision, we suppose that the user's usage follows the delta distribution. We then derive the following necessary condition on users' utility scaling factor θ in the usage utility function (1):

Corollary 3: A user sells data when the scaling factor θ satisfies $\theta \le \hat{\theta}$ and buys data otherwise, where

$$\hat{\theta} = \left(\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{(p-\rho)d_j^s - \rho d_l^b}{\rho^{\frac{\alpha-1}{\alpha}} - (p-\rho)^{\frac{\alpha-1}{\alpha}}}\right)\right)^{\alpha}. \quad (9)$$

Thus, users with high utility scaling θ become buyers, while those with low θ become sellers.

[4]Here we assume the user can always sell or buy all the bid data. More generally, the user could estimate the maximum amount of data he or she could sell or buy at a given price using past experience (Section IV); the amount of data sold is the minimum of this quantity and the optimal bid amount. Users buy (resp. sell) data if the resulting utility is higher for buying (selling) data at the prices maximizing these utilities.

III. ISP Trading Policies

The ISP will match buyers and sellers so as to optimize its revenue, subject to constraints imposed by user bids. We analyze the optimal matching in Section III-B before considering whether the resulting revenue exceeds that of the primary data market in Section III-C.

A. ISP Optimization

The ISP will often encounter sellers' and buyers' bids that are not exactly aligned: for instance, if a seller offers more data than any single buyer is willing to purchase. To facilitate the matching of such bids, we suppose that the ISP can match multiple buyers to multiple sellers. Since the ISP acts as a middleman, this flexibility is transparent to all users. All required accounting can be done internally by the ISP.

We denote the matching between buyers and sellers with a matrix $\mathbf{\Omega} = [\Omega_{lj}]_{l,j=1}^{L,J} \ge 0$. Each (l, j) entry of $\mathbf{\Omega}$ represents the percentage of the lth buyer's demand (i.e., amount of data bid) b_l that is satisfied by the jth seller's data supply s_j; thus, $\Omega_{lj} b_l$ represents the amount of data that buyer l purchases from seller j. Note that the ISP can take any bids from users (e.g., $s_j = s_j^\star(\sigma_j)$ and $b_l = b_l^\star(\pi_l)$) in the matching optimization.

1) Matching Constraints: The ISP's matching is primarily constrained by the buyer and seller bids. Buyer l's bid of a price π_l and amount of data b_l constrains the ISP matching in two ways: first, the buyer will buy at most b_l amount of data, leading to the feasible set

$$\mathcal{B} = \left\{ \mathbf{\Omega} \,\Big|\, \sum_{j=1}^{J} \Omega_{lj} \le 1, \quad l = 1, \ldots, L \right\}. \quad (10)$$

We thus suppose that the buyer will accept matchings in which her bid is only partially matched (Section II).

Second, the buyer's price π_l gives an upper bound on the average purchase price of her data. We assume that the buyer will pay this bid price π_l for all data purchased; the amount paid, $\pi_l \sum_j \Omega_{lj} b_l$, must be at least as much as the data cost specified by sellers' bid prices (i.e., a cost $\sigma_j \Omega_{lj} b_l$ for each seller j). Mathematically, we have the feasible set

$$\Pi = \left\{ \mathbf{\Omega} \,\Big|\, \sum_{j=1}^{J} \Omega_{lj}\sigma_j \le \pi_l \sum_{j=1}^{J} \Omega_{lj}, \ l = 1, \ldots, L \right\}. \quad (11)$$

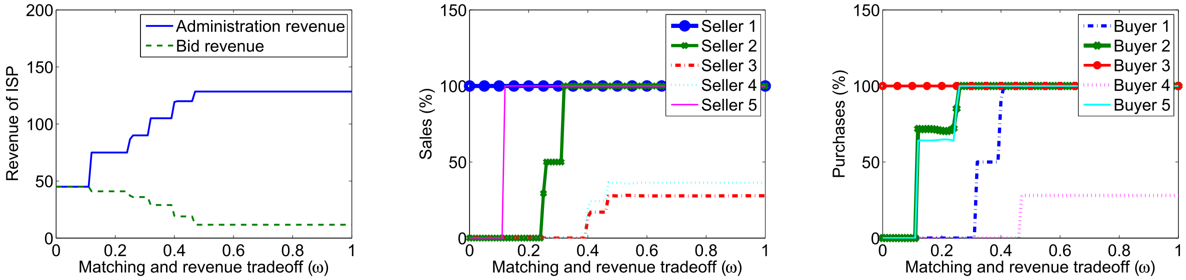
If the total amount paid by the buyer exceeds the data cost, the ISP keeps the excess as part of its bid revenue.

Similarly, seller j's bid of a price σ_j and amount of data s_j implies that he will sell at most s_j amount of data:

$$\mathcal{S} = \left\{ \mathbf{\Omega} \,\Big|\, \sum_{l=1}^{L} \Omega_{lj} b_l \le s_j, \quad j = 1, \ldots, J \right\}. \quad (12)$$

In return, the total money paid by all buyers for seller j's data $\sum_l \Omega_{lj} b_l \pi_l$ must be at least the cost of the data $\sigma_j \sum_l \Omega_{lj} b_l$:

$$\Sigma = \left\{ \mathbf{\Omega} \,\Big|\, \sum_{l=1}^{L} \pi_l \Omega_{lj} b_l \ge \sigma_j \sum_{l=1}^{L} \Omega_{lj} b_l, \ j = 1, \ldots, J \right\}. \quad (13)$$



(a) ISP Revenue. (b) Matched percentage of sellers' data. (c) Matched percentage of buyers' data.

Fig. 3. ISP revenue and user matching with $\rho = 15$, $p = 60$, $\mathbf{s} = (3, 2, 3, 2, 2)^\top$, $\boldsymbol{\sigma} = (35, 45, 48, 48, 42)^\top$, $\mathbf{b} = (2, 1, 3, 2, 2)^\top$ and $\boldsymbol{\pi} = (35, 45, 50, 35, 40)^\top$. Seller 1 and Buyer 3 offer the lowest and highest prices respectively, and can always trade all their data. Users with the highest selling price (Seller 3 and 4) and the lowest purchasing price (Buyer 4) can trade data when ω is sufficiently large (Proposition 4).

Thus, the ISP must choose $\boldsymbol{\Omega} \in \mathcal{B} \cap \Pi \cap \mathcal{S} \cap \Sigma$, which can be written as a set of linear constraints as in (10)-(13).

Intuitively, if sellers and buyers bid sufficiently low and high prices respectively, they can be matched to at least one other user. We derive these price thresholds using (11) and (13):

Proposition 3 (Price Feasibility): If seller j sells data to at least one buyer ($\sum_l \Omega_{lj} b_l > 0$), then his selling price σ_j is not higher than all buyers' purchasing prices: $\sigma_j \leq \max_l \pi_l$.

Analogously, if buyer l purchases data from at least one seller (i.e., $\sum_j \Omega_{lj} > 0$), then her purchasing price is not lower than all sellers' selling prices, i.e., $\pi_l \geq \min_j \sigma_j$.

2) ISP Objective: The ISP's objective in choosing a matching $\boldsymbol{\Omega}$ is to maximize its revenue from the secondary market. We identify two sources of ISP revenue: "administration revenue" and "bid revenue" (Figure 1).

The ISP's revenue from the administration fee is proportional to the volume of data traded, i.e., $\rho \sum_{l,j} \Omega_{lj} b_l$. To calculate the bid revenue, we add the differences between each buyer's payment and each seller's income: $\sum_l \left(\pi_l \sum_j \Omega_{lj} b_l - \sum_j \sigma_j \Omega_{lj} b_l \right)$. From (11), this gap is always positive. The ISP thus maximizes its revenue by solving the linear program

$$\begin{aligned} &\text{maximize } \omega\rho \sum_{j=1}^{J} \sum_{l=1}^{L} \Omega_{lj} b_l \\ &\qquad + (1-\omega) \sum_{l=1}^{L} \sum_{j=1}^{J} (\Omega_{lj} b_l \pi_l - \Omega_{lj} b_l \sigma_j) \\ &\text{subject to } \boldsymbol{\Omega} \in \mathcal{B} \cap \mathcal{S} \cap \Pi \cap \Sigma, \\ &\qquad\quad \boldsymbol{\Omega} \geq \mathbf{0}, \\ &\text{variable: } \boldsymbol{\Omega}. \end{aligned} \tag{14}$$

The parameter ω trades off between administration revenue and bid revenue; its effect is our next subject of discussion.[5] We use $\boldsymbol{\Omega}^\star$ to denote the optimal solution to (14). We then characterize the optimal solution $\boldsymbol{\Omega}^\star$ affected by the ISP's choice of ω and user bids in the discussion below.

We note that, if a seller bids $\tilde{s}_j > s_j^\star$, this does not improve his chance of having $\sum_l \Omega_{lj}^\star b_l^\star = s_j^\star$ at the optimal point of (14), since buyers may bid lower than the price that the seller is willing to accept or they may not have a large enough demand to accommodate the seller's offer. If buyers do have sufficient demand to meet the seller's bid, the seller may have $s_j^\star < \sum_l \Omega_{lj}^\star b_l^\star = \tilde{s}_j$, yielding suboptimal utility for him. Similarly, buyers do not bid more than their optimal amounts $b_l^\star$.

B. Matching Buyers and Sellers

Solving (14) with $\omega = 0.5$, i.e., weighting the bid revenue and administration revenue equally, maximizes the ISP's total revenue in the secondary market. However, changing ω can lead to different matching outcomes. The ISP can thus incorporate other considerations into its matching objective.

Taking $\omega < 0.5$, i.e., preferentially weighting the bid revenue, is equivalent to reducing the administration fee ρ. When the ISP preferentially weights its bid revenue, it attempts to match buyers with high prices to sellers with low prices, increasing the difference in the amount paid by buyers and sellers. In contrast, when maximizing its administration revenue, the ISP wishes to maximize the total amount of data traded. Thus, for higher ω (i.e., preferential weight to administration revenue) the ISP might match a seller to buyers with both higher and lower prices; buyers' prices π_l would then average out to equal the seller's price σ_j, and the seller would be able to trade more data than if he had only been matched to buyers with higher π_l. Indeed, we can derive a necessary condition on ω under which such matchings occur:

Proposition 4 (Matching Feasibility): If $\pi_l < \sigma_j$ and

$$\omega < \frac{\max_j \sigma_j - \min_l \pi_l}{\rho + \left(\max_j \sigma_j - \min_l \pi_l\right)}, \tag{15}$$

then the ISP will not match buyer l to seller j.

If $\omega = 1$, then ω never satisfies (15), and low-price buyers can be matched to sellers with higher prices. The amount matched of a user's bid thus depends on both the other users' bids as well as the ISP's matching objective.

Figure 3 illustrates the effect of varying ω with the matching outcomes for five users. When the ISP preferentially weights bid revenue, i.e., ω is small, only the seller with the lowest

[5] We will show in Section III-C that the expected total data consumed in the secondary market decreases with the volume of data traded. Thus, maximizing the administration revenue is equivalent to minimizing the cost of handling network traffic as well as to maximizing the ISP's profit.

TABLE I

COMPARISON OF USER UTILITY AND ISP REVENUE IN THE PRIMARY MARKET AND THE SECONDARY MARKET

	Primary market	Secondary market
Buyer l	$E(U_l^b \mid b_l^\star(p)) = \int_{d_l^b-o_l^b}^{d_l^b+b_l^\star(p)} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b - pb_l^\star(p)$	$E(U_l^b \mid b_l^\star(\pi_l)) = \int_{d_l^b-o_l^b}^{d_l^b+b_l^\star(\pi_l)} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b - \pi_l b_l^\star(\pi_l)$
Seller j	$E(U_j^s \mid s_j^\star = 0) = \int_{d_j^s-o_j^s}^{d_j^s} V_j^s(c_j^s)f(c_j^s)\mathrm{d}c_j^s$	$E(U_j^s \mid s_j^\star(\sigma_j)) = \int_{d_j^s-o_j^s}^{d_j^s-s_j^\star(\sigma_j)} V_j^s(c_j^s)f(c_j^s)\mathrm{d}c_j^s + (\sigma_j - \rho)s_j^\star(\sigma_j)$
Data consumed	$\sum_{j=1}^{J}\left(d_j - \frac{1}{2}o_j^s\right) + \sum_{l=1}^{L}\left(d_l^b - \frac{1}{2}(o_l^b - b_l^\star(p))\right)$	$\sum_{j=1}^{J}\left(d_j - \frac{1}{2}o_j^s\right) + \sum_{l=1}^{L}\left(d_l^b - \frac{1}{2}\left(o_l^b - \max\left\{b_l^\star(p) - \sum_{j=1}^{J}\Omega_{lj}^\star b_l^\star(\pi_l), 0\right\}\right)\right)$
Revenue	$p\sum_{l=1}^{L} b_l^\star(p)$	$\rho\sum_{l=1}^{L}\Omega_{lj}^\star b_l^\star(\pi_l) + \sum_{l=1}^{L}\sum_{j=1}^{J}(\pi_l - \sigma_j)\,\Omega_{lj} b_l^\star(\pi_l) + p\sum_{l=1}^{L}\hat{b}_l^\star(p)$

price (Seller 1) and the buyer with highest price (Buyer 3) are matched. However, as ω increases, more users are matched; in fact, for $\omega > 0.44$, buyers 1 and 4 both purchase data, even though their bid prices are lower than all the sellers' bid prices. Furthermore, as ω increases, the administration revenue $\rho \sum_j \sum_l \Omega_{lj} b_l$ increases, but the bid revenue $\sum_l \sum_j (\Omega_{lj} b_l \pi_l - \Omega_{lj} b_l \sigma_j)$ decreases.

Even for $\omega = 1$, some buyers and sellers may not be matched to any user. We can in fact derive price thresholds for buyers and sellers above (resp. below) which all the buyers (sellers) can trade some data, and below (above) which no buyer (seller) trades any data:

Proposition 5 (Price Competition): Suppose that the sellers are sorted with price ascending ($\sigma_{j+1} \geq \sigma_j$) and the buyers are sorted with price descending ($\pi_{l+1} \leq \pi_l$). Then $\mathbf{\Omega}^\star$ is a block matrix with all the non-zero entries in the northwest corner:

1) If the mth buyer is not matched with any seller ($\sum_j \Omega_{mj} = 0$), then all buyers $l > m$ (i.e., whose bid prices are lower than that of buyer m) are also unmatched: all the entries below an all-zero row in $\mathbf{\Omega}^\star$ are zero.
2) If the nth seller is not matched with any buyer ($\sum_l \Omega_{ln} = 0$), then all sellers $j > n$ (i.e., whose bid prices are higher than that of seller n) are also unmatched: all the entries to the right of an all-zero column in $\mathbf{\Omega}^\star$ are zero.

The buyers and sellers compete with one another on the basis of price. Buyers paying higher prices and sellers accepting lower prices thus have more opportunities to trade data.

C. Comparison to the Primary Market

In the absence of a secondary market, the buyers would buy overage data from the ISP instead of purchasing from other users. Thus, in the secondary market, the buyers purchase more data due to lower prices, while sellers consume less data to gain revenue by selling data to buyers. Since the ISP receives administration revenue in proportion to the amount of data sold in the secondary market, its revenue might be larger than the revenue earned in the primary market. We mathematically formulate these differences in Table I, which compares users' utilities, expected total data consumed, and ISP revenue in both markets.

Figure 4 illustrates ISP and user behavior in the primary and secondary markets for the simplified case of one buyer and one seller. In the primary market, the buyers purchase

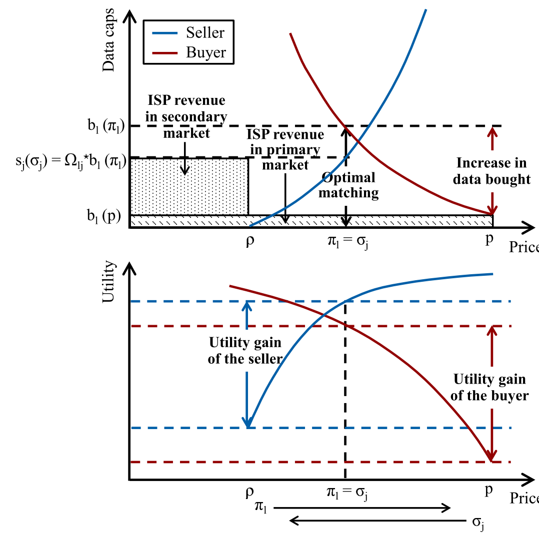


Fig. 4. As illustrated here for one seller and one buyer, users always increase their utilities and the ISP can earn more revenue in the secondary market.

data from the ISP at the maximum price p. The lth buyer thus maximizes her utility by purchasing $b_l^\star(p)$ data from the ISP. Hence, the revenue of the ISP in the primary market is $p \sum_{l=1}^{L} b_l^\star(p)$. Sellers do not participate in the primary market, which is equivalent to letting $\sigma_j = \rho$ in the secondary market: at this price, the seller does not earn any revenue from selling data and loses utility if he sells data; therefore, the jth seller's utility in the primary market is calculated when $s_j^\star = 0$. Hence, the expected amount of users' data caps consumed in the primary market is $\sum_{j=1}^{J} \int_{d_j^s-o_j^s}^{d_j^s} c_j^s f(c_j^s)\mathrm{d}c_j^s + \sum_{l=1}^{L} \int_{d_l^b-o_l^b}^{d_l^b+b_l^\star(p)} c_l^b f(c_l^b)\mathrm{d}c_l^b$, as shown in the first column and third row of Table I (if $f(c_j^s)$ and $f(c_l^b)$ are uniform distributions).

In Figure 4, the one seller and the one buyer are matched when their prices align ($\sigma_j = \pi_l$). Thus, in the secondary market, the lth buyer purchases $b_l^\star(\pi_l)$ amount of data, where $\pi_l < p$. Since $b_l^\star(\cdot)$ monotonically decreases with respect to the price (Corollary 2), $b_l^\star(p) < b_l^\star(\pi_l)$ and the buyer bids more data in the secondary than she would purchase in the primary market. However, not all sellers' and buyers' bids are always fully satisfied, i.e., the constraints in (10) and (12) may not be tight at optimality. For example, the amount of data offered by the seller in Figure 4 is less than the amount of data bid by the buyer, and then the buyer's bid will be only partially satisfied, i.e., $\Omega^\star_{lj} b_l(\pi_l) < b_l(\pi_l)$. However, she may still purchase $\hat{b}^\star_l(p)$ amount of overage data from the ISP, with $\hat{b}^\star_l(p)$ chosen by maximizing the utility:

$$E(U^b_l \mid \hat{b}_l) = \int_{d^b_l - o^b_l}^{d^b_l + \sum_{j=1}^J \Omega^\star_{lj} b^\star_l(\pi_l) + \hat{b}_l} V^b_l(c^b_l) f(c^b_l) \mathrm{d}c^b_l - \hat{b}_l p, \tag{16}$$

leading to (cf. the proof of Proposition 6)

$$\hat{b}^\star_l(p) = \max\left\{ b^\star_l(p) - \sum_{j=1}^J \Omega^\star_{lj} b^\star_l(\pi_l), 0 \right\}. \tag{17}$$

The buyer is then expected to consume the amount of data $\int_{d^b_l - o^b_l}^{d^b_l + \sum_{j=1}^J \Omega^\star_{lj} b^\star_l(\pi_l) + \hat{b}^\star_l(p)} c^b_l f(c^b_l) \mathrm{d}c^b_l$ in the secondary market, where after trading, the buyer's distribution of data usage f may change. For instance, we have $f(c^b_l) = \left(o^b_l + \sum_{j=1}^J \Omega^\star_{lj} b^\star_l(\pi_l) + \hat{b}^\star_l(p)\right)^{-1}$ if it is a uniform distribution. On the other hand, if only part of seller j's bid is matched, he is expected to consume the amount of data $\int_{d^s_j - o^s_j}^{d^s_j - \sum_{l=1}^L \Omega^\star_{lj} b^\star_l(\pi_l)} c^s_j f(c^s_j) \mathrm{d}c^s_j$ in the secondary market. Similarly, if it is a uniform distribution, the seller's usage distribution becomes $f(c^s_j) = \left(o^s_j - \sum_{l=1}^L \Omega^\star_{lj} b^\star_l(\pi_l)\right)^{-1}$. Hence, the expected total data consumed in the secondary market is computed as in Table I.

Although buyers would purchase more data in the secondary market, the data is transferred from sellers to buyers. Even though buyers may still purchase overage data from the ISP when $\sum_{j=1}^J \Omega^\star_{lj} < 1$, we can see from (17) that they purchase less overage data than in the primary market. This is because after trading, buyers purchase overages starting from larger data caps:

Proposition 6 (Traffic decrease): If user usage follows a uniform distribution, the expected total data consumed in the secondary market is less than that in the primary market.

Furthermore, if the users' usage distributions are delta distributions as in Sections II-A and II-B, they always use up all of the data that they have after trading, and thus the statement in Proposition 6 also holds. From Table I, we can see that the expected data consumed in the secondary market is a piecewise linear decreasing function of $\sum_{l=1}^L \sum_{j=1}^J \Omega^\star_{lj} b^\star_l(\pi_l)$, i.e., the volume of data traded, and that the data trading can in fact avoid the extra traffic due to overage purchases if buyers can purchase at least the amount of overage data that they would buy in the primary market in the secondary market. Thus, in order to account for the traffic cost in (14), we can simply adjust the weight of the administration revenue.

We notice that ISP revenue when offering data trading consists of administration revenue, bid revenue, and the buyers' overage purchase if their bids are not fully matched, while ISP revenue in the primary market only has the buyers' overage charge. However, the unit price of the overage data is higher than the price that the buyers pay to the sellers, leading the buyers to purchase more data in the secondary market than they would in the primary market. Thus, although the ISP loses revenue from overage fees, it may gain more revenue from the administration fees and bid revenue. If we suppose the best matching result, i.e., all constraints (10)-(13) are tight, the secondary market may allow the ISP to recover the revenue lost from the primary market:

Proposition 7 (Revenue benefit): A necessary condition for the ISP to earn more revenue in the secondary market than in the primary market is

$$\frac{p}{\rho} \le \min_{l,\dots,L} \frac{b^\star_l(\rho)}{b^\star_l(p)}. \tag{18}$$

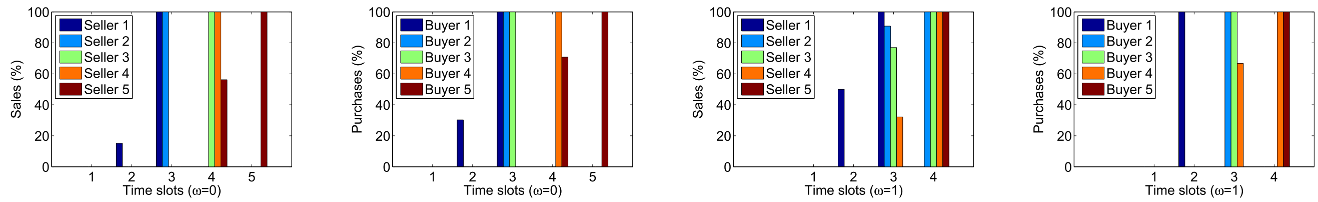
For instance, if the buyers' future usage distributions are delta distributions as in Section II-B, then $b^\star_l(\pi_l) = (\pi_l/\theta^b_l)^{-1/\alpha^b_l} - d^b_l$. Thus, $b^\star_l(\rho)/b^\star_l(p) > (p/\rho)^{1/\alpha^b_l} > p/\rho$, and the ISP can earn more revenue in the secondary market. Proposition 7 show that the administration fee has an impact on the ISP revenue, and it should not be set too low.

IV. Dynamic Data Trading

A sustainable secondary market must allow buyers and sellers to submit new bids at any time, thus necessitating the ISP to run multiple matchings in a given month. Moreover, buyers and sellers can *actively learn* from each matching outcome: for instance, if a seller is not matched to any buyer, this seller can lower his price in the next bid to attract buyers.[6] We incorporate these initial prices and price adjustments responding to the matching outcome in the following dynamics, which are formalized in Algorithm 3:

1) Users can submit their bids to the ISP at any time during the month. Initially, any users who are joining the secondary market for the first time choose $\pi_l(0)$ and $\sigma_j(0)$. They then calculate the optimal amounts of data to bid $b_l(0) = b^\star_l(\pi_l)$ and $s_j(0) = s^\star_j(\sigma_j)$ as in Algorithms 1 and 2. Optimistic buyers (resp. sellers) might choose their initial prices as $\pi_l(0) = \rho$ (resp. $\sigma_j(0) = p$), though buyers and sellers would likely have to raise and lower their prices respectively before they could be matched. Risk-averse sellers and buyers, on the other hand, would respectively abide by the minimum and maximum prices to ensure that they will be matched. Other users would leverage their past experience in prior months, choosing a price in $[\rho, p]$.
2) Upon receiving bids from at least one seller and one buyer, the ISP runs the matching optimization (14). The participants in each iteration k may be different: users with their bids fulfilled in the last matching outcome, i.e., $\sum_l \Omega_{lj}(k+1) b_l(k) = s_j(k)$ or $\sum_j b_l(k) \Omega_{lj}(k+1) = b_l(k)$, will quit the matching, and new users may join the data trading. The ISP can update all user bids that it receives before rerunning the matching. Thus, the ISP must run many matchings over a month. As such,

[6]Note that this learning is not perfect; the set of user bids can and likely will change from matching to matching, so the user can only estimate, not deterministically predict, his or her matching outcome from past experience. Moreover, the user's feedback from past matchings is limited to the amount of data successfully bought or sold, which is likely insufficient to deduce the distributions of other users' bids or the ISP's matching algorithm.



(a) Seller $(\omega = 0)$ (b) Buyer $(\omega = 0)$ (c) Seller $(\omega = 1)$ (d) Buyer $(\omega = 1)$

Fig. 5. Illustration of the data trading framework in Algorithm 3 with five sellers and five buyers. The parameter setting is as follows: $\rho = 15$, $\epsilon = 5$, $p = 60$, $\mathbf{s}(0) = (2, 2, 2, 2, 2)^\top$, $\boldsymbol{\sigma}(0) = (52, 54, 56, 58, 60)^\top$, $\mathbf{b}(0) = (1, 1, 2, 3, 3)^\top$ and $\boldsymbol{\pi}(0) = (42, 39, 36, 33, 30)^\top$. We plot the percentage of each buyer/seller's total amount of data bid that has been successfully matched at each iteration. In (a) and (b), $\omega = 0$; while in (c) and (d), $\omega = 1$.

Algorithm 3 Data Trading Dynamics

At $k = 0$, the jth seller initializes $\sigma_j(0)$ and $s_j^\star(\sigma_j(0))$, and the lth buyer initializes $\pi_l(0)$ and $b_l^\star(\pi_l(0))$.

while $L(k) > 0$ **and** $J(k) > 0$ **do**

1) Upon receiving bids $\big(b_l(k), \pi_l(k)\big)$ and $\big(s_j(k), \sigma_j(k)\big)$ from $L(k)$ buyers and $J(k)$ sellers, the ISP updates the constraint sets $\mathcal{B} \equiv \mathcal{B}(k)$, $\Pi \equiv \Pi(k)$, $\mathcal{S} \equiv \mathcal{S}(k)$ and $\Sigma \equiv \Sigma(k)$.

2) The ISP computes $\mathbf{\Omega}(k+1)$ by solving (14) with $L(k)$, $J(k)$, $s_j(k)$, $\sigma_j(k)$, $b_l(k)$, $\pi_l(k)$ as L, J, s_j, σ_j, b_l, π_l.

3) Each seller j of all $J(k+1) = J(k)$ sellers updates the bid price and amount of data:

if $\sum_l \Omega_{lj}(k+1) b_l(k) < s_j(k)$ **then**
$d_j^s(k+1) \leftarrow d_j^s(k) - \sum_l \Omega_{lj}(k+1) b_l(k)$,
$o_j^s(k+1) \leftarrow o_j^s(k) - \sum_l \Omega_{lj}(k+1) b_l(k)$,
$\sigma_j(k+1) \leftarrow \max\{\sigma_j(k) - \epsilon_j^s(k), \rho\}$,
Run Algorithm 1 to obtain $s_j(k+1)$.
end if
if $\sum_l \Omega_{lj}(k+1) b_l(k) = s_j(k)$ **then**
Transaction is successful: $J(k+1) \leftarrow J(k+1) - 1$.
end if

4) Each buyer l of all $L(k+1) = L(k)$ buyers updates the bid price and amount of data:

if $\sum_j b_l(k) \Omega_{lj}(k+1) < b_l(k)$ **then**
$d_l^b(k+1) \leftarrow d_l^b(k) + \sum_j b_l(k) \Omega_{lj}(k+1)$,
$o_l^b(k+1) \leftarrow o_l^b(k) + \sum_j b_l(k) \Omega_{lj}(k+1)$,
$\pi_l(k+1) \leftarrow \min\{\pi_l(k) + \epsilon_l^b(k), p\}$,
Run Algorithm 2 to obtain $b_l(k+1)$.
end if
if $\sum_j b_l(k) \Omega_{lj}(k+1) = b_l(k)$ **then**
Transaction is successful: $L(k+1) \leftarrow L(k+1) - 1$.
end if

5) New sellers and buyers submit bids to the ISP:
A new seller submits bid: $J(k+1) \leftarrow J(k+1) + 1$.
A new buyer submits bid: $L(k+1) \leftarrow L(k+1) + 1$.

6) $k \leftarrow k + 1$.

end while

the number of matchings can vary from month to month depending on how frequently users submit bids as well as how fast their bids are fulfilled.

3) Users respond to the matching outcome from the ISP. In each iteration k, if only a portion of the bid is matched, the buyers and sellers increase and decrease their prices by $\epsilon_l^b(k)$ and $\epsilon_j^s(k)$ respectively, subject to the constraints that $\pi_l(k), \sigma_j(k) \in [\rho, p]$. They then recompute the amounts of data to bid with these new prices and submit the new bids to the ISP. Users can set ϵ based on their transaction history and their risk preferences: a larger ϵ changes the price more, increasing the likelihood of being matched but lowering their utilities.

V. Numerical Evaluation

A. Trading Dynamics

We now analyze the data trading dynamics in Algorithm 3 with a five-user example. Figure 5 shows the fractions of their total bids that each seller and buyer trades in each iteration. In Figures 5(a) and 5(b), $\omega = 0$ (i.e., the ISP optimizes its bid revenue), while in Figures 5(c) and 5(d), $\omega = 1$ (optimizing the administration revenue). The sellers and buyers are ordered respectively in increasing and decreasing order of price.

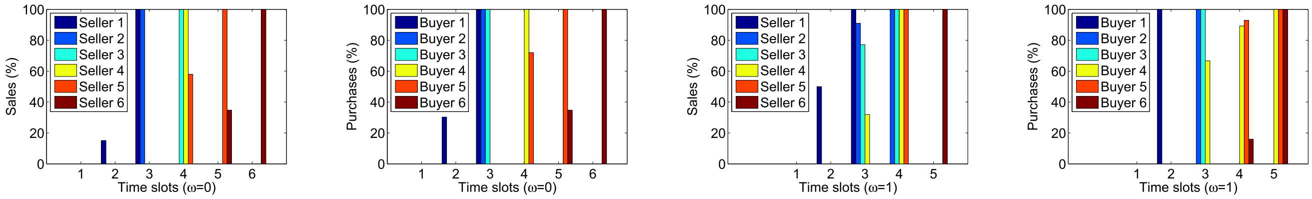
As shown in Figures 5(a) and 5(c), the matching optimization always matches the sellers with lower prices first; the sellers finish selling their bids in increasing order of their prices. Conversely, as shown in Figures 5(b) and 5(d), the matching optimization always matches the buyers with higher prices first, and the buyers finish purchasing their bids in decreasing order of their prices. Thus, buyers with higher price bids and sellers with lower price bids are more likely to be matched. Moreover, the users in Figures 5(c) and 5(d) ($\omega = 1$) are all matched one iteration earlier than those in Figures 5(a) and 5(b) ($\omega = 0$): the ISP matches more users when optimizing administration revenue rather than bid revenue.

In Figure 6, we suppose that new users enter the market at the third time slot. One new seller submits a 2GB bid with a price higher than the other sellers' highest price; at the same time, one new buyer submits a 2GB bid, with a price lower than the other buyers' lowest price. These prices reflect the fact that new participants do not have the experience to realistically estimate the amount of data they can buy or sell at different prices. However, by adjusting their prices the users adapt quickly: the new seller and buyer finish their trading within three time slots.

B. Experiments With User Data

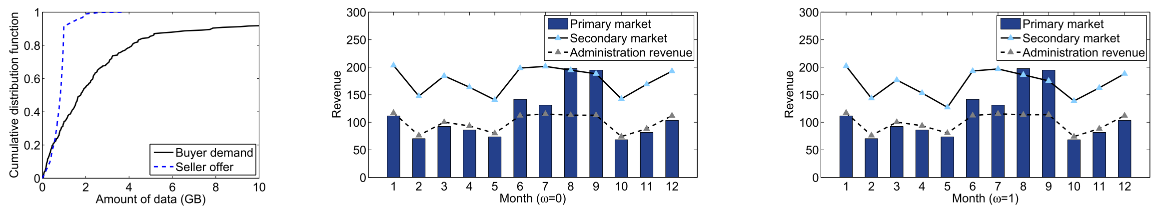
We now simulate Algorithm 3 using real-world data usage. Our data comes from 100 mobile users of a U.S. ISP from January to December 2013. The data contains in-network RADIUS records at a session level for each user and their monthly data plans (i.e., data caps and overage fees).

We classify the users as sellers and buyers using (9). Each user is assumed to have a uniform distribution of future usage.



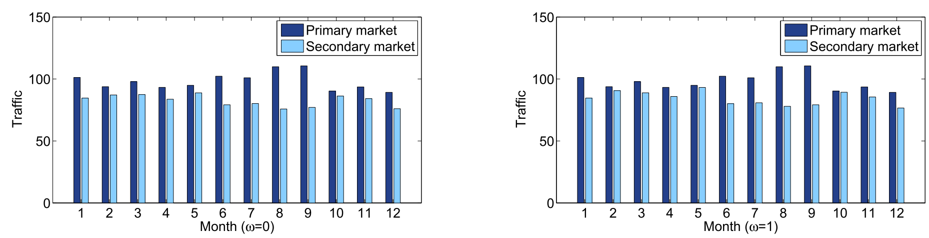
(a) Seller ($\omega = 0$) (b) Buyer ($\omega = 0$) (c) Seller ($\omega = 1$) (d) Buyer ($\omega = 1$)

Fig. 6. Matching in successive timeslots (Algorithm 3) with parameters as in Figure 5. One new buyer and one new seller join the market at the third time slot. In (a) and (b), $\omega = 0$; while in (c) and (d), $\omega = 1$.



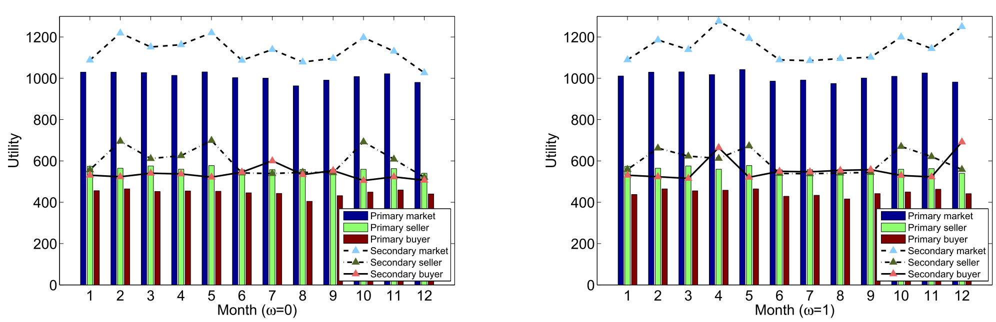
(a) CDF of user bids. (b) Comparison of the revenue ($\omega = 0$). (c) Comparison of the revenue ($\omega = 1$).

Fig. 7. The ISP usually but not always gains revenue in the secondary compared to the primary market ($\rho = 2$, $p = 4$, $\alpha_j^s = \alpha_l^b = 0.6$, $\theta_j^s = 3.3$, $\theta_l^b = 9$).



(a) Comparison of the network traffic ($\omega = 0$). (b) Comparison of the network traffic ($\omega = 1$).

Fig. 8. The overall network traffic in the secondary market is less than that in the primary market (parameters as in Figure 7).



(a) Comparison of the utility ($\omega = 0$). (b) Comparison of the utility ($\omega = 1$).

Fig. 9. Buyers and sellers always increase their utilities in the secondary market (parameters as in Figure 7).

Figure 7(a) shows the distribution of buyer and seller bids over all twelve months; we see that sellers' bids are generally much smaller than buyers', as some buyers bid an enormous amount of data (e.g., for regular HD video streaming). However, there are fewer buyers than sellers. In each month, the users keep trading as described in Algorithm 3 until either sellers' or buyers' bids are all met. We do not consider the case that buyers may purchase overage data if their bids can only be partially satisfied (except in Figure 8).

We calculate the total bid and administration revenue and the resulting total traffic in the network for each month, and compare them to those of the primary market in Figures 7 and 8 respectively. For the purpose of simulation, we suppose a uniform distribution for user usage in both primary and secondary markets. As expected, Figure 8 shows that overall network traffic in the secondary market is reduced compared to the primary market. In most months, the ISP's administration revenue alone is larger than the revenue from the primary market due to a large increase in sellers' and buyers' bids matched (Proposition 6). In a few months, e.g., June to September, the primary market yields more revenue and interestingly, traffic in the secondary market drops more:

although the sellers do not bid enough data to completely satisfy buyers' demand in the secondary market, they fulfill the buyers with the amount of their overage purchase in the primary market, leading to $\hat{b}_l^\star(p) = 0$. We can also observe from Figure 7 that the gap between the total revenue and administration revenue (i.e., the bid revenue) in the secondary market is slightly larger when $\omega = 0$ as in Figure 7(b) than when $\omega = 1$ as in Figure 7(c). We observe a similar pattern for the gap between the total amounts of network traffic in the primary and secondary markets as in Figure 8: at $\omega = 0$, the ISP explicitly maximizes its bid revenue and makes less effort to maximize the total amount of data traded.

Figure 9 shows the total utilities, the utilities of the buyers and the utilities of the sellers in the primary market and secondary market. As we would expect, the utilities of the sellers and buyers in the secondary market are always larger than those in the primary market. The amount of this differential, however, varies from month to month.

VI. Conclusion

Mobile data trading brings forth new challenges in network economics, since traded data plans in the secondary market affect how users and the ISP behave strategically in conjunction with pricing in the primary market. We first derive the optimal amounts of data in users' bids, depending on the bidding prices that sellers and buyers choose to bid in the secondary market. We take into account uncertainty in users' usage and the amounts of data caps that they need. We then establish a necessary condition under which a user will choose to buy or sell data in the secondary market.

The ISP matches the buyers and sellers in the secondary market by solving a linear program to maximize its revenue subject to users' bid constraints. We compare the optimal matchings when bid or administration revenue is emphasized and derive a necessary condition under which the ISP gains revenue in the secondary market as compared to the primary market. Furthermore, we show that the total amount of data consumed in the secondary market is less than that of the primary market, thus benefitting ISPs' operational costs, but that user demands are better matched to their preferred data caps at the prices that they are willing to pay. With dynamic matchings over multiple times in the month, we examine how users adapt their bids over time to increase the chance of being matched. Finally, we simulate these dynamics over one year of usage data from a U.S. ISP, demonstrating a unique and sustainable market for trading mobile data that is beneficial both for the ISP and for users who desire customized pricing schemes.

Appendix

A. Nonlinear Perron-Frobenius Theory in [26]

Let $\|\cdot\|$ be a monotone norm on $\mathbb{R}^L$. For a concave mapping $f : \mathbb{R}_+^L \to \mathbb{R}_+^L$ with $f(\mathbf{z}) > 0$ for $\mathbf{z} \geq \mathbf{0}$, the following statements hold. The conditional eigenvalue problem $f(\mathbf{z}) = \lambda \mathbf{z}$, $\lambda \in \mathbb{R}$, $\mathbf{z} \geq \mathbf{0}$, $\|\mathbf{z}\| = 1$ has a unique solution $(\lambda^*, \mathbf{z}^*)$, where $\lambda^* > 0$, $\mathbf{z}^* > \mathbf{0}$. Furthermore, $\lim_{k\to\infty} \tilde{f}(\mathbf{z}(k))$ converges geometrically fast to $\mathbf{z}^*$, where $\tilde{f}(\mathbf{z}) = f(\mathbf{z})/\|f(\mathbf{z})\|$.

B. Proof of Proposition 1

Proof: Taking the second-order derivative of $E(U_j^s \mid s_j)$ in (2) with respect to s_j, we have:

$$\mathrm{d}^2 E(U_j^s \mid s_j)/\mathrm{d}s_j^2 = \frac{(d_j^s - s_j)^{2-\alpha_j^s}}{(1-\alpha_j^s)(2-\alpha_j^s)(o_j^s - s_j)^3}\Psi(o_j^s),$$

where

$$\Psi(o_j^s) = \frac{2(d_j^s - s_j)^2}{(d_j^s - s_j)^2} - \frac{2(2-\alpha_j^s)(o_j^s - s_j)(d_j^s - s_j)}{(d_j^s - s_j)^2} + \frac{(1-\alpha_j^s)(2-\alpha_j^s)(o_j^s - s_j)^2}{(d_j^s - s_j)^2} - 2\left(\frac{d_j^s - o_j^s}{d_j^s - s_j}\right)^{2-\alpha_j^s}.$$

Next, we show that $\Psi(o_j^s)$ decreases with o_j^s by taking the first-order derivative of $\Psi(o_j^s)$, given by:

$$\begin{aligned}
&\mathrm{d}\Psi(o_j^s)/\mathrm{d}o_j^s \\
&= \frac{2(2-\alpha_j^s)}{(d_j^s - s_j)^2}\Big(-(d_j^s - s_j) + (1-\alpha_j^s)(o_j^s - s_j) \\
&\quad + (d_j^s - o_j^s)^{1-\alpha_j^s}(d_j^s - s_j)^{\alpha_j^s}\Big) \\
&= \frac{2(2-\alpha_j^s)}{(d_j^s - s_j)^2}\Big(-\alpha_j^s(d_j^s - s_j) - (1-\alpha_j^s)(d_j^s - o_j^s) \\
&\quad + (d_j^s - o_j^s)^{1-\alpha_j^s}(d_j^s - s_j)^{\alpha_j^s}\Big) \leq 0,
\end{aligned}$$

where the inequality holds due to the inequality of arithmetic-geometric means that $\alpha_j^s(d_j^s - s_j) + (1-\alpha_j^s)(d_j^s - o_j^s) \geq (d_j^s - o_j^s)^{1-\alpha_j^s}(d_j^s - s_j)^{\alpha_j^s}$ for all $\alpha_j^s \in (0,1)$. Since $o_j^s \in [s_j, d_j^s]$, we have $\Psi(o_j^s) \leq \Psi(o_j^s = s_j) = 0$, which also means $\mathrm{d}^2 E(U_j^s \mid s_j)/\mathrm{d}s_j^2 \leq 0$. Thus, $E(U_j^s \mid s_j)$ is concave. ■

C. Proof of Corollary 1

Proof: Consider two prices for seller j, σ_j^1 and σ_j^2, with $\sigma_j^1 < \sigma_j^2$. Then from (4), the optimal amounts sold $s_j^\star$ satisfy

$$\frac{\mathrm{d}}{\mathrm{d}s_j}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s\right)\Bigg|_{s_j^\star(\sigma_j^i)} = \rho - \sigma_j^i.$$

for $i = 1, 2$. Since $\rho - \sigma_j^1 > \rho - \sigma_j^2$, we have

$$\frac{d}{ds_j}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s\right)\Bigg|_{s_j^\star(\sigma_j^1)} > \frac{d}{ds_j}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s\right)\Bigg|_{s_j^\star(\sigma_j^2)}.$$

Since $\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s$ is a concave function by Proposition 1, its first derivative is a decreasing function of s_j. Thus, $s_j^\star\left(\sigma_j^2\right) > s_j^\star\left(\sigma_j^1\right)$ as desired. ■

D. Proof of Lemma 1

Proof: We first prove below that the self-mapping function at Step 1 of Algorithm 1 is concave when $s_j \leq d_j^s - \left(\frac{1+\alpha_j^s o_j^s/d_j^s}{2(\sigma-\rho)}\right)^{1/\alpha_j^s}$. From (4), we have the following self-mapping function:

$$s_j = g(s_j) = o_j^s + \frac{1}{\sigma_j - \rho}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s - V_j^s(d_j^s - s_j)\right),$$

i.e., the self-mapping function at Step 1 of Algorithm 1. Hence, $g(s_j)$ is a concave self-mapping function if the following function $h(s_j)$ is concave:

$$h(s_j) = \int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s - V_j^s(d_j^s - s_j).$$

Taking the second-order derivative of $h(s_j)$ with respect to s_j, we have:

$$\begin{aligned} h''(s_j) =& \ 2(o_j^s - s_j)^{-2} \int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s \\ & - 2(o_j^s - s_j)^{-2} V_j^s(d_j^s - s_j) + (o_j^s - s_j)^{-1} V_j^{s\prime}(d_j^s - s_j) \\ & - V_j^{s\prime\prime}(d_j^s - s_j). \end{aligned}$$

Combining (7) with (19), we can obtain:

$$\begin{aligned} h''(s_j) =& -2(o_j^s - s_j)^{-1}(\sigma_j - \rho) \\ & + (o_j^s - s_j)^{-1} V_j^{s\prime}(d_j^s - s_j) - V''(d_j^s - s_j) \\ =& \ \frac{(o_j^s - s_j)^{-1}}{(d_j^s - s_j)^{\alpha_j^s}}\Bigg(-2(\sigma_j - \rho)(d_j^s - s_j)^{\alpha_j^s} \\ & + \theta_j^s + \theta_j^s \alpha_j^s \frac{o_j^s - s_j}{d_j^s - s_j}\Bigg). \end{aligned} \tag{19}$$

Due to that $s_j \leq d_j^s - \left(\frac{\theta_j^s(1+\alpha_j^s o_j^s/d_j^s)}{2(\sigma-\rho)}\right)^{1/\alpha_j^s}$, we have

$$\begin{aligned} (\sigma - \rho)(d_j^s - s_j)^{\alpha_j^s} \ \geq& \ \frac{\theta_j^s}{2}\left(1 + \alpha_j^s \frac{o_j^s}{d_j^s}\right)^{1/\alpha_j^s} \\ \Rightarrow& \ (\sigma - \rho)(d_j^s - s_j)^{\alpha_j^s} \\ \geq& \ \frac{\theta_j^s}{2}\left(1 + \alpha_j^s \frac{o_j^s - s_j}{d_j^s - s_j}\right)^{1/\alpha_j^s}, \end{aligned}$$

which implies that $h''(s_j) \leq 0$. Therefore, $h(s_j)$ is concave so that $g(s_j)$ is a concave self-mapping. Furthermore, the normalization at Step 2 of Algorithm 1 is a monotone norm constraint of s_j. Then, the nonlinear Perron-Frobenius theory (cf. Appendix A) can be leveraged for the algorithm design. ∎

E. Proof of Proposition 2

Proof: Taking the second-order derivative of $E(U_l^b \,|\, b_l)$ in (5) with respect to b_l, we have:

$$\mathrm{d}E^2(U_l^b \,|\, b_l)/\mathrm{d}b_l^2 = \frac{(d_l^b + b_l)^{2-\alpha_l^b}}{(1-\alpha_l^b)(2-\alpha_l^b)(o_l^b + b_l)} \Psi(o_l^b),$$

where

$$\begin{aligned} \Psi(o_l^b) =& \ \frac{2(d_l^b + b_l)^2}{(d_l^b + b_l)^2} - \frac{2(2-\alpha_l^b)(o_l^b + b_l)(d_l^b + b_l)}{(d_l^b + b_l)^2} \\ & + \frac{(1-\alpha_l^b)(2-\alpha_l^b)(o_l^b + b_l)^2}{(d_l^b + b_l)^2} - 2\left(\frac{d_l^b - o_l^b}{d_l^b + b_l}\right)^{2-\alpha_l^b}. \end{aligned}$$

Next, we show that $\Psi(o_l^b)$ decreases with o_l^b by taking the first-order derivative of $\Psi(o_l^b)$, given by:

$$\begin{aligned} & \mathrm{d}\Psi(o_l^b)/\mathrm{d}o_l^b \\ &= \frac{2(2-\alpha_l^b)}{(d_l^b + b_l)^2}\Big(-(d_l^b + b_l) + (1-\alpha_l^b)(o_l^b + b_l) \\ &\quad + (d_l^b - o_l^b)^{1-\alpha_l^b}(d_l^b + b_l)^{\alpha_l^b}\Big) \\ &= \frac{2(2-\alpha_l^b)}{(d_l^b + b_l)^2}\Big(-(1-\alpha_l^b)(d_l^b - o_l^b) - \alpha_l^b(d_l^b + b_l) \\ &\quad + (d_l^b - o_l^b)^{1-\alpha_l^b}(d_l^b + b_l)^{\alpha_l^b}\Big), \end{aligned}$$

where the inequality holds due to the inequality of arithmetic-geometric means that $(1-\alpha_l^b)(d_l^b - o_l^b) + \alpha_l^b(d_l^b + b_l) \geq (d_l^b - o_l^b)^{1-\alpha_l^b}(d_l^b + b_l)^{\alpha_l^b}$ for all $\alpha_l^b \in (0,1)$. Since $o_l^b \in [0, d_j^s]$, we have $\Psi(o_l^b) \leq \Psi(o_l^b = 0) \leq \Psi(o_l^b = -b_l) = 0$, which also means $\mathrm{d}E^2(U_l^b \,|\, b_l)/\mathrm{d}b_l^2 \leq 0$. Thus, $E(U_l^b \,|\, b_l)$ is concave. ∎

F. Proof of Corollary 2

Proof: Consider two prices for buyer l, π_l^1 and π_l^2, with $\pi_l^1 < \pi_l^2$. Then from (4), the optimal amounts sold $b_l^\star$ satisfy

$$\frac{\mathrm{d}}{\mathrm{d}b_l}\left(\int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b\right)\Bigg|_{b_l^\star(\pi_l^i)} = \pi_l^i.$$

for $i = 1, 2$. Since $\pi_l^1 < \pi_l^2$, we have

$$\begin{aligned} & \frac{d}{db_l}\left(\int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b\right)\Bigg|_{b_l^\star(\pi_l^1)} \\ & \quad < \frac{d}{db_l}\left(\int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b\right)\Bigg|_{b_l^\star(\pi_l^2)}. \end{aligned}$$

Since $\int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b$ is a concave function by Proposition 1, its first derivative is a decreasing function of b_l. Thus, $b_l^\star\left(\pi_l^1\right) > b_l^\star\left(\pi_l^2\right)$ as desired. ∎

G. Proof of Lemma 2

Proof: Similar to the proof in Appendix D, we first prove below that the self-mapping function at Step in of Algorithm 2 is concave when $b_l \leq \left(\frac{1+\alpha_l^b o_l^b/d_l^b}{2\pi_l}\right)^{1/\alpha_l^b}$. From (7), we have the following self-mapping function:

$$\begin{aligned} b_l &= g(b_l) \\ &= \frac{1}{\pi_l}\left(V_l^b(d_l^b + b_l) - \int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b\right) - o_l^b, \end{aligned}$$

i.e., the self-mapping function at Step of Algorithm 2. Hence, $g(b_l)$ is a concave self-mapping function if the following function $h(b_l)$ is concave:

$$h(b_l) = V_l^b(d_l^b + b_l) - \int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b)\mathrm{d}c_l^b.$$

Taking the second-order derivative of $h(b_l)$ with respect to b_l, we have:

$$\begin{aligned} h''(b_l) = & -2(o_l^b + b_l)^{-2} \int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b)\mathrm{d}c_l^b \\ & + 2(o_l^b + b_l)^{-2} V_l^b(d_l^b + b_l) - (o_l^b + b_l)^{-1} V_l^{b'}(d_l^b + b_l) \\ & + V_l^{b''}(d_l^b + b_l). \end{aligned} \tag{20}$$

Combing (7) with (20), we can obtain:

$$\begin{aligned} h''(b_l) &= 2(o_l^b + b_l)^{-1}\pi_l - (o_l^b + b_l)^{-1} V_l^{b'}(d_l^b + b_l) \\ &\quad + V_l^{b''}(d_l^b + b_l) \\ &= \frac{(o_l^b + b_l)^{-1}}{(d_l^b + b_l)^{\alpha_l^b}} \Bigg(2\pi_l (d_l^b + b_l)^{\alpha_l^b} \\ &\qquad - \upsilon\theta_l^b - \theta_l^b \alpha_l^b \frac{o_l^b + b_l}{d_l^b + b_l} \Bigg). \end{aligned}$$

Due to that $b_l \le \left(\frac{\theta_l^b(1+\alpha_l^b o_l^b/d_l^b)}{2\pi_l}\right)^{1/\alpha_l^b} - d_l^b$, we have

$$\begin{aligned} 2\pi_l(d_l^b + b_l)^{\alpha_l^b} &\le \theta_l^b\left(1 + \alpha_l^b \frac{o_l^b}{d_l^b}\right) \\ &\Rightarrow 2\pi_l(d_l^b + b_l)^{\alpha_l^b} \le \theta_l^b\left(1 + \alpha_l^b \frac{o_l^b + b_l}{d_l^b + b_l}\right), \end{aligned}$$

which implies that $h''(b_l) \le 0$. Therefore, $h(b_l)$ is concave so that $g(b_l)$ is a concave self-mapping. Furthermore, the normalization at Step 2 of Algorithm 2 is a monotone norm constraint of b_l. Then, the fixed-point algorithm converges to the unique optimal solution by leveraging the nonlinear Perron-Frobenius theory (cf. Appendix A). ■

H. Proof of Corollary 3

Proof: We consider the special cases of a delta distribution that are respectively centered at $d_j^s - s_j$ if the user is a seller and $d_l^b + b_l$ if the user is a buyer. Unlike uniform distribution, users could gain the most usage utility by consuming their entir data. As the utility functions for a seller and a buyer are given in (3) and (6) respectively, the optimality conditions in (4) and (7) can be rewritten respectively as $s_j^\star = d_j^s - \left(\frac{1}{\theta}(\sigma_j - \rho)\right)^{-\frac{1}{\alpha}}$ and $b_l^\star = \left(\frac{\pi_l}{\theta}\right)^{-\frac{1}{\alpha}} - d_l^b$. Then, we obtain the maximum utilities for the seller and the buyer, given respectively by

$$E_\delta(U_j^s \,|\, s_j^\star(\sigma_j)) = \frac{\alpha}{1-\alpha}(\sigma_j - \rho)^{1-\frac{1}{\alpha}}\theta^{\frac{1}{\alpha}} + (\sigma_j - \rho)d_j^s,$$

and

$$E_\delta(U_l^b \,|\, b_l^\star(\pi_l)) = \frac{\alpha}{1-\alpha}\pi_l^{1-\frac{1}{\alpha}}\theta^{\frac{1}{\alpha}} - \pi_l d_l^b.$$

By substituting $E_\delta(U_j^s \,|\, s_j^\star(p))$ and $E_\delta(U_l^b \,|\, b_l^\star(\rho))$ back into (8), we can obtain (9). ■

I. Proof of Proposition 3

Proof: Due to $\sum_{j=1}^J \Omega_{lj} b_l \ge 0$ and $\sum_{l=1}^L \Omega_{lj} b_l \ge 0$, the inequality constraints in (11) and (13) can be rewritten respectively, as:

$$\xi_1\sigma_1 + \xi_2\sigma_2 + \cdots + \xi_J\sigma_J \le \pi_l,$$

where $\xi_j = \Omega_{lj} b_l / \left(\sum_{j=1}^J \Omega_{lj} b_l\right)$ and we have $\xi_1 + \xi_2 + \ldots, \xi_J = 1$, and,

$$\eta_1\pi_1 + \eta_2\pi_2 + \cdots + \eta_L\pi_L \ge \sigma_j,$$

where $\eta_l = \Omega_{lj} b_l / \left(\sum_{l=1}^L \Omega_{lj} b_l\right)$ and we have $\eta_1 + \eta_2 + \cdots + \eta_L = 1$. In other words, π_l should be higher than at least one nonnegative linear combination of all the selling prices $\sigma_1, \ldots, \sigma_J$, and σ_j should be lower than at least one nonnegative linear combination of all the purchasing prices $\pi_l, \ldots, \pi_L$. Since we also have $\min_{\mathbf{1}^\top \boldsymbol{\xi}=1, \boldsymbol{\xi} \ge \mathbf{0}} \{\xi_1\sigma_1 + \xi_2\sigma_2 + \cdots + \xi_J\sigma_J\} = \min_{j=1,\ldots,J}\{\sigma_j\}$ and $\max_{\mathbf{1}^\top \boldsymbol{\eta}=1, \boldsymbol{\eta} \ge \mathbf{0}} \{\eta_1\pi_1 + \eta_2\pi_2 + \cdots + \eta_L\pi_L\} = \max_{l=1,\ldots,L}\{\pi_l\}$, this completes the proof. ■

J. Proof of Proposition 4

Proof: We form the Lagrangian for (14) by introducing the dual variables $\mathbf{Z} \in \mathbb{R}_+^{L\times J}$, $\mathbf{x} \in \mathbb{R}_+^L$, $\mathbf{y} \in \mathbb{R}_+^J$, $\boldsymbol{\mu} \in \mathbb{R}_+^L$ and $\boldsymbol{\nu} \in \mathbb{R}_+^J$ respectively for the constraints $\Omega_{lj} \ge 0$, $l = 1, \ldots, L$, $j = 1, \ldots, J$, $\sum_{j=1}^J \Omega_{lj} b_l \le b_l$, $l = 1, \ldots, L$, $\sum_{l=1}^L \Omega_{lj} b_l \le s_j$, $j = 1, \ldots, J$, $\sum_{j=1}^J \Omega_{lj} b_l \sigma_j \le \pi_l\left(\sum_{j=1}^J \Omega_{lj} b_l\right)$, $l = 1, \ldots, L$, and $\sum_{l=1}^L \Omega_{lj} b_l \pi_l \ge \sigma_j\left(\sum_{l=1}^L \Omega_{lj} b_l\right)$, $j = 1, \ldots, J$. Then, we can obtain the Lagrangian for (14), given by:

$$\begin{aligned} &\mathcal{L}(\boldsymbol{\Omega}, \mathbf{Z}, \mathbf{x}, \mathbf{y}, \boldsymbol{\mu}, \boldsymbol{\nu}) \\ &= \omega\rho \sum_{j=1}^J \sum_{l=1}^L \Omega_{lj} b_l \\ &\quad + (1-\omega) \sum_{l=1}^L \sum_{j=1}^J (\Omega_{lj} b_l \pi_l - \Omega_{lj} b_l \sigma_j) + \sum_{l=1}^L \sum_{j=1}^J Z_{lj}\Omega_{lj} \\ &\quad - \sum_{l=1}^L \sum_{j=1}^J x_l(\Omega_{lj} b_l - b_l) - \sum_{l=1}^L \sum_{j=1}^J y_j(\Omega_{lj} b_l - s_j) \\ &\quad - \sum_{l=1}^L \sum_{j=1}^J \mu_l(\Omega_{lj} b_l \sigma_j - \Omega_{lj} b_l \pi_l) \\ &\quad - \sum_{l=1}^L \sum_{j=1}^J \nu_j(\Omega_{lj} b_l \sigma_j - \Omega_{lj} b_l \pi_l). \end{aligned} \tag{21}$$

Taking first-order derivative of (21) with respect to Ω_{lj} and setting it to zero, we have the following equation at optimality:

$$Z_{lj}^\star = (x_l^\star + y_j^\star - \omega\rho)b_l + \left(\mu_l^\star + \nu_j^\star + (1-\omega)\right)(\sigma_j - \pi_l)b_l. \tag{22}$$

If $\exists l, j$ such that $\Omega_{lj}^\star > 0$ and $\sigma_j > \pi_l$, we have $Z_{lj}^\star = 0$ and (22) can then be rewritten as $\omega = \frac{(x_l^\star + y_j^\star) + (\mu_l^\star + \nu_j^\star + 1)(\sigma_j - \pi_l)}{\rho + (\sigma_j - \pi_l)}$, which leads to the inequality:

$$\omega \geq \frac{\sigma_j - \pi_l}{\rho + (\sigma_j - \pi_l)}, \tag{23}$$

due to the nonnegativity of the dual variables $x_l^\star$, $y_j^\star$, $\mu_l^\star$ and $\nu_j^\star$. Then, (15) is sufficient for (23). ■

K. Proof of Proposition 5

Proof: By inspecting the Lagrangian for (14) formed in (21) and the optimality condition derived in (22), we establish the following proof. If the lth buyer is not matched with any seller, we have $\sum_{j=1}^{J} \Omega_{lj} b_l = 0$. By using the complementary slackness at optimality, we have $\Omega_{lj}^\star Z_{lj}^\star = 0$ and $\Omega_{lj}^\star = 0,\ j = 1, \ldots, J \ \Rightarrow\ Z_{lj}^\star > 0,\ j = 1, \ldots, J$. We can also derive from $x_l^\star (b_l - \sum_{j=1}^{J} \Omega_{lj}^\star b_l) = 0$ and $\mu_l^\star \sum_{j=1}^{J} (\Omega_{lj}^\star b_l \pi_l - \Omega_{lj}^\star b_l \sigma_j) = 0$ that $x_l^\star = 0$ and $\mu_l^\star > 0$. For the mth buyer where $\pi_m < \pi_l$, the price constraint $\sum_{j=1}^{J} \Omega_{mj}^\star b_m \sigma_j \leq \pi_m \left(\sum_{j=1}^{J} \Omega_{mj}^\star b_m \right)$ is tighter than the price constraint for the lth buyer so we have $\mu_m^\star > \mu_l^\star$. Since $x_m^\star \geq x_l^\star = 0$ always holds for all the dual variables, we can conclude from the above derivation that $Z_{mj}^\star / b_m \geq Z_{lj}^\star / b_l > 0,\ j = 1, \ldots, J$ (cf. (22)). Hence, we have $Z_{mj}^\star > 0,\ j = 1, \ldots, J$, i.e., the mth buyer, whose purchasing price π_m is lower than π_l, is also unmatched. Similar proof can be applied to the second bullet point for the sellers in Proposition 5. ■

L. Proof of Proposition 6

Proof: We first show how to calculate the result in (17). To maximize the utility in (16), the overage data purchased when $\sum_{j=1}^{J} \Omega_{lj}^\star < 1$ satisfies

$$\frac{\mathrm{d}}{\mathrm{d}\hat{b}_l} \left(\int_{d_l^b - o_l^b}^{d_l^b + \sum_{j=1}^{J} \Omega_{lj}^\star b_l^\star(\pi_l) + \hat{b}_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b \right) = p$$

$$\Rightarrow \frac{\mathrm{d}}{h(\hat{b}_l)} \frac{h(\hat{b}_l)}{\mathrm{d}\hat{b}_l} \left(\int_{d_l^b - o_l^b}^{d_l^b + h(\hat{b}_l)} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b \right) = p,$$

where $h(\hat{b}_l) = \sum_{j=1}^{J} \Omega_{lj}^\star b_l^\star(\pi_l) + \hat{b}_l$ and $h(\hat{b}_l)/\mathrm{d}\hat{b}_l = 1$. Thus, at optimality of (16), we have $h(\hat{b}_l^\star(p)) = b_l^\star(p)$, leading to (17). From (17), we can observe that $\hat{b}_l^\star(p) < b_l^\star(p)$. Comparing the expected total data consumed in primary and secondary markets shown in Table I, we can easily find that the expected total data consumed in the secondary market is less than that in the primary market due to $\hat{b}_l^\star(p) < b_l^\star(p)$. ■

M. Proof of Proposition 7

Proof: Suppose we have the best matching result by solving (14), which means all constraints (10)-(13) are tight, then we have $\sum_{j=1}^{J} s_j^\star(\sigma_j) = \sum_{l=1}^{L} b_l^\star(\pi_l) \leq \sum_{l=1}^{L} b_l^\star(\rho)$ and no revenue from the buyer/seller price difference. If revenue of the secondary market is higher than the revenue of the primary market, we have $\rho \sum_{j=1}^{J} s_j^\star(\sigma_j) \geq p \sum_{l=1}^{L} b_l^\star(p)$. Then, $\rho \sum_{l=1}^{L} b_l^\star(\rho) \geq p \sum_{l=1}^{L} b_l^\star(p)$ implies (18). ■

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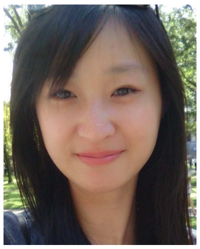

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