

due by Thursday, March 8

Problem 1. Suppose the production function in a firm is given by $F(x_1, x_2)$ (ie, given two inputs x_1 and x_2 , the firm can produce the final output in the amount $y = F(x_1, x_2)$). Suppose F is concave, increasing in each of its inputs and differentiable (as many times as you need). The elasticity of substitution between the two inputs is defined as

$$(1) \quad \sigma = - \frac{\partial \log x_1(p_1, p_2, y) / x_2(p_1, p_2, y)}{\partial \log p_1 / p_2},$$

where p_1, p_2 are prices (in terms of the final good y) of inputs, and $x_i(p_1, p_2, y)$ is the derived demand for input x_i (ie, how much of this input the firm will need if it is maximizing profits, needs to produce final output y , and faces prices p_1 and p_2 of inputs). Assume also that the cost function of producing final output y is given by $C(p_1, p_2, y)$ (which is also differentiable as many times as you need).

- (1) Make sure the elasticity of substitution is well-defined.
- (2) What is the elasticity of substitution in terms of the cost function C ?
- (3) What is the elasticity of substitution in terms of the production function F ?

Problem 2. Assume that independent research discovers that consumption of your company's daily vitamin (at one tablet per day dosage) reduces the contemporaneous annual risk of cancer by about 2% and that other research has established that people are willing to pay about \$200 per year to reduce their contemporaneous annual risk of cancer by 10%. Assume that sales of vitamins are currently 100,000 bottles per year.

- (1) How much do you estimate you could increase the price per bottle and still sell as many bottles after people learn about the health benefits of your vitamins?
- (2) How much would wide-spread knowledge of this research increase the demand for your company's product if a 50 tablet bottle of vitamins sells for \$80 and the elasticity of demand for your company's vitamins is -2.0 ?
- (3) What is the social value of this new information? How much will health improve as a result of this new information?
- (4) How much should your company be willing to spend on R&D to develop an improved vitamin that costs 8 cents more per tablet to

produce but would cut cancer deaths by an additional 3% (for a total reduction of 5%)?

- (5) Now assume that people respond to this new information by changing their diets in a way that offsets the cancer reducing benefits of your product. How would this affect your analysis? Does this reduce the value of the innovation?