Abstract

I study the effects of disclosing financial information on the occurrence of bank runs and on management risk-taking activities. The main trade-off is between the risk of bank runs, which increases with a disclosure delay, and managerial incentives for risk taking, which runs discipline. I find that the main policy consideration is the growth rate of bank assets. If bank assets grow sufficiently slowly, then the optimal policy is to disclose with a lag, in order to balance managerial risk taking and creditors’ coordination problems. When bank assets have high-growth rates, a disclosure lag increases the occurrence of runs and decreases bank value.
1 Introduction

Disclosure of information for financial institutions highlights two sides of the same coin: bank runs are costly when they occur, but they serve as an ex-ante disciplining device. Is there an optimal way to trade off the potential of disclosure to trigger bank runs against the benefits of providing information on risk-taking activities? It is known that disclosure can provide creditors with valuable information on a manager’s risk taking, which can enable creditors to find ways to restrict manager activity, e.g., Diamond and Rajan (2001). However, banks operate in environments with multiple frictions, and increasing transparency might be sub-optimal, e.g., Goldstein and Sapra (2014). For instance, disclosures might destabilize the banking system due to strategic interaction among creditors. If a disclosure suggests that a bank is solvent but in a precarious position, creditors may become more inclined to run on the bank. Two recent events where costs and benefits of information disclosure manifest are the publication of borrowers from the Federal Reserve’s discount window and the disclosure of banks’ stress tests results.

Given the costs and benefits of information disclosure for financial institutions, the optimal timing for financial-information disclosure remains unclear. As such, I study the effects of lagged financial-information disclosure on the occurrence of bank runs and manager risk-taking activities. I develop a model of a bank that is subject to runs in the form of debt rollover freezes as in He and Xiong (2012), in which a manager chooses asset risk and creditors receive delayed performance information about the asset.¹

The main contribution of this paper is that requiring disclosure with a lag maximizes bank value when the bank holds a low-growth rate asset. Under these circumstances, introducing a disclosure lag generates two opposing effects. On the one hand, increased opacity causes creditors to run more frequently. On the other hand, the increase in runs by creditors forces the manager to reduce risk, in order to reduce the run probability. The risk reduction effect dominates, leading to a decrease in the run probability. However, for a large enough disclosure lag, bank runs worsen and the increase

¹A bank in this paper is a financial intermediary that has short-term runnable liabilities: MMF, repo, ABCP, and large uninsured deposits.
the resulting run probability. The bank value is a concave function of the disclosure lag. There is an optimal lag that trades-off the cost of bank runs introduced by coordination problems and risk taking.

Another contribution of this paper is that the model reproduces empirical facts of discount window disclosures. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis. Specifically, she found that following discount window disclosures, banks that accessed the facility exhibited positive stock returns, positive bond returns, and lower asset risk. Through the lens of the model, I interpret these mandatory disclosures as shortening of the disclosure lag, from an infinite lag to 2-4 years. The model qualitatively reproduces these empirical facts and suggests that the asset risk reduction documented could be optimal, as opposed to the result of a managerial short-termism inefficiency.

A final contribution of this paper shows that bank value decreases as the disclosure lag increases for a bank with a high-growth-rate asset. The introduction of a disclosure lag generates two opposing effects. First, a disclosure lag decreases creditors’ incentives to run, because the asset has a high growth rate relative to the risk. As a result, the manager optimally reacts to the run probability reduction by increasing asset risk. The run probability increases because the manager’s extra risk-taking dominates the reduction in the run frequency by creditors.

Costs and benefits of information disclosure manifest in important situations such as the disclosure of banks’ stress tests results, and the publication of borrowers from the Federal Reserve’s discount window. Emergency lending facilities like the discount window became important tools used by the Federal Reserve during the financial crisis of 2007-2009. The Fed encourages banking institutions to borrow from these emergency funding facilities by keeping these banks’ emergency funding confidential. This confidential borrowing by financial institutions has both social benefits and costs. Benefits include the control of bank runs; costs include potential worsening of risk-taking activities by bank managers. In a similar way, costs and benefits of information disclosure

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are at the core of banks’ stress test results publication. Stress tests evaluate whether banks have sufficient capital to absorb losses resulting from adverse economic conditions. Having sufficient capital concerns bank creditors, and it is a useful piece of information for creditors to enhance manager performance evaluation. With better information, creditors can enhance manager performance by restricting the money deposits before it is too late. If all creditors decide to restrict the flow of funding at the same time, however, a bank run occurs.\(^3\) Bank runs are costly because they force premature liquidation of the asset, possibly at a large discount.

How long should a disclosure lag be? I study this question in a model of lagged disclosure based on the rollover freeze model of He and Xiong (2012). The model features two type of agents, a manager and a collection of creditors. Each creditor is small and concerned only with getting repaid. An individual creditor sees his chance of triggering a run as negligible. But creditors are aware of the possibility of runs, and they use the information they have to form beliefs about how likely a run might be. The strategic concern among creditors creates a coordination problem that may result in a debt rollover freeze, a bank run. The bank manager controls the asset’s risk and holds the equity of the firm. As an equity holder, the manager’s payoff is equal to a call option on the bank’s asset. That call option induces the manager to increase asset risk for low asset values. Finally, creditors receive delayed information about the current value of the bank’s asset. A disclosure lag is the amount of time that information is kept confidential. Creditors are aware of the fact that they are receiving outdated information and form rational expectations about the current asset value.

The creditors’ equilibrium run threshold changes when a disclosure lag is implemented. The change in the run threshold is a function of the bank asset’s growth-to-risk ratio. With a low growth-to-risk ratio for the asset, increasing opacity with the implementation of a disclosure lag causes creditors to run more frequently. Creditors run more frequently because they anticipate that a risky asset could have decreased in value since the disclosure date. On the other hand, creditors’ equilibrium run frequency decreases when the asset has a high growth-to-risk ratio, because a high

\(^3\)See "Lenders Stress Over Test Results" [http://on.wsj.com/1PWhsOh](http://on.wsj.com/1PWhsOh)
growth rate increases the probability of asset value appreciation since the last disclosure date.

A reinterpretation of the model gives an application to accounting. Immediate disclosure or no information delay is reminiscent of mark to market accounting. A positive disclosure lag corresponds to historical cost accounting, where asset value is updated after some time. My model predicts that when switching from historical cost to mark to market accounting, value for banks with high growth rate assets will increase, management will reduce asset risk, and stocks and bonds will exhibit positive returns. Moreover, when bank assets have low growth rates, bank value is maximized under a historical cost regime as it balances bank runs and risk taking.4

1.1 Related literature

This paper is related to the literature on disclosure for financial firms. Gigler et al. (2013) studied the frequency of disclosure that should be required for public firms. They showed that when the bank manager can endogenously make decisions, frequent disclosure does not necessarily imply economic efficiency. Similarly, my model shows that a shorter disclosure lag does not necessarily improve bank value, as the manager might take excessive risk. Both Gigler et al. (2013) and this paper illustrate that in a model with multiple frictions and when the bank’s management decision is endogenous, price efficiency does not imply economic efficiency. My paper is also related to Williams (2015), who studied how informative bank stress tests should be and concluded that stress tests should give enough failing grades to keep passing grades credible and avoid runs. Faria-e-Castro et al. (2016) studied the tradeoff between a market breakdown caused by adverse selection and bank runs triggered when information is disclosed. Shapiro and Skeie (2015) studied the optimal bail-out policy that balances bank runs and risk-taking activities.

My paper also relates to the literature on the balance of bank runs and managerial incentives. Cheng and Milbradt (2012) studied how debt maturity affects the tradeoff between incentive provision and bank runs. Diamond and Rajan (2001) studied how runs serve as a commitment device

4Guillaume et al. (2008) analyzed the tradeoff between imprecise accounting information provided by historical cost and price distortions for illiquid assets that are marked to market.
for bank management. He and Xiong (2012) analyzed a dynamic coordination problem among creditors of a firm with staggered debt structure. Morris and Shin (2002) showed the trade-off between market discipline and strategic concerns that cause coordination problems. I provide a bank-specific structure that builds on He and Xiong (2012) and explicitly models the information disclosure lag. The modeling choice makes it possible to identify what conditions justify the use of a lag by looking at bank characteristics, such as the bank’s underlying assets.

Several empirical studies relate to the considerations studied in this paper. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis. Armantier et al. (2015) provided evidence for the existence and magnitude of stigma in the interbank market associated with banks borrowing from the discount window. Kleymenova (2016) concluded that there is no stigma in capital markets whereas Armantier et al. (2015) documented the existence of stigma in the interbank market. My model does not study stigma, but qualitatively reproduces the market reaction of stocks, bonds, and the reduction in asset risk after disclosure reported in Kleymenova (2016).

2 Model

The model builds on He and Xiong (2012) and is set in continuous time with an infinite horizon. The bank invests in a long-term asset by rolling over short-term debt financed by a continuum of small creditors.

2.1 Asset and risk taking

The bank’s asset holding is normalized to one unit. The bank borrows $1 at time $t = 0$ to buy the asset. Once the asset is in place, it generates a constant stream of cash flow $r dt$ over the interval $[t, t + dt]$. At a random time $\tau_\phi$, which arrives according to a Poisson process with intensity $\phi > 0$, the asset matures with a final payoff of $y_{\tau_\phi}$. The advantage of assuming a random asset maturity is that it makes the expected life of the asset constant and equal to $1/\phi$. 
The final payoff of the asset evolves according to a geometric Brownian motion with drift $\mu$ and volatility $\sigma$,

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t,$$

where $W(t)$ is a standard Brownian motion, $\mu$ is the growth rate of the final payoff, and $\sigma$ is the instantaneous volatility. The initial value of the process, $y_0$, is given and observed by all participants.

The bank’s asset generates a constant cash flow $rdt$ and the random final payoff $y_{\tau_\phi}$. The value of the asset is the expected discounted future cash flows

$$F(y_t) = E_t \left[ \int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_\phi-t)} y_{\tau_\phi} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t,$$

where $\frac{r}{\rho + \phi}$ is the present value of the constant cash flow and $\frac{\phi}{\rho + \phi - \mu} y_t$ is the present value of the final payoff. Because the fundamental value of the bank is a linear function of $y_t$, I refer to $y_t$ as the bank fundamental.

The bank manager controls the risk of the asset; specifically, she chooses the asset’s final payoff volatility, $\sigma$. The bank manager chooses at time $t = 0$ a risk level $\sigma \in [\sigma_L, \sigma_H]$. Equivalently, the banker chooses a combination of an asset with low risk and one with high risk. The value of $\sigma$ is observable by all agents, but assumed not contractible. The manager’s decision is a one time choice and is kept constant afterwards. Endogenous risk taking is a feature not present in He and Xiong (2012). A different version of endogenous risk taking was studied by Cheng and Milbradt (2012) to investigate optimal debt maturity that balances bank runs and incentive provision.

### 2.2 Debt financing, runs, and liquidation

The bank financing, runs, and liquidation closely follow He and Xiong (2012). I present a brief review for completeness: The bank finances the asset by issuing short-term debt. Each debt contract lasts for an exponentially distributed amount of time with mean $1/\delta$. Creditors are paid interest payments at rate $rdt$ until the contract expires. Once an individual contract expires, the creditor
chooses whether to roll over the debt or withdraw his funds. The debt maturity times are independent across creditors so that each creditor expects some other creditors’ debt to mature before his.

In aggregate, a fraction $\delta dt$ of the bank’s debt matures over the time interval $[t, t + dt]$. When these creditors decide to withdraw their funds, a bank run, the bank must find financing from other sources or it will be forced into bankruptcy. I assume that the bank has access to a credit line that supplies the required financing. When a run occurs, there is a probability $\theta \delta dt$ that the credit line will fail to provide the required financing. The parameter $\theta > 0$ measures the reliability of the credit line. A low value of $\theta$ means that the credit line will sustain the run with high probability. If the credit line fails before the bank has recovered, the bank is forced to liquidate the asset in an illiquid secondary market. The asset’s liquidation value is a fraction $0 < \alpha < 1$ of the fundamental value of the asset

$$L(y_t) = \alpha F(y_t)$$

$$= L + l y_t,$$ (3)

with the constants $L = \frac{\alpha}{\rho + \phi}$ and $l = \frac{\alpha \phi}{\rho + \phi - \mu}$.

### 2.3 Information sets

The model departs from He and Xiong (2012) in the creditors’ information structure. A regulator has control over the disclosure of the bank’s fundamental, $y_t$. The regulator discloses the value of the bank’s fundamental with a lag $I \geq 0$. At time $t > 0$ creditors observe the bank’s fundamental value at time $t - I$, $y_{t-I}$. Creditors know that they are receiving outdated information and form rational expectations about the bank’s fundamental value. Define $x_t$, the lagged fundamental process as

$$x_t = y_{t-1}$$ (4)
Figure 1: Information structure for a disclosure lag \( I > 0 \). The solid line represents the fundamental process, \( y(t) \), observed by the manager. The dashed line represents the lagged process, \( x(t) \), which is the information available to the creditors at time \( t \).

with \( I \geq 0 \). The lagged fundamental process \( x_i \) follows a geometric Brownian motion with the same drift and variance as in equation (1)

\[
\frac{dx_t}{x_t} = \mu dt + \sigma dW_t, \ x_I = y_0.
\] (5)

I assume that the bank manager is an insider and observes the bank’s fundamental, \( y_t \), with no lag. Moreover, she is aware of the fact that creditors observe a lagged fundamental value, and she will use this information in making optimal choices. Management closeness to the bank’s daily operations motivates this assumption.

Figure 1 describes the information structure of the model. The lag in disclosure is set at a value \( I > 0 \), and is common knowledge to all parties. The solid line represents a realization of the fundamental process, \( \{y_u\}_{u \leq t} \), which is observed by the manager as it occurs. The dashed line represents the corresponding lagged process \( \{x_u\}_{u \leq t} \), which is the information available to the creditors at each point. Denote by \( E^I_t \{ \cdot \} = E \{ \cdot | x_t \} \) the conditional expectation used by creditors when the disclosure lag is set to \( I \). The manager receives immediate information and, hence, the
The appropriate conditional expectation is

$$E_t^0 \left[ \cdot \right] = E \left[ \cdot | y_t \right].$$

### 2.3.1 Financing stage

At $t = 0$, financing occurs and the manager chooses the asset risk. Creditors choose an optimal run threshold. A run threshold is a value of the fundamental, $y_*$, such that creditors’ strategy is to run whenever the lagged fundamental is below $y_*$, $x_t < y_*$, and to rollover when $x_t \geq y_*$. The equilibrium determination of this threshold is explained in the next section.

The fundamental process $y_t$ starts at $t = 0$, but no coupons are paid and no actions are allowed until $t = I$. At $t = I$, coupons start to be paid and delayed information starts to be disclosed to creditors. Debt contracts also start to mature, and a bank run could occur. Figure 2 shows a timeline of the financing stage of the model.
3 A creditor’s and the manager’s problems

3.1 An individual creditor’s problem

I follow He and Xiong (2012) and analyze the rollover decision for an individual creditor by taking as given that other creditors use a monotone strategy. A monotone strategy is a strategy in which all creditors whose debt matures decide to rollover if the bank’s lagged fundamental \( x_t \) is greater than a threshold \( y_s \). If the bank’s fundamental is lower than the threshold, \( x_t \leq y_s \), the creditors’ optimal decision is to run.

There are two possible outcomes for the bank. Either the asset’s final payoff is realized or the bank is prematurely liquidated after a run. These events are not controlled directly by an individual creditor. However, once an individual’s debt matures, he can decide whether to rollover the debt or not. Each creditor receives interest payments at a rate \( r \) per unit of time until

\[
\tau = \min(\tau_0, \tau_\delta, \tau_\delta),
\]

which is the earliest of the following three events: asset maturity, forced liquidation after a run, and debt expiration without rollover, respectively. When debt matures, creditors receive the face value of the debt back and have the option to rollover their position by buying the new debt.

With a risk neutral creditor, the value of one unit of debt is given by the value function

\[
V(x_t) = E^F_t \left[ \int_t^\tau e^{-\rho(s-t)} r ds + e^{-\rho(\tau-t)} \left( \min \{1, y_\tau\} \mathbf{1}_{\{\tau = \tau_\rho\}} \right) \right.
\]

\[
+ \min \{1, \mathcal{L}(y_\tau)\} \mathbf{1}_{\{\tau = \tau_\delta\}} + \max_{\text{rollover or run}} \{V(x_\tau; y_\tau), 1\} \mathbf{1}_{\{\tau = \tau_\delta\}} \left],
\]

where \( \mathbf{1}_{\{} \) takes the value 1 when the statement in brackets is true and 0 otherwise. The value for an individual creditor has four components, represented by the four terms in the right hand side of equation (8). First term, coupon payments are at rate \( r \). Second term, the creditor will receive at most $1 when the asset matures, \( \min (1, y_\tau) \). Third term, creditors’ will be paid at most $1 after a
run forces asset liquidation, \( \min (1, \mathcal{L} (y_t)) \). Fourth term, the creditor decides whether to rollover or run when the debt matures. The expectation in (8) is conditional on the information available to the creditor at time \( t \), that is, the lagged fundamental value process \( x_t \).

The Appendix derives the HJB equation for the value function \( V(x_t) \),

\[
pV(x_t; y_t, \sigma) = \mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx} + r + \phi \left( E_t^I \left[ \min \{1, y_t\} \right] - V(x_t; y_t) \right) + \theta \delta \mathbf{1}_{\{x_t < y_t\}} \left( E_t^I \left[ \min \{1, \mathcal{L} (y_t)\} \right] - V(x_t; y_t) \right) + \delta \max \{0, 1 - V(x_t; y_t)\}
\]

with boundary conditions

\[
V(0) = \frac{\rho + \phi \delta L + \delta}{\rho + \phi + \delta + \theta \delta}
\]

\[
\lim_{x \to \infty} V(x) = \frac{r + \phi}{\rho + \phi},
\]

where \( E_t^I [\cdot] \) is the expectation taken with the information available to the creditor at time \( t \), see equation (6). Even though the creditor knows that the current time is \( t \), he needs to estimate the current value of \( y_t \) from the information provided by the lagged process, \( x_t = y_{t-I} \). Hence, a creditor’s estimate of the payoff when the asset matures is \( E_t^I \left[ \min \{1, y_t\} \right] = E_t^I \left[ \min \{1, x_{t-I}\} \right] \). Similarly, the creditor’s estimate of the payoff in case of bankruptcy at time \( t \) is \( E_t^I \left[ \min \{1, \mathcal{L} (y_t)\} \right] = E_t^I \left[ \min \{1, \mathcal{L} (x_{t-I})\} \right] \). The information structure of the model leads to a generalized version of the HJB equation in He and Xiong (2012), such that I recover their equation when the disclosure lag is set to zero, \( I = 0 \).

The left hand side of equation (9), \( pV(x_t; y_t) \), represents the creditor’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, \( \mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx} \), capture the change in continuation value caused by the fluctuation in the bank’s fundamental. The next four terms represent the components that appeared in the integral form of the value function, equation (8) - that is, the coupon payment rate, \( r \), the value change when the as-
set matures at time \( \tau \phi \), \( \phi (E_t \left[ \min \{ 1, y_t \} \right] - V (x_t; y_*)) \), the value change caused by a forced liquidation, \( \theta \delta 1_{\{x_t < y_*\}} \left( E_t \left[ \min \{ 1, \mathcal{L} (y_t) \} \right] - V (x_t; y_*) \right) \), and the option to run when the debt contract expires, \( \delta \max_{\text{run or rollover}} \{ 1 - V (x_t; y_*), 0 \} \), respectively.

A creditor whose debt matures will choose to rollover whenever the value of doing so is higher than the debt’s face value of $1. In other words, if the value function only crosses 1 at the point \( x' \), \( V (x'; y_*) = 1 \), then \( x' \) is the optimal run threshold for the creditor. If debt matures and \( x_t < x' \), the creditor will not rollover the debt. On the other hand, when \( x_t \geq x' \), the creditor finances a new debt contract. I compute symmetric monotone equilibria for which the threshold used by all creditors is the same, \( y_* \). Therefore, a condition to determine the equilibrium threshold \( y_* \) is that \( V (y_*; y_*) = 1 \).

### 3.2 The bank manager’s problem

The bank manager holds the firm’s equity, the residual claim after creditors are paid. Given a run threshold, \( y_* \), the manager maximizes the total value of the residual claim by choosing the asset risk level, \( \sigma \). The manager’s choice is made at time \( t = 0 \) and is held constant afterwards. The value of equity at time \( t = 0 \) is

\[
Q (y_0; y_*, \sigma) = E^0_{t=0} \left[ e^{-\rho (\tau - 0)} \left( (y_\tau - 1)^+ 1_{\{\tau = \tau \phi \}} + (\mathcal{L} (y_\tau) - 1)^+ 1_{\{\tau = \tau \phi \}} \right) \right] \tag{11}
\]

where \( (\cdot)^+ = \max \{\cdot, 0\} \) and \( E^0_{t=0} [\cdot] \) is the expectation conditional on the information available to the manager at time \( t = 0 \), the non-delayed fundamental value \( y_0 \). The value function of the manager has two components, represented by the two terms on the right hand side of (11). The first represents the equity payoff when the asset matures, \( \max \{y_\tau - 1, 0\} \), and the second represents the payoff after a forced liquidation, \( \max \{\mathcal{L} (y_\tau) - 1, 0\} \).

Figure 2 presents a timeline describing the information available to the manager at time \( t = 0 \). The manager observes the fundamental value with no delay, \( \{y_\tau\}_{\tau \leq t} \). Also, she is aware that creditors are making optimal choices based on lagged information.
The Appendix presents the derivation of the HJB equation for the equity value \( Q(y_0; y_*, \sigma) \),

\[
\rho Q(y_0; y_*, \sigma) = \mu y_0 Q_y + \frac{\sigma^2}{2} y_0^2 Q_{yy} + \phi \left( E_{t=0}^0 \left[ \max \{ y_I - 1, 0 \} \right] - Q \right) \\
+ \theta \delta 1_{\{y_0 < y_*\}} \left( E_{t=0}^0 \left[ \max \{ \mathcal{L}(y_I) - 1, 0 \} \right] - Q \right),
\]

with boundary conditions

\[
Q(0) = 0 \\
Q(y_0) = \frac{-\phi}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_0 \text{ for } y_0 \gg 0.
\]

The left hand side of equation (12), \( \rho Q(y_0; y_*, \sigma) \), represents the banker’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, \( \mu y_0 Q_y + \frac{\sigma^2}{2} y_0^2 Q_{yy} \), capture the change in continuation value caused by the fluctuation in the bank fundamental. The next two terms capture the expected continuation value change after the asset matures or a run. The terms \( E_{t=0}^0 \left[ \max \{ y_I - 1, 0 \} \right] \) and \( E_{t=0}^0 \left[ \max \{ \mathcal{L}(y_I) - 1, 0 \} \right] \) represent the manager’s best estimate of the final payoff if the asset ends at time \( t = I \). Even though she will see the asset fundamental in real time, she needs to make a decision at time \( t = 0 \) knowing only the initial value of the fundamental. The manager’s estimate of her final payoff is equal to the value of a call option on the bank’s asset, with time to maturity equal to the disclosure lag.

Given a run threshold, \( y_* \), and the initial value of the process, \( y_0 \), the manager chooses the asset risk level that maximizes the value of equity at time \( t = 0 \),

\[
\sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q(y_0; y_*, \sigma),
\]

subject to (12) and (13).
4 Asset risk and run equilibrium

The analysis requires some parameter restrictions in order to be meaningful. I work with the same parameter restrictions as in He and Xiong (2012). First,

\[ \rho < r < \rho + \phi, \] (15)

which says that interest payments at rate \( r \) are higher than the discount rate but not so high that they justify automatic rollover. Second, the growth rate of the asset is bounded,

\[ \mu < \rho + \phi, \] (16)

which ensures that the value of the asset is finite. Third, the liquidation value of the asset when the fundamental \( y_t = 1 \) is lower than the initial capital raised from creditors,

\[ \frac{\alpha r}{\rho + \phi} + \frac{\alpha \phi}{\rho + \phi - \mu} = L + l < 1, \] (17)

which ensures that the creditors have incentives to run when the fundamental deteriorates.

**Definition.** A pair of asset risk and run threshold, \((\sigma_*, y_*)\), is an equilibrium if:

1. Given the asset risk \( \sigma_* \), each creditor chooses and optimal run threshold \( y_* \). The run threshold is symmetric among creditors. Hence, a condition for \( y_* \) to be optimal is \( V(y_*; y_*; \sigma_*) = 1 \).
2. Given the run threshold \( y_* \), the manager maximizes utility by choosing the asset risk \( \sigma \). The manager solves equation (14) subject to (12) and (13).

5 Equilibrium Analysis

I solve the model numerically. Table 1 presents the baseline parameter values which are calibrated to a typical financial firm during the recent financial crisis, see He and Xiong (2012). I use a
lower drift for the final payoff, $\mu$, and a higher cash flow rate $r$, compared to the calibration in He and Xiong (2012). Also, the volatility of the final payoff is now endogenous and limited to $\sigma \in [\sigma_L, \sigma_H]$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Interpretation</th>
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</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>1.5%</td>
<td>Discount rate</td>
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<tr>
<td>$r$</td>
<td>8.42%</td>
<td>Cash flow rate from asset</td>
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<tr>
<td>$\phi$</td>
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<td>Intensity of terminal value realization</td>
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<tr>
<td>$\alpha$</td>
<td>55%</td>
<td>Liquidation discount</td>
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<tr>
<td>$\mu$</td>
<td>0.5%</td>
<td>Asset’s final payoff drift</td>
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<td>$\delta$</td>
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<td>Intensity at which debt matures</td>
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<td>$\theta$</td>
<td>5</td>
<td>Intensity of credit line failure</td>
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<tr>
<td>$y_0$</td>
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<td>Fundamental value at time 0</td>
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<td>Low-risk volatility of final payoff</td>
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<tr>
<td>$\sigma_H$</td>
<td>12%</td>
<td>High-risk volatility of final payoff</td>
</tr>
</tbody>
</table>

Table 1. Baseline parameters.

### 5.1 Immediate disclosure equilibrium, no lag

Consider a situation in which there is immediate disclosure, or $I = 0$. Immediate disclosure implies that the lagged and the actual fundamental process coincide, $x_t = y_t$, and, hence, creditors use the real-time value of the asset. Figure 3 plots the creditors’ run threshold and the manager’s asset-risk choice, respectively. The solid line plots the optimal asset risk chosen by the manager, $\sigma_*$, as a function of the creditors’ threshold $y_*$. As the creditors’ run threshold increases, the manager optimally chooses lower asset risk. This relationship can be understood by studying equation (12), which gives the equity value given a run threshold. The manager faces a tradeoff when choosing the asset risk; higher levels of asset risk transfer value from creditors to the manager but also increase the chance of reaching the run threshold, where costly liquidation occurs. As a result, for
high values of the run threshold, the manager chooses low levels of asset risk to avoid a run. On the other hand, for low values of the run threshold, the manager chooses high asset-risk because the run probability is low.

The dashed line in Figure 3 plots creditors’ equilibrium optimal run threshold, \( y^* \), as a function of the asset’s risk, \( \sigma \). The threshold chosen by creditors increases with asset risk. This relation can be intuitively understood by studying creditors’ final payoff, \( \min \{ 1, y_t \} \), which is equal to the sum of the payoff of a risk free bond and a short position on a put option,

\[
\min \{ 1, y_t \} = 1 - \max \{ 1 - y_t, 0 \}.
\]  

(18)

When the manager increases asset risk, the value of the short position on the put option decreases. Creditors react by endogenously increasing the run threshold. A similar analysis applies to the final payoff after a run, \( \min \{ 1, \mathcal{L}(y_{r}) \} \).

An equilibrium is a pair \( (\sigma^*, y^*) \) where \( \sigma^* \) is consistent with \( y^* \). The intersection of the solid
line and dashed line determines the equilibrium in Figure 3. The equilibrium is given by the pair \((\sigma_*, y_*) = (12\%, 0.96)\), which indicates that \(\sigma_*\) takes the maximum value and runs will occur if the fundamental value \(y_t\) gets below 0.96.

### 5.2 Equilibrium with disclosure lags, \(I > 0\)

Figure 4 plots the manager’s and creditors’ optimal decisions for immediate disclosure and a two-year lag, \(I \in \{0, 2\}\). The black-solid line and the black-dashed line reproduce the optimal responses of the manager and creditors’ when there is immediate disclosure, as in Figure 3. As before, the equilibrium with immediate disclosure is given by the intersection of the lines at the point \((\sigma_*, y_*) = (12\%, 0.96)\). The grey-solid line and the grey-dashed line reproduce the optimal choices of the manager and creditors when there is a two-year lag, respectively. The equilibrium occurs at the intersection of these lines, where \(\sigma_*\) is consistent with \(y_*\) and is given by the pair \((\sigma_*, y_*) = (10.2\%, 0.98)\). The equilibrium for a two-year disclosure lag has a higher run threshold and lower asset risk compared to the immediate disclosure equilibrium.

The equilibrium changes after the introduction of a disclosure lag because it changes both the manager’s and creditors’ expected payoff. The creditors’ final payoff, \(E^I_t \left[ \min(1, y_t) \right] \), is the sum of the payoff of a risk free bond and a short position on a put option written on the asset’s final payoff,

\[
E^I_t \left[ \min \{1, y_t \} \right] = 1 - E^I_t \left[ \max \{1 - y_t, 0 \} \right].
\]  
(19)

The derivative of the short put option value with respect to the lag \(I\) is

\[
\frac{\partial E^I_t \left[ - \max \{1 - y_t, 0 \} \right]}{\partial I} = -N'(-d_-) \frac{\sigma}{\sqrt{2I}} + x_t e^{\mu I} N(-d_+),
\]  
(20)

where

\[
d_\pm = \frac{1}{\sigma \sqrt{I}} \left( \ln [x_t] + \left( \mu \pm \frac{1}{2} \sigma^2 \right) I \right),
\]  
(21)

\(N(\cdot)\) is the cumulative of the normal standard distribution and \(I\) indicates the first derivative. The
Figure 4: Two equilibriums: Immediate disclosure and disclosure lags of 2 years. The solid lines plot optimal asset risk as a function of the run threshold $y^*$. The dashed lines plot the optimal run threshold as a function of the asset risk. An equilibrium requires asset risk to be consistent with the run threshold.

The first term in the expected payoff’s change, equation (20), is always negative and the second is always positive. Hence, when the asset risk $\sigma$ is high relative to the drift $\mu$, the derivative is negative. A negative derivative indicates that the creditors’ payoff decreases as the lag $I$ increases. When the creditors’ expected payoff decreases, they have more incentives to run. In the opposite scenario, when the asset risk is low relative to the drift, the expected payoff increases with the lag and creditors run less frequently.

Figure 4 illustrates that for risk values higher than $\sigma = 6\%$, creditors increase the run threshold when the disclosure lag increases. The partial derivative (20) is negative. Intuitively, there is extra uncertainty created by the delay in information, and creditors protect themselves by running sooner. On the other hand, when the asset risk is lower than $\sigma = 6\%$, creditors’ optimal threshold decreases with a disclosure lag. The creditors’ payoff increases as the lag increases, which reduces their incentives to run. The partial derivative (20) is positive.

A disclosure lag also changes the manager’s optimal risk choice, as illustrated in Figure 4. The manager takes advantage of the fact that creditors are unable to take action for the first $I$ periods.
Formally, the manager’s payoff is equal to the value of a call option written on the fundamental, \( y_t \), with maturity equal to the disclosure lag, \( I \). The derivative of the call option value with respect to the disclosure lag is given by

\[
\frac{\partial E_0 \left[ \max \{ y_I - 1, 0 \} \right]}{\partial I} = N' \left( d_- \right) \frac{\sigma}{2\sqrt{I}} + y_0 \mu e^{\mu I} N \left( d_+ \right),
\]

with \( d_{\pm} \) as in equation (21). Since the derivative of the call option is always positive, the manager’s payoff is increasing with the lag, which allows him to increase asset risk.

The economic mechanism when the disclosure lag increases from 0 to 2 years illustrated in Figure 4 can be described as follows. A higher disclosure lag creates extra uncertainty for creditors who, in turn, react by running more frequently. The manager has an incentive to increase risk with more opacity, as her actions are not controlled in the first \( I \) periods. However, the increase in the run threshold by creditors dominates and actually makes the manager reduce risk in equilibrium.

The bank value changes as the information environment changes from immediate disclosure to a two-year lag. Holding everything else constant, a lower asset risk increases the bank value because it decreases the probability of the fundamental value crossing the run threshold. In addition, a higher run threshold decreases bank value because runs become more likely. The net change in bank value depends on how the change in \( \sigma_s \) and \( y_s \) affects the run probability. With a geometric Brownian motion for \( y_t \), the probability of a run is given by

\[
P_{y_0} \left( \inf_{0 \leq s \leq \tau_\delta} y_s \leq y_s \right) = \left( \frac{y_s}{y_0} \right)^{\sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{\sigma^2}{\alpha^2} + \left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)}}.
\]

The run probability is increasing in the run threshold, \( y_s \), and in asset risk, \( \sigma \).

### 5.3 Optimal Disclosure Lags

Figure 5 plots the bank value and run probability for disclosure lags from 0 to 6 years. The left plot shows that the bank value is a concave function of the disclosure lag. The bank value increases
Figure 5: Bank value and run probability for several disclosure lags. An optimal disclosure lag occurs between 2 and 3 years.

when the lag increases from 0 to 2 years, because the reduction in asset risk dominates the increase of the run threshold, the mechanism illustrated in Figure 4. The bank value attains a maximum around 2.5 years and decreases afterwards, where the risk reduction benefit does not compensate the increase in the run threshold.

The right plot in Figure 5 shows the run probability as a function of the disclosure lag. The run probability decreases initially as the manager’s risk reduction dominates the increased run threshold. It attains a minimum around 2.5 years and then increases. The probability of a run is a sufficient statistic for the bank value. Increasing the run threshold increases the probability of a run and decreases bank value. Decreasing asset risk decreases the run probability and increases bank value.

6 Disclosure lags and the data

The model is able to reproduce qualitatively some empirical facts reported in the literature of discount window disclosures. Kleymenova (2016) studied the capital market consequences of
Figure 6: Equity and debt values for different lags on the top plot. Asset risk as chosen by the manager for different lags on the bottom plot.

mandatory disclosures of discount window participants during the financial crisis. Banks accessed the discount window from 2007 to 2009, thinking that their participation would remain confidential. Kleymenova (2016) provided evidence that even after disclosing the information with a lag, in 2011, the disclosures contained new information. Specifically, the study found that following the discount window disclosures, banks that accessed the facility had positive cumulative abnormal stock returns of 1.10%. The study also found that after the discount window disclosures, banks that used the discount window placed their bonds at lower yields, on average 1.16% lower. Regarding asset risk, the study found that following discount window disclosures, banks decreased risk taking on their assets.

In the context of my model, I can qualitatively reproduce Kleymenova’s findings by interpreting the mandatory disclosures as a disclosure lag shortening. Under this interpretation, a non-disclosure environment corresponds to a very long disclosure lag, possibly infinite. The disclosure that took place in 2011 corresponds to a disclosure with a lag of 2-4 years. Therefore, by changing the disclosure lag, I can study what the model predicts about asset risk, equity prices, and debt prices and compare these predictions with the empirical evidence.

The top plot in Figure 6 shows the equilibrium equity and debt prices for disclosure lags that
range from 0 to 10 years. The bottom plot presents asset risk as chosen by the manager for the same range of disclosure lags. These plots are generated by setting the growth rate to $\mu = 1\%$ and the cash flow rate $r = 7.72\%$. The top panel shows that when disclosure lags decrease, implying more frequent disclosure, equity and debt prices increase, consistent with the finding in Kleymenova (2016) about abnormal returns in equity and lower yields in debt. The bottom plot shows that for a shorter disclosure lag, the manager chooses less asset risk in equilibrium, which is also consistent with the empirical evidence.

7 Comparative Statics

The baseline parameters of Table 1 produce an optimal lag of around 2.5 years. What other parameters produce an optimal lag? In order to answer this question, I conduct a comparative statics analysis and derive regions in which the lag is optimal and regions in which it is not. I evaluate whether a lag is optimal for different combinations of the growth rate $\mu$ and cashflow $r$ while keeping the initial value of the asset, $F(y_0)$ in equation (2), fixed. Increasing $\mu$ and decreasing $r$ transfers value from the coupon payments to the final payoff and effectively increases the average cash-flow maturity of the asset. The increase in maturity is accompanied by more risk, since a greater proportion of the cash flow comes from the final payoff, which is subject to random shocks from the Brownian motion.

7.1 Banks with a high growth asset

Figure 7 presents the equilibrium analysis for a parametrization with $\mu = 1.5\%$ and $r = 7\%$. This is the parametrization used by He and Xiong (2012). It has a higher growth rate, $\mu$, compared to the baseline parameters of Table 1. The manager’s and creditors’ optimal choice follow the same general behavior as in the baseline calibration, Figure 4. That is, the creditors threshold increases with asset risk and the manager’s risk allocation decreases with the run threshold.

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5Keeping the asset value constant and increasing the drift to $\mu = 1\%$ gives $r = 7.72\%$. 

23
Figure 7: Equilibrium with immediate disclosure and with a lag of 2 years. The solid lines represent the manager’s best response and the dashed lines represent the creditors’ run cutoff.

There are two equilibrium effects when introducing a disclosure lag for a bank with relatively higher growth rate asset. The first is to increase asset risk from 3.24% to 3.84%. The second is to decrease the run threshold from 0.9 to 0.89. The economic mechanism is as follows. Under immediate disclosure and a high growth asset, the manager chooses a low level of asset risk and creditors choose a low run threshold. The introduction of a lag eases creditors’ incentives to run, lowering the equilibrium run threshold. However, a lower run threshold gives room to the manager to increase asset risk. The resulting equilibrium with a two-year lag has a higher asset risk and a lower run threshold when compared to the immediate disclosure case. The higher asset risk effect dominates and decreases bank value, despite the lower run threshold.

## 7.2 Asset characteristics and optimal disclosure lags

The previous example illustrates that a disclosure lag is not beneficial for banks with a high-growth asset. However, disclosure lags are beneficial when the bank’s asset has low growth rates. Figure 8 plots equilibrium pairs \((\sigma_s, y_s)\) for two disclosure lags, \(I \in \{0, 2\}\), while varying \(\mu\) and \(r\). The equilibrium with baseline parameters is located on the top right side of the plot. The introduction of a lag causes the equilibrium to change from a high asset risk with a high run threshold \((\sigma_s, y_s) = \)
Figure 8: Equilibrium with Constant Asset Values and Disclosure Lag. High-growth assets are in the bottom left of the graph and low-growth assets are in the top right.

(12\%, 0.95), black line, to a lower asset risk and higher run threshold \((\sigma_s, y_s) = (10\%, 0.98)\), grey line. Equilibriums using the He and Xiong (2012) parameters are located at the bottom left side of Figure 8. The introduction of a lag decreases the run threshold and increases the asset risk, black to grey line.

More generally, Figure 8 shows that a lag is beneficial when the bank’s asset has relatively low growth rates or low average cashflow maturity. An bank asset with short duration is typically highly liquid, and, hence, another interpretation from Figure 8 is that a disclosure lag balances risk taking and runs when banks hold liquid assets. The economic mechanism is a follows; liquid assets have low growth opportunities that induce the bank manager to increase asset risk. This produces an asset with a low growth-to-risk ratio. As the disclosure lag increases, creditors become nervous and tend to run more frequently because of the bad characteristics of the asset. The manager optimally reduces asset risk to compensate for the increased run probability. The risk reduction effect dominates, and the bank value increases with a disclosure lag.
7.3 Liquidity, bank value, and run probability

The black line in Figure 8 plots equilibrium pairs for different value of $\mu$ while keeping the asset value constant. The equilibrium with baseline parameters is the black circle with coordinates $(\sigma_*, y_*) = (12\%, 0.95)$, top right side of the plot. When $\mu$ increases, increasing the duration of the asset, the equilibrium moves along the black line. The manager optimally chooses lower risk, and creditors choose a lower run threshold. A lower asset risk and lower run threshold increase the bank value. The bank value increases when duration increases. Similarly, if long duration assets are relatively illiquid, the bank value increases with illiquid assets.

8 Disclosure lag as an alternative to bailouts

Another way to affect the balance between runs and risk-taking activities is for the regulator to change the bailout policy. The parameter $\theta$ controls the reliability of bailouts. A higher value for $\theta$ means a lower probability of a bailout. In the model, the regulator could decrease the run
probability by decreasing the reliability of bailouts, a situation shown in Figure 9. When the regulator decreases the reliability of bailouts, creditors optimally react by running more frequently, a first order effect. Seeing the increased run probability, the manager reduces the equilibrium asset risk. The risk effect dominates, and the resulting run probability decreases.

Figure 9 presents an example of this situation where parameter $\theta$, which measures the intensity of failure during a bank run, increases from 5 to 7. A increase in parameter $\theta$ means that the government sustains the bank run for a shorter period of time. The grey-dashed line shows that creditors increase the run threshold in response to the new, stricter bailout policy. To attain an equilibrium, the manager reduces asset risk. The bank value increases as the run probability decreases.

Under the baseline parameters of Table 1, the regulator has two options to decrease the run probability: First, increase opacity by implementing a disclosure lag, the focus of this paper. Second, decrease the reliability of bailouts. The two options attain the same results in terms of bank value, run probability, and bailout costs. I argue that increasing opacity via a disclosure lag is relatively easier.

Compared to changing the bailout policy, introducing a disclosure lag might face much less scrutiny and opposition from Wall Street firms and Congress, while attaining the same results. The disclosure lag policy also might be easier to commit to. For example, since the Dodd Frank Act, discount window disclosures are made with a two-year lag, and changing this would require an act of Congress. My paper shows that decreasing the reliability of bailouts and implementing a disclosure lag are two complementary regulatory tools. A world with no bailouts could be equivalent to a world with some bailouts and a disclosure lag, according to the model. This suggests a rationale for the coexistence of bailout and disclosure lags.
9 Conclusion

This paper studies the effects of disclosing financial information on the occurrence of bank runs and management risk-taking activities. In particular, I study how a balance between bank runs and asset risk-taking activities can be attained by delaying information release. I develop a model of a bank that is subject to runs in the form of debt rollover freezes as in He and Xiong (2012), in which a manager chooses asset risk and creditors receive delayed performance information about the asset.

The main contribution of this paper is that a positive disclosure lag that maximizes bank value when the bank holds a low-growth-rate asset. Under this condition, implementing a disclosure lag generates two opposing effects. First, increased opacity causes creditors to run more frequently. As a result, the increase in runs by creditors forces the manager to reduce risk, in an attempt to lower the run probability. The risk reduction effect dominates and the bank value increases. When the disclosure lag is large enough, bank runs worsen and the bank value decreases. There is an optimal lag that trades off the cost of bank runs introduced by coordination problems and risk taking.

Another contribution is that the model also reproduces features of the data. Kleymenova (2016) studied the capital market consequences of mandatory disclosures of discount window participants during the financial crisis. My model suggests that the asset risk reduction documented in Kleymenova (2016) could be optimal as opposed to the result of managerial short-termism inefficiency.

A final contribution shows that bank value decreases as the disclosure lag increases for a bank with a high-growth-rate asset. Under these circumstances, the introduction of a disclosure lag generates two opposing effects. First, a disclosure lag decreases creditors’ incentives to run. Second, the manager optimally reacts to the run probability reduction by increasing asset risk. The risk increase effect dominates, and the bank value decreases.

The model has applications to accounting. A zero disclosure lag corresponds to a situation in which a bank’s assets are marked to market. A positive disclosure lag corresponds to historical cost accounting regime, where the asset value is updated after some time. The model predicts that when switching from historical cost to mark to market accounting, value for banks with high growth rate
assets will increase, management will reduce asset risk, and stocks and bonds will exhibit positive returns. Moreover, when bank assets have low growth rates, bank value is maximized under a historical cost regime as it balances bank runs and risk-taking.

I also analyze how bailout policies interact with disclosure lags. My model shows that good incentives can be provided by either a stricter bailout policy or by implementing a disclosure lag. I argue that the implementation of a disclosure lag is much easier to commit to. My model suggests that disclosure lags are a way of tightening the bailout policy. This suggests a rationale for the coexistence of bailout and disclosure lags.

This paper contributes to understanding how bank runs and incentive provision balance in equilibrium. The model serves as a benchmark to evaluate disclosure policies for financial institutions. However, the model does not address the behavior of the interbank lending market. By abstracting from the interbank lending market, I am able to study the specificities of the disclosure lag at the cost of other considerations such as adverse selections among heterogeneous banks. These could be important considerations that future research will address, in order to better inform policy makers.

References


10 Appendix

10.1 Creditor’s HJB Equation

In this section I review the creditor’s problem and some of its properties. Recall that the creditor’s problem can be written as the solution of the following value function equation:

\[
V(x_t) = E_t\left[ \int_t^\tau e^{-\rho(s-t)}r ds + e^{-\rho(\tau-t)}\min(1, y_\tau) 1_{\{\tau=\tau^0\}} + \min(1, \mathcal{L}(y_\tau)) 1_{\{\tau=\tau^0\}} + \max\text{run or rollover} (1, V(x_\tau; y_\tau)) 1_{\{\tau=\tau^0\}} \right]
\]

Now, I rewrite the problem in a different way. Fix a threshold \(y^*_u\). At each point in time \(u \geq t\), the creditor receives interest payments \(r\), and when the asset matures he receives \(\min(1, x_{u+I})\) or \(\min(1, \mathcal{L}(x_{u+I}))\) when the bank is forced into bankruptcy. Therefore, the creditor’s expected payoff at time \(u \geq t\) is given by

\[
 r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y^*_u\}}} E_u[\min(1, x_{u+I})] + \frac{\theta \delta 1_{\{x_u < y^*_u\}}}{\phi + \theta \delta 1_{\{x_u < y^*_u\}}} E_u[\min(1, \mathcal{L}(x_{u+I}))]
\]

where \(E_u[\min(1, x_{u+I})]\) and \(E_u[\min(1, \mathcal{L}(x_{u+I}))]\) have Black-Scholes like formulas. Let \(p_1(x, u + I) = E[\min(1, x_{u+I}) | x_u = x]\) and \(p_2(x, u + I) = E[\min(1, \mathcal{L}(x_{u+I})) | x_u = x]\) and integrate the discounted payoff across possible values of \(x\) to get

\[
p(t, x_t) = E_{t, x_t} \left[ e^{-\rho(u-t)} \left( r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y^*_u\}}} p_1(x_u, u + I) + \frac{\theta \delta 1_{\{x_u < y^*_u\}}}{\phi + \theta \delta 1_{\{x_u < y^*_u\}}} p_2(x_u, u + I) \right) \right].
\]
I show that $e^{-\rho t} p(x_t, t)$ is a martingale under $P$. Let $s \leq t \leq u$

$$
E \left[ e^{-\rho t} p(t, x_t) \mid F(s) \right] = E \left[ e^{-\rho s} \left( r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_1(x_u, u + I) + \frac{\theta \delta 1_{\{x_u < y_u\}}}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_2(x_u, u + I) \right) \mid F(t) \right] \mid F(s) 
$$

$$
= E \left[ E \left[ e^{-\rho u} \left( r + \frac{\phi}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_1(x_u, u + I) + \frac{\theta \delta 1_{\{x_u < y_u\}}}{\phi + \theta \delta 1_{\{x_u < y_u\}}} p_2(x_u, u + I) \right) \mid F(t) \right] \mid F(s) \right] 
$$

$$
= e^{-\rho s} p(s, x_s),
$$

by the tower property. Hence, $e^{-\rho t} p(x_t, t)$ is a martingale and $p(x_t, t)$ satisfies the Black-Scholes formula.

The creditor’s value can be written as the Laplace transform of $p(t, x_t)$, or

$$
V(x_t; y_s) = \int_0^\infty (\phi + \theta \delta 1_{\{x_u < y_u\}}) e^{-u(\phi + \theta \delta 1_{\{x_u < y_u\}})} p(t, x_t) du
$$

and, hence, I can apply the Laplace transform to both sides of the Black-Scholes PDE to get equation (9).

### 10.2 Manager’s HJB Equation

In this section I show that $Q(y_t; y_s, \sigma)$, satisfies equation (12). Recall that

$$
Q(y_t; y_s, \Gamma) = E_t^0 \left[ e^{-\rho(\tau - t)} \left( (y_{\tau - 1})^+ 1_{\{\tau = \tau_o\}} + (\mathcal{L}(y_{\tau}) - 1)^+ \right) 1_{\{\tau = \tau_o\}} \right].
$$

Fix a time $u \geq I$. At time $u$, the asset could expire with payoff $(y_u - 1)^+$, the bank could be forced into bankruptcy with payoff $(\mathcal{L}(y_u) - 1)^+$, or it could continue.
Creditors’ receive information with a lag and, hence, the intensity relevant at time \( u \) depends on the value of the fundamental at time \( u - I \). The expected payoff at time \( u \) can be therefore written as

\[
\phi (y_u - 1)^+ + \theta \delta \mathbf{1}_{\{y_u < y_s\}} (L(y_u) - 1)^+
\]

where \( \phi \) represents the intensity of the asset expiring at time \( u \) and \( \theta \delta \mathbf{1}_{\{y_u < y_s\}} \) represents the intensity of the bank failing at time \( u \).

Consider the discounted expected payoff at time \( u - I \)

\[
payoff (u - I, y_{u-I}) = E_{u-I} \left[ e^{-\rho I} (\phi (y_u - 1)^+ + \theta \delta \mathbf{1}_{\{y_u < y_s\}} (L(y_u) - 1)^+) \right]
\]

which can be written as

\[
payoff (u - I, y_{u-I}) = \phi E_{u-I} [e^{-\rho I} (y_u - 1)^+] + \theta \delta \mathbf{1}_{\{y_u < y_s\}} E_{u-I} [e^{-\rho I} (L(y_u) - 1)^+]
\]

The expected discounted payoff at time \( t = 0 \) is therefore

\[
c(t = 0, y_t) = E_{t=0} [e^{-\rho(u-I)} payoff(0, y_0)].
\]

I show that \( e^{-\rho t} c(t, y_t) \) is a martingale under \( P \). Let \( s \leq t \leq u - I \) and consider

\[
E \left[ e^{-\rho t} c(t, y_t) | F(s) \right] = E \left[ E[e^{-\rho(u-I)} payoff(u - I, y_{u-I}) | F(t)] | F(s) \right] = E \left[ e^{-\rho(u-I)} payoff(u - I, y_{u-I}) | F(s) \right] = e^{-\rho s} c(s, y_s)
\]

which indeed means that \( e^{-\rho t} c(t, y_t) \) is a martingale. Therefore \( c(s, x) \) satisfies the Black-Scholes
formula

\[-\rho c (s, x) + c_t (s, x) + \mu x c_x (s, x) + \frac{1}{2}\sigma^2 x^2 c_{xx} (s, x) = 0\]

with boundary conditions

\[c (u - I, x) = \text{payoff} (u - I, x) \quad \forall x \in [0, \infty)\]

\[c (s, 0) = 0 \quad \forall s \in [t, u - I]\]

\[\lim_{x \to \infty} c (s, x) = \phi (xe^{(u-\rho)(u-s)} - e^{-\rho(u-s)}) \quad \forall s \in [0, u - I].\]

It is convenient to change the time variable to the time to expiration \(\tau = u - I - t\) so that \(g (\tau, x) = c (u - I - t, x)\) satisfies the PDE

\[-\rho g (\tau, x) - g_\tau (\tau, x) + \mu x g_x (\tau, x) + \frac{1}{2}\sigma^2 x^2 g_{xx} (\tau, x) = 0\]  \quad (24)

with boundary conditions

\[g (0, x) = \text{payoff} (u - I, x) \quad \forall x \in [0, \infty)\]

\[g (s, 0) = 0 \quad \forall s \in [0, u - I]\]

\[\lim_{x \to \infty} g (s, x) = \phi (xe^{(u-\rho)(s+I)} - e^{-\rho(s+I)}) \quad \forall s \in [0, u - I].\]

Since the asset maturity and bank failure after a run are exponentially distributed, the manager’s value is the Laplace transform of \(g (\tau, x)\), or

\[Q (x; y) = \int_0^\infty e^{-(\phi + \theta 1_{\{y-u-I<y+s\}}) (\tau)} g (\tau, x) \, d\tau\]

and, hence, I can apply the Laplace transform to both sides of the Black-Scholes PDE, equation (24), to get

\[-\rho Q (x) - \left( (\phi + \theta 1_{\{y-u-I<y+s\}}) Q (x) - g (0, x) \right) + \mu x Q_x (x) + \frac{1}{2}\sigma^2 x^2 Q_{xx} (x) = 0\]
where \( Q(\cdot) \) is the Laplace transform of \( g(\tau, x) \).

Finally, replace the boundary condition for \( g(0, x) \) to get equation (12)

\[
\rho Q(x) = \mu Q_x(x) + \frac{1}{2} \sigma^2 x^2 Q_{xx}(x) + \phi \left( E_{u-I}[e^{-\rho t}(y_u - 1)^+] - Q(x) \right) + \theta \delta 1_{\{y_{u-I} < y_u\}} \left( E_{u-I}[e^{-\rho t}(L(y_u) - 1)^+] - Q(x) \right)
\]

10.3 Government’s expected bailout costs

Following Cheng and Milbradt (2012) I can compute the Government’s expected bailout costs by solving the following ODE.

\[
\rho G(y_t; y_*) = \mu y_t G_y + \frac{\sigma^2}{2} y_t^2 G_{yy} - \phi G(y_t; y_*) - \theta \delta 1_{\{y_t < y_*\}} G(y_t; y_*) + \delta 1_{\{y_t < y_*\}} [1 - V(y_t; y_*)]
\]

In standard form

\[
\frac{\sigma^2}{2} y_t^2 G_{yy} + \mu y_t G_y - G(\rho + \phi + 1_{\{y_t < y_*\}} \theta \delta) = -\delta 1_{\{y_t < y_*\}} [1 - V(y_t; y_*)]
\]

with boundary conditions

\[
\lim_{y \to \infty} G(y) = 0 \quad G(0) = \frac{\delta (1 - V(0))}{\rho + \phi + \theta \delta}
\]

where \( V(0) \) is the value for the creditors when the fundamental is 0.
10.4 Call and put options value and time to maturity

In this section I present a review of the behavior of call and put options prices with respect to time to maturity. This review will be useful in understanding the results of the model.

Recall the formula

\[ E \left[ \left( K - xe^{\sqrt{\tau}Z - \frac{1}{2}v} \right)^+ \right] = KN (-z_-) - e^{\mu I} N (-z_+) \]  \hspace{1cm} (25)

where

\[ z_{+/-} = \frac{1}{\sqrt{v}} \left( \log \left[ \frac{x}{K} \right] \pm \frac{1}{2} v \right) \]  \hspace{1cm} (26)

and \( Z \sim N (0, 1) \)

Consider the value of a put option on \( y_t \) with strike 1, time to expiration \( I \) and no discounting. Mathematically, the value of this option is \( E_t [(1 - y_{t+I})^+] \). I can compute this expectation by using equation (25) with \( K = 1, x = y_t e^{\mu I} \) and \( v = \sigma^2 I \). I get

\[ p_1 (I, y_t) = E \left[ (1 - y_{t+I})^+ \right] = N (-z_{-1}) - y_t e^{\mu I} N (-z_{+1}) \]

where

\[ z_{+/-1} = \frac{1}{\sigma \sqrt{I}} \left( \log [y_t] + \left( \mu \pm \frac{1}{2} \sigma^2 \right) I \right) \]  \hspace{1cm} (27)

Compute the derivative of \( p_1 (I, y_t) \) with respect to \( I \),

\[ \frac{\partial p_1}{\partial I} = -\mu y_t e^{\mu I} N (-z_{+1}) + \frac{N' (-z_{-1}) \sigma}{2\sqrt{I}}. \]

Now consider the value of a put option on \( y_t \) with strike \( \frac{1-L}{I} \), time to expiration \( I \) and no discounting. Mathematically, the value of this option is \( E_t \left[ \left( \frac{1-L}{I} - y_{t+I} \right)^+ \right] \). I can compute this expectation by using equation (25) with \( K = \frac{1-L}{I}, x = y_t e^{\mu I} \) and \( v = \sigma^2 I \). I get

\[ p_2 (I, y_t) = E \left[ \left( \frac{1-L}{I} - y_{t+I} \right)^+ \right] = \frac{1-L}{I} N (-z_{-2}) - y_t e^{\mu I} N (-z_{+2}) \]
where
\[ z_{+/-2} = \frac{1}{\sigma \sqrt{I}} \left( \log \left[ \frac{ly_t}{1 - L} \right] + \left( \mu \pm \frac{1}{2} \sigma^2 \right) I \right). \]  
(28)

Compute the derivative of \( p_2 (I, y_t) \) with respect to \( I \),
\[ \frac{\partial p_2}{\partial I} = -\mu y_t e^{\mu I} N (-z_{+2}) + \frac{(1 - L) N' (-z_{-2}) \sigma}{2 \sigma \sqrt{I}}. \]

Recall the formula
\[ E \left[ \left( x e^{\sqrt{\sigma} (z - \frac{\sigma}{2} \sigma) - K} \right)^+ \right] = x N(z_+) - K N(z_-) \]  
(29)

where \( z_+ \) and \( z_- \) are given by equation (14). Consider the value of a call option on \( y_t \) with strike 1 and time to expiration \( I \). Mathematically, the value of this option is \( E_t \left[ e^{-\mu I} (y_{t+I} - 1)^+ \right] = e^{-\mu I} E_t \left[ (y_{t+I} - 1)^+ \right] \). I can compute this expectation by using equation (29) with \( K = 1, x = y_t e^{\mu I} \) and \( v = \sigma^2 I \). I get
\[ c_1 (I, y_t) = E \left[ e^{-\mu I} (y_{t+I} - 1)^+ \right] = y_t N(z_{+1}) - e^{-\mu I} N(z_{-1}) \]
where \( z_{+1/-1} \) are give by equation (27). The partial derivative of \( c_1 \) with respect to \( I \) is given by
\[ \frac{\partial c_1}{\partial I} = \mu e^{-\mu I} N(z_{-1}) + \frac{y_t N'(z_{+1}) \sigma}{2 \sigma \sqrt{I}} \]
which is always always positive.

Finally, consider the value of a call option on \( y_t \) with strike \( \frac{1 - L}{I} \) and time to expiration \( I \). Mathematically, the value of this option is \( E_t \left[ e^{-\mu I} (y_{t+I} - \frac{1 - L}{I})^{+} \right] = e^{-\mu I} E_t \left[ (y_{t+I} - \frac{1 - L}{I})^{+} \right] \). I can compute this expectation by using equation (29) with \( K = \frac{1 - L}{I}, x = y_t e^{\mu I} \) and \( v = \sigma^2 I \). I get
\[ c_2 (I, y_t) = E \left[ e^{-\mu I} \left( y_{t+I} - \frac{1 - L}{I} \right)^+ \right] = y_t N(z_{+2}) - \frac{1 - L}{I} e^{-\mu I} N(z_{-2}) \]
where \( z_{+2} \) are given by equation (27). The partial derivative of \( c_2 \) with respect to \( I \) is given by

\[
\frac{\partial c_2}{\partial I} = \mu - \frac{L}{l} e^{-\mu t} N(z_{-2}) + \frac{y_I N'(z_{+2}) \sigma}{2\sqrt{I}}
\]

which is always positive.