Disclosure Lags in the Presence of Bank Runs and Risk-Shifting

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Abstract

I study the effects of asset disclosure lags on incentive provision and runs for a bank financed with short-term debt. Runs occur because of intertemporal coordination problems and bank management has discretion over the asset riskiness. Creditors receive lagged information about the asset’s value. A disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank holds liquid assets. When the bank’s assets are illiquid, a disclosure lag increases the occurrence of runs and hence decreases the value of the bank. Illiquid assets decrease the manager’s risk appetite and this translates into a reduced run probability.

1 Introduction

Emergency lending facilities became an important tool used by the Federal Reserve during the financial crisis. Stress tests have become an important component of the supervisory toolkit. However, it remains controversial to what extent there should be detailed disclosure of information about emergency lending facilities and stress tests results. As argued by Goldstein and Sapra (2014), improved market discipline is seen as a benefit of disclosure, but there are potential social costs because banks face an environment with multiple frictions.

One of the benefits of information disclosure is that it improves market discipline. More disclosure is desirable because it allows investors to take corrective action, either by restricting the

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1Goldstein and Sapra (2014)
flow of funding or actively intervening before it is too late. For banks, short-term debt can serve as a disciplining device for moral hazard and risk shifting\(^2\). The fragility of the capital structure provides incentives for creditors to monitor bank management and thus mitigates agency issues, which in turn is value increasing.

There are costs associated with disclosure however. Goldstein and Sapra (2014) summarize some of the disclosure costs that have been documented both in theoretical and empirical research. One of the documented costs says that if depositors have fundamental and strategic concerns, their actions could fail to coordinate in an efficient way after disclosure. Recent research has focused on the role that runs of short-term creditors had as a leading cause of the financial crisis 2007-2009\(^3\). The basic idea is that the fragile capital structure financed by a very short debt suffered from a rollover freeze. The capital structure in the shadow banking system was prone to inefficient runs due to a coordination problem introduced by debt that was similar in characteristics and function to demand deposits.

To the benefits and costs of disclosure, a third dimension can be added. There is debate in what should be an appropriate lag (if at all) for information to be disclosed. The Dodd-Frank Act required discount window participants to be disclosed with a two year lag. This was a change from a non-disclosure policy that the Fed had since the creation of the discount window. Disclosure with a lag is seen as a way to balance the benefits of market discipline while mitigating the costs of a bank run. Some of this debate has been reported in the press\(^4\).

This paper studies how lags in asset information disclosure affect the benefits and costs associated with disclosure. In particular, we study the effects of disclosure lags on runs and incentive provision for a bank financed with short-term debt. Runs occur because there is an intertemporal coordination problem among creditors as in He and Xiong (2012). A risk-shifting problem occurs


\(^3\)See He and Xiong (2012), Morris and Shin (2009), Acharya (2010), Gorton and Metrick (2011), and Brunnermeier (2009).

because bank management has discretion over the asset riskiness. Creditors receive lagged information about the asset’s value. Our contribution is threefold. First, we show that a disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank holds liquid assets. A disclosure lag is beneficial because the bank manager reduces risk as creditors get nervous with opacity. The risk reduction increases the bank value.

Second, when the bank’s assets are illiquid, a disclosure lag increases the occurrence of bank runs and hence decreases the value of the bank. In the presence of a disclosure lag, creditors run less frequently but the manager increases the project risk. The increase in risk makes the bank value to decrease. Third, illiquid assets decrease the manager’s appetite for risk and this translates into a reduced run probability. Illiquid assets have relatively high growth opportunities that decrease the need of risky bets by the manager.

We present a model that studies the effect of asset disclosure lags on incentive provision and bank runs by incorporating three key components. First, there is a risk-shifting problem because the bank manager chooses the riskiness of projects held on the asset side to maximize her utility. The bank raises funds in the form of short term debt and uses the resources to finance the project. Without debt financing, the bank value is independent of the asset risk. However, in the presence of debt financing, the bank manager has incentives to risk-shift when the asset value is low.

The second component is that a bank run occurs when creditors decide not to rollover the debt. The bank finances the project by raising funds in the form of staggered short-term debt. As debt matures, creditors must decide whether to rollover or withdraw. The staggered short-term debt creates an intertemporal coordination problem that causes bank runs in the same way as in He and Xiong (2012). If the bank cannot sustain a run, it is forced to liquidate the project inefficiently. An increase in the asset’s volatility makes it more likely for fundamentals to deteriorate before the debt matures, and hence it increases the creditors’ incentive to run today.

The third component is the regulator’s control of information disclosure to bank creditors. In particular, creditors receive lagged information about the current value of the bank’s project. Depositors are aware of the fact that they are receiving outdated information and form rational
expectations about the value of the project today. Creditors’ response to a disclosure lag depends on the project’s growth-to-risk ratio. Creditors run less frequently with a disclosure lag if the project has a high growth-to-risk ratio. The reason is that low realizations of the project’s value observed with a lag are expected to recover given the high growth. On the other hand, creditors’ incentives to run increase when the asset has a low growth-to-risk ratio.

A lag in disclosure also affects the optimal behavior for the bank manager however. A longer disclosure lag makes risk shifting more attractive because it increases the marginal value of risk. The manager’s payoff is equal to a call option on the assets of the bank with time to maturity being the disclosure lag. A longer disclosure lag increases the value for the bank manager.

The manager risk-shifting problem, the bank run and the asymmetric information induced by the regulator’s policy make the effect of disclosure lags on bank value non-trivial. First, a disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank holds liquid assets. Liquid assets have low growth opportunities that motivate the bank manager to compensate by taking on a risky project. This produces an asset with a low growth-to-risk ratio. As the disclosure lag increases, creditors become nervous and tend to run more frequently because of the bad characteristics of the project. The manager optimally reduces risk to compensate the increased probability a bank run. The risk reduction effect dominates and the bank value increases with a disclosure lag.

Second, when the bank’s assets are illiquid, a disclosure lag increases the occurrence of runs and hence decreases the value of the bank. Illiquid assets have high growth opportunities that prevent excessive risk taking by the manager. With low-risk taking, the project has a high growth-to-risk ratio. As the disclosure lag increases, creditors’ incentive to run decreases given the good quality of the project. The bank manager sees that the run probability decreases and takes advantage of this by increasing risk. The increase in risk effect dominates and the bank value decreases. A disclosure lag increases the occurrence of bank runs when bank assets are illiquid.

Third, illiquid assets decrease the manager’s risk appetite that reduces the bank run probability. Keeping the value of the assets constant, the bank manager selects low risk projects when the
project has relatively higher growth opportunities (illiquid assets). This occurs because a high growth rate increases the manager’s payoff and hence she depends less on risk-shifting to increase her utility. With low risk, creditors’ aggregate response is to run less frequently. Hence, as growth opportunities increase we see banks that have lower probability of default.

This paper is related to the disclosure in the financial industry literature. Gigler, Kanodia, Sapra, Venugopal (2013) (hereafter GKSV) study the frequency of disclosure that should be required for public firms. They show that when the bank manager can endogenously take decisions, frequent disclosure does not necessarily imply economic efficiency. In a similar flavor, our model says that a shorter disclosure lag does not necessarily improve bank value as the manager might risk-shift. In a model with frictions and when the bank’s decision is endogenous, price efficiency does not imply economic efficiency. Morris and Shin (2002) show the trade-off between market discipline and strategic concerns that cause coordination problems. We provide special structure to the problem, which allows us to identify whether disclosure lags are beneficial by looking at the bank assets.

The study presented in this paper applies to various situations. First, it could be applied to the analysis of disclosure of participants, volumes and collateral at the discount window of the Federal Reserve. Before the Dodd-Frank act, identities of borrowers at the discount window where kept confidential. The Dodd-Frank act mandated that disclosure should take place with a two year lag maximum. Our model applies to this situation since the tradeoffs analyzed are present in the discount window disclosure debate5. Second, disclosure of stress tests has been controversial for several reasons. This has been documented both in academic papers (Goldstein and Sapra (2014)) and in the press6. A lag in disclosure is a tool that bank regulators might consider in order to balance the benefits and costs.

There are other contexts for which this framework might be relevant. In accounting, we see a debate between mark to market and historical valuation. A shorter lag implies a situation in which the accounting is closer to a mark to market model whereas a longer lag relates to a historical

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5See "Banks Face Borrowing Stigma" http://on.wsj.com/1L5CXb6
6See "Lenders Stress Over Test Results" http://on.wsj.com/1PWhsOh
based accounting. This model could shed light into understanding the impact of accounting rules when taking into consideration the distribution of assets held by banks. Finally, central banks have been using disclosure lags in a variety of situations. In many situations it has shifted from a no disclosure policy to a disclosure with a lag policy. For example, the decisions of the Federal Open Market Committee (FOMC) were not announced twenty years ago. The Fed gradually increased the degree of communication and currently it releases the minutes of the meeting with a lag of three weeks\(^7\).

This paper is organized in 6 sections, the first one being this introduction. In the following section, we describe the environment were the bank lives and what actions can be taken by creditors and bank management. Section 3 presents the optimization problems faced by creditors and bank management. Section 4 presents the equilibrium analysis, while Section 5 introduces an example of an optimal disclosure lag. Section 6 characterizes the optimal disclosure lag policy by studying the asset characteristics. Conclusion follows.

## 2 Model

The model builds on He and Xiong (2012). As mentioned in the introduction, we explicitly model the bank fundamentals disclosure lag and we allow the manager to endogenously choose the riskiness of the assets\(^8\). The model is written in continuous time with and infinite horizon. The bank invests in a long-term asset by rolling over short-term debt, which is financed by a continuum of small creditors.

### 2.1 Assets, the banker and moral hazard

We normalize the bank asset holding to one unit. The bank borrows $1 at time \( t = 0 \) to buy the asset. Once the asset is in place, it generates a constant stream of cash flow \( r dt \) over the interval \([t, t + dt]\). At a random time \( \tau_\phi \), which arrives according to a Poisson process with intensity \( \phi > 0 \),

\(^7\)See “A Short History of FOMC Communication” http://www.dallasfed.org/research/eclett/2013/el1308.cfm
\(^8\)This is a bank in the sense that it is financed by short-term debt that is prone to runs.
the asset matures with a final payoff of $y_{r_0}$. The advantage of assuming a random asset maturity is that it makes the expected life of the project constant and equal to $1/\phi$.

The final payoff of the asset evolves according to a geometric Brownian motion with drift $\mu$ and volatility $\sigma$. Mathematically, $y_t$ evolves according to the process

$$dy_t = \mu y_t dt + \sigma y_t dW_t$$

where $W(t)$ is a standard Brownian motion, $\mu$ is the growth rate of the final payoff and $\sigma$ is the instantaneous volatility.

The bank’s asset generates the constant cash flow $rdt$ and the random final payoff $y_{r_0}$. The value of the project is given by the expected discounted future cash flows

$$F(y_t) = E_t \left[ \int_t^{\tau_0} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_0-t)} y_{r_0} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t$$

where $\frac{r}{\rho + \phi}$ is the present value of the constant cash flow and $\frac{\phi}{\rho + \phi - \mu} y_t$ is the present value of the final payoff. Since the fundamental value of the bank is a linear function of $y_t$, we refer to $y_t$ as the bank fundamental.

A. Risk-shifting

The bank manager controls the project riskiness, in particular, she chooses the asset’s final-payoff volatility ($\sigma$). The bank manager chooses a risk level $\sigma$ such that $\sigma_L \leq \sigma \leq \sigma_H$. Equivalently, the banker chooses a combination of a project with low risk and a project with high risk. The value of $\sigma$ is observable, but assumed not contractible. Risk-shifting is a feature not present in He and Xiong (2012). A different version of risk-shifting is studied by Cheng and Milbradt (2011).

2.2 Debt financing, runs and liquidation

The bank financing, runs and liquidation follow closely He and Xiong (2012). We present a brief review for completeness. The bank finances the asset by issuing short-term debt. That is, each debt
contract lasts for an exponentially distributed amount of time with mean $1/\delta$. Once an individual contract expires, the creditor chooses whether to roll over the debt or run. The maturity shocks are independent across creditors so that each creditor expects some other creditors’ contracts to mature before his.

In aggregate, a fraction $\delta dt$ of the banks debt matures over the time interval $[t, t + dt]$. When maturing creditors choose to run, the bank must find financing from other sources or it will be forced into bankruptcy. We assume that the bank has access to a credit line that supplies the financing required. When a run occurs, there is a probability $\theta \delta dt$ that the credit line will fail to provide the required financing. The parameter $\theta > 0$ measures the reliability of the credit line. If the credit line fails, the bank is forced to liquidate assets in an illiquid secondary market. We assume that the asset’s liquidation value is a fraction $0 < \alpha < 1$ of the fundamental value of the project

$$L (y_t) = \alpha F (y_t)$$

$$= \frac{\alpha r}{\rho + \phi} + \frac{\alpha \phi}{\rho + \phi - \mu} y_t$$

$$= L + l y_t$$

where $L = \frac{\alpha r}{\rho + \phi}$ and $l = \frac{\alpha \phi}{\rho + \phi - \mu}$.

2.3 Information sets

Our model departs from He and Xiong (2012) in the setup of the information structure. A regulator has control over the disclosure of the bank’s fundamental, $y_t$. The regulator discloses the value of the bank fundamental with a lag $I \geq 0$. This means that at time $t > 0$ creditors learn about the bank’s fundamental value at time $t - I$, $y_{t-I}$. Creditors are aware of the fact that they are receiving outdated information and will form rational expectations about the bank’s fundamental value today.
Mathematically, creditors observe $x_t$, the bank fundamental process with lag, where

$$x_t = y_{t-I}$$

with $I \geq 0$. Knowing that $y_t$ evolves according to equation (1), creditors anticipate that $x_t$ evolves according to a geometric Brownian motion with the same drift and variance as in equation (1),

$$dx_t = \mu x_t dt + \sigma x_t dW_t$$

and $x_t = y_{t-I}$.

We assume that the bank manager is an insider and hence observes the bank fundamental, $y_t$, with no lag. Moreover, she is aware of the fact that creditors observe a lagged fundamental value and she will use this information in making optimal choices.

2.3.1 Timeline

Figure 1 presents a timeline describing the information structure of the model. The lag in disclosure is set at a value $I \geq 0$, and is common knowledge to all parties. The solid line represents a realization of the fundamental process, $\{y_u\}_{u \leq t}$, which is observed by the manager. The dashed line represents the corresponding lagged process $x(t)$, which is the information available to the creditors at time $t$.

At time $t - I$, the bank manager chooses (a one-time decision) the riskiness of the project, $\sigma \in [\sigma_L, \sigma_H]$, based on the current value of the fundamental, $y_{t-I}$.
3 Creditor’s and manager’s problem

3.1 An individual creditor’s problem

We follow He and Xiong (2012) and analyze the rollover decision for an individual creditor by taking as given that other creditors use a monotone strategy. A monotone strategy indicates that all creditors whose debt is maturing decide to rollover when the bank’s fundamental $x_t$ is greater than a threshold $y_\ast$. On the other hand, when $x_t \leq y_\ast$ all maturing creditors decide to run.

There are two possible outcomes for the bank. Either the project’s final payoff is realized or the bank is prematurely liquidated after a run. These events are not controlled directly by an individual creditor. However, once an individual’s debt matures, he can decide whether to rollover the debt or not. Each creditor receives interest payments at a rate $r$ per unit of time until

$$\tau = \min \left( \tau_\phi, \tau_{\theta \delta}, \tau_{\delta} \right)$$

which is the earliest of the following three events: Project completion, forced liquidation after a run and debt expiration without rollover, respectively. When debt matures, creditors receive the face value of the debt back and have the option to rollover their position by buying the new debt.
With a risk neutral creditor, the value of one unit of debt is given by the value function

$$V (x_t) = E_t \left[ \int_t^\tau e^{-\rho(s-t)} r ds + e^{-\rho(\tau-t)} \left\{ \min (1, y_\tau) 1_{\{\tau=\tau_0\}} + \min (1, \mathcal{L} (y_\tau)) 1_{\{\tau=\tau_\delta\}} + \max_{\text{run or rollover}} (1, V (x_\tau; y_\tau)) 1_{\{\tau=\tau_\delta\}} \right\} \right]$$

where $1_{\{}$ takes the value 1 when the statement in brackets is true and 0 otherwise. The value for an individual creditor has four components, represented by the four terms in the right hand side of equation (4). First, he will receive coupon payments at rate $r$ while the bank is alive. Second, when the asset matures, the creditor will be paid in full if the final payoff is greater than 1, and $y_t$ if it is lower than 1 ($\min (1, y_\tau)$). Third, if the bank is forced into bankruptcy, the creditor will be paid in full only when the asset value after liquidation costs is greater than 1, otherwise he will receive the asset which can be liquidated for $\mathcal{L} (y_\tau)$ ($\min (1, \mathcal{L} (y_\tau))$). The last term represents the option that each creditor has to decide whether to rollover or run when the debt matures. The expectation in (4) is conditional on the information available to the creditor at time $t$, that is, the lagged fundamental value process $x_t$.

The Appendix derives the HJB equation for the value function $V (x_t)$,

$$\rho V (x_t; y_\tau) = \mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx} + r + \phi [E_t [\min(1, y_t)] - V (x_t; y_\tau)] + \theta \delta 1_{\{x_t < y_\tau\}} [E_t [\min(1, \mathcal{L} (y_t))] - V (x_t; y_\tau)] + \delta \max_{\text{run or rollover}} (1 - V (x_t; y_\tau), 0)$$

where $E_t [\cdot]$ is the expectation taken with the information available to the creditor at time $t$. Even though the creditor knows that the current time is $t$, he needs to estimate the current value of $y_t$ from the information provided by the lagged process, $x_t = y_{t-1}$. Hence, a creditor’s estimate of the payoff in case of project completion is $E_t [\min(1, y_t)] = E_t [\min(1, x_{t+1})]$. Similarly, the creditor’s estimate of the payoff in case of bankruptcy at time $t$ is $E_t [\min(1, \mathcal{L} (y_t))] = E_t [\min(1, \mathcal{L} (x_{t+1}))]$. 

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The information structure of the model produces a generalized version of the HJB equation in He and Xiong (2012). Of course, when \( I = 0 \), we recover the value function in He and Xiong (2012).

The left hand side of equation (5), \( \rho V (x_t; y_*) \), represents the creditor’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, \( \mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx} \), capture the change in continuation value caused by the fluctuation in the bank fundamental. The next four terms represent the components that appeared in the integral form of the value function, equation (4). That is, the coupon payment rate, \( r \), the value change when the project matures at time \( \tau \phi \) (\( E_t [\min(1, y_t)] - V (x_t; y_*) \)), the value change caused by a forced liquidation, \( \theta \delta 1_{\{x_t < y_*\}} [E_t [\min(1, L(y_t))] - V (x_t; y_*) \]), and the option to run when the debt contract expires, \( \delta \max_{\text{run or rollover}} (1 - V (x_t; y_*), 0) \), respectively.

A creditor whose debt is maturing will choose to rollover whenever the value of doing so is higher than the debt’s face value of 1. In other words, if the value function only crosses 1 at the point \( x' \) (\( V (x'; y_*) = 1 \)), then \( x' \) is the optimal cutoff for the creditor. Whenever debt matures and \( x_t < x' \), the creditor will not rollover the debt. On the other hand, when \( x_t \geq x' \), the creditor finances a new debt contract.

In this paper we solve for symmetric monotone equilibria for which the threshold \( y_* \) is the same for all creditors. Therefore, a condition to determine the threshold \( y_* \) is that \( V (y_*; y_*) = 1 \).

### 3.2 The bank manager’s problem

The bank manager holds the firm’s equity. Given a rollover threshold, \( y_* \), the manager maximizes the total value of the residual claim by choosing the project riskiness, \( \sigma \). The manager’s choice is made at time \( t - I \) and is held constant from \( t - I \) onwards. Mathematically, the value of equity at time \( t \) can be written as

\[
Q (y_t; y_*, \sigma) = E_{t-I} [e^{-\rho(t-(t-I))} \{ \max (y_T - 1, 0) \delta \{\tau - \tau_T \} + \max (L(y_T) - 1, 0) \delta \{\tau = \tau_T \} \}]
\]

where \( E_{t-I} [\cdot] \) is the expectation conditional on the information available to the manager at
time $t - I$, the non-delayed fundamental value $y_{t-I}$. The value function of the manager has two components, represented by the two terms on the right hand side of (6). The first one represents the equity payoff when the project ends, $(y_t - 1)^+$ and the second one represents the payoff after a forced liquidation, $(L(y_I) - 1)^+$.

Figure 2 presents a timeline describing the information available to the manager at time $t - I$. As an insider of the bank, we assume that the manager can observe the fundamental value with no delay, $\{y_s\}_{s \leq t-I}$. This is shown in Figure 2 as the black solid-line. The manager is also aware that creditors are making optimal choices based on lagged information. This means that at time $t - I$, the manager can anticipate how creditors will behave (rollover the debt or not) from $t - I$ until $t$. It also means that the value function depends on the path of the fundamental process from $t - 2I$ to $t - I$, $\{y_s\}_{t-2I \leq s \leq t-I}$.

We can write the manager’s value function as the sum of two terms: Events that happen from $t - I$ until $t$ (path dependent) and events that happen from $t$ onwards (independent of the path). Mathematically,

$$Q\left(\{y_s\}_{t-2I \leq s \leq t-I} ; y_t, \sigma\right) = Q^1 \left(\{y_s\}_{t-2I \leq s \leq t-I} ; y_t, \sigma\right) + e^{-\rho I} P(\tau \geq t)Q^2 (y_{t-I} ; y_t, \sigma)$$

(7)

where $Q^1 \left(\{y_s\}_{t-2I \leq s \leq t-I} ; y_t, \sigma\right)$ represents the part of the value function that is path dependent and $Q^2 (y_{t-I} ; y_t, \sigma)$ represents the value function from $t$ onwards which is not path dependent.
We study the steady state situation in which the value function for the manager is path independent. Specifically, we study the behavior of \( Q^2(y_{t-I}; y_*, \sigma) \). This modelling choice allows us to focus on how the manager and creditors change behavior as the delay in information changes while keeping the path dependence complications at a minimum. Moreover, this scenario is consistent with a case in which there has been a disclosure lag policy in effect for a long time, and changes are being considered.

The value for the manager at time \( t - I \) from events that occur after \( t \), \( Q^2(y_{t-I}; y_*, \sigma) \), can be computed as

\[
Q^2(y_{t-I}; y_*, \sigma) = E[e^{-\rho(\tau-t)}((y_T - 1)^+ + \mathcal{L}(y_T) - 1)^+]1_{\{\tau_0 \leq \tau\}}|A_2]
\]

with \( A_2 = \{y_{t-I}, \tau \geq t\} \). The set \( A_2 \) represents the information for events that occur after \( t \).

The Appendix presents the derivation of the HJB equation for the equity value conditional on the bank surviving until time \( t \), \( Q^2(y_{t-I}; y_*, \sigma) \),

\[
\rho Q^2(y_{t-I}; y_*, \sigma) = \mu y_{t-I} Q^2_y + \frac{\sigma^2}{2} y_{t-I}^2 Q^2_{yy} + \phi \left[ E_{t-I}[(y_T - 1)^+] - Q^2 \right] + \theta \delta 1_{\{y_{t-I} < y_*\}} \left[ E_{t-I} \left[ (\mathcal{L}(y_T) - 1)^+ \right] - Q^2 \right].
\]

The left hand side of equation (9), \( \rho Q^2(y_t; y_*, \sigma) \), represents the banker’s required return. The right hand side represents the expected increments on the continuation value. The first two terms, \( \mu y_{t-I} Q^2_y + \frac{\sigma^2}{2} y_{t-I}^2 Q^2_{yy} \), capture the change in continuation value caused by the fluctuation in the bank fundamental. The next two terms capture the final value for the banker. The third term, \( \phi \left[ E_{t}[(y_{t+I} - 1)^+] - Q^2 \right] \), captures the expected continuation value change when the project matures at time \( \tau_\phi \). The last term, \( \theta \delta 1_{\{y_T < y_*\}} \left[ E_{t} \left[ (\mathcal{L}(y_{t+I}) - 1)^+ \right] - Q^2 \right] \), represents the expected continuation value change caused by a forced liquidation.
3.2.1 Optimization problem

Given an initial value of the process, \( y_{t-1} \), and an optimal creditor’s cutoff, \( y_* \), the manager chooses the project riskiness that maximizes the value of equity at time \( t \). Mathematically,

\[
\sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q^2 (y_{t-1}; y_*, \sigma)
\]

subject to (1)and (8).

4 Bank Runs and Asset-Risk Equilibrium

Given a project riskiness, \( \sigma \), we limit attention to monotone equilibria in which each creditor’s rollover strategy is monotone with respect to the bank fundamental observed at time \( t \), \( x_t \). This means that we are interested in deriving a cutoff, \( y_* \), such that whenever \( x_t < y_* \), the optimal decision for maturing creditors is to run on the bank and when \( x_t \geq y_* \), the optimal strategy is to rollover. Moreover, we explore symmetric monotone equilibria in which each creditor’s optimal choice, \( x' \), must be equal to the other creditors’ threshold \( y_* \). Therefore, a condition for determining the equilibrium threshold is \( V (y_*; y_*) = 1 \).

The analysis requires some parameter restrictions in order to be meaningful. We work with the same parameter restrictions as in He and Xiong (2012).

**Definition 1** A pair \((\sigma_*, y_*)\), project volatility and run threshold, is a equilibrium if \( V (y_*; y_*) = 1 \) and \( \sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q^2 (y_{t-1}; y_*, \sigma) \). That is, the threshold is optimal for a creditor that takes \( \sigma_* \) as given and the manager maximizes the equity value (8) taking the optimal cutoff \( y_* \) as given.

4.1 Equilibrium Analysis

We perform a numerical analysis that will serve to understand the basic properties of the model.

A. Parameter values
Table 1 presents the baseline parameter values. Except for the volatility (which is endogenously chosen by the manager), we use the *same* parameters values as in He and Xiong (2012).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Interpretation</th>
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</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>1.5%</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$r$</td>
<td>7%</td>
<td>Cash flow rate from project</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.077</td>
<td>Intensity of terminal value realization</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>55%</td>
<td>Liquidation discount</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.5%</td>
<td>Drift of asset final payoff</td>
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<td>$\delta$</td>
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<td>Intensity at which debt matures</td>
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<tr>
<td>$\theta$</td>
<td>5</td>
<td>Intensity of credit line failure</td>
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<td>$y_t$</td>
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<td>Fundamental value at time $t$</td>
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<td>Minimum volatility</td>
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<tr>
<td>$\sigma_H$</td>
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<td>Maximum volatility</td>
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</table>

4.2 **Benchmark: No runs and no risk shifting**

It is informative to conduct and analysis of the model when there are no runs or risk shifting frictions.

4.2.1 **No Runs**

If there are no runs in the model but a single creditor that holds a perpetual bond, then the manager faces no trade off when choosing volatility. The bank manager benefits with higher risk and hence she chooses the maximum risk possible, $\sigma_H$. If at the riskiness level $\sigma_H$ the creditors value is greater than the initial investment required of $\$1$, then financing will occur. Otherwise, there is no financing. The firm value is independent of $\sigma$, but a transfer of value occurs from the creditor to the manager as the manager chooses more risky projects.
4.2.2 No Risk Shifting

If we do not allow the manager to choose asset riskiness and there is no delay in the information we are back at the model by He and Xiong (2012). Runs are costly because of illiquid asset values and hence a lower run threshold increases bank value. That means that the lower the threshold the higher the bank value.

4.3 Equilibrium with no lag, \( I = 0 \)

We start the analysis of equilibrium in the model. Consider first a situation in which there is no lag in disclosure, or \( I = 0 \).

Figure 3 plots the manager’s and creditors’ optimal behavior. The solid line plots the optimal volatility \( \sigma_* \) as a function of the cutoff \( y_* \). The higher the run cutoff \( y_* \), the lower the optimal volatility \( \sigma_* \). For low values of \( y_* \), the probability of the fundamental hitting \( y_* \) is low, and since the manager is long a call option on the fundamental of the bank, he chooses the maximum volatility possible. As the run cutoff increases, the marginal value of volatility for the manager decreases because the probability of a run (where the manager gets nothing after liquidation) increases.

The dash-dot line in Figure 3 shows the optimal threshold, \( y_* \), as a function of the fundamental’s volatility, \( \sigma \). The running cutoff is an increasing function of project risk. The creditor’s final payoff, \( E_t \left[ \min(1, y_t) \right] \), is equal to the payoff obtained by a short position on a put option and long position on a bond, that is, \( E_t \left[ \min(1, y_t) \right] = E_t \left[ 1 - \max(1 - y_t, 0) \right] \). A higher volatility makes the short position on the put less valuable, harming creditors, which in aggregate choose a higher running threshold.

A pair \((\sigma_*, y_*)\) is an equilibrium if \( \sigma_* \) is consistent with \( y_* \). This means that the intersection of the solid line and dash-dot line determines an equilibrium in Figure 3. The equilibrium is given by the pair \((\sigma_*, y_*) = (3.24\%, 0.9)\). This in an equilibrium where there is some risk-shifting and we will observe a run whenever the fundamental value \( x_t \) gets below 0.9.
4.4 Equilibrium with disclosure lag, $I \geq 0$

Now we turn our attention to the equilibrium analysis when there is a disclosure lag, $I > 0$. Figure 5 plots the banker’s and creditors’ optimal strategy using the baseline parameters for two lag values, $I \in \{0, 2.5\}$. The black-solid line plots the bank manager’s optimal volatility ($\sigma_*$) as a function of the cutoff $y_*$ for $I = 0$. The black-dash line plots the creditors’ optimal run threshold, $y_*$, as a function of the volatility $\sigma$ for $I = 0$ (Same lines as in Figure 3). The equilibrium is given by the intersection of the lines at the point $(\sigma_*, y_*) = (3.24\%, 0.9)$.

Figure 5 also plots the optimal choice taken by the bank creditors and manager when there is a disclosure lag of $I = 2.5$ years. The grey-solid line plots the optimal risk level chosen by the bank manager ($\sigma_*$) as a function of the cutoff $y_*$ for $I = 2.5$. The grey-dash line plots the optimal run threshold chosen by creditors, $y_*$, as a function of the volatility $\sigma$ for $I = 2.5$. The intersection of these lines determines the equilibrium, which is given by the pair $(\sigma_*, y_*) = (3.84\%, 0.89)$. When compared to the zero lag case, this equilibrium has a lower roll-over threshold and higher asset risk.

By studying equation (5), we can understand why the rollover threshold ($y_*$) decreases with an increase in the disclosure lag. The expected continuation value change when the project ends is
given by the term $\phi [E_t [\min(1, y_t)] - V (x_t; y_*)]$. As noted previously, $\min(1, y_t) = 1 - \max(1 - y_t, 0)$, so that the creditors final payoff is equal to having a long position in a bond and a short position in a put option written on the final payoff ($y_t$). The derivative of the put option value, $p(t, x_t) = E_t [\max(1 - y_t, 0)] = E_t [\max(1 - x_{t+1}, 0)]$, with respect to the lag $I$ is given by

$$\frac{\partial p(t, x_t)}{\partial I} = N' (-d_-) \frac{\sigma}{2\sqrt{I}} - x_t \mu e^{\mu I} N (-d_+),$$

where

$$d_+ = \frac{1}{\sigma \sqrt{I}} \left[ \ln [x_t] + \left( \mu + \frac{1}{2} \sigma^2 \right) I \right]$$

and $N (\cdot)$ is the cumulative distribution of the standard normal distribution and $\tau$ indicates the first derivative.

When the volatility of the project $\sigma$ is low relative to the drift $\mu$, equation (10) indicates that the put option value decreases as the lag $I$ increases (the derivative is negative). In other words, creditors, who are short a put option, see an increase in value as the lag $I$ increases. This is intuitive since with a positive drift, a longer lag makes it more likely for the final asset payoff to end above the face value of the debt. In this situation, bad news that occurred in the past are smoothed thanks to the expectation that the process recovers on average.

If the volatility of the project $\sigma$ is high relative to the drift $\mu$, the opposite effect takes place. Since the put option increases in value with the delay of information, creditors are worse off and hence choose to run sooner (a larger threshold). Figure 5 illustrates that this is the case for the higher volatilities in the allowed range.

A lag in disclosure also affects the optimal behavior for the manager. A longer lag in disclosure makes risk-shifting more attractive because the marginal value of volatility increases. The expected continuation value change when the project ends is given by the term $\phi [E_{t-I} [(y_t - 1)^+] - Q^2 (y_t; y_*)]$ which indicates that the manager is long a call option on the final payoff of the asset with maturity equal to $I$. The derivative of the call option value, $c(t, y_{t-I}) = E_{t-I} [\max(y_t - 1, 0)]$, with respect
to the lag $I$ is given by

$$\frac{\partial c(t,y_{t-1})}{\partial I} = N'(d_-) \frac{\sigma}{2\sqrt{I}} + y_t \mu e^{\mu I} N(d_+),$$

(11)

which is always positive.

It is not clear whether the bank value is higher with a disclosure lag. Holding everything else constant, a higher value for $\sigma_s$ decreases the value of the bank. This occurs because a higher volatility rate increases the likelihood of the fundamental value crossing the roll over threshold $y_s$, which will cause creditors to run and costly liquidate the bank.

On the other hand, a lower run cutoff $y_s$ increases the bank value. This occurs because a rollover freeze is less likely and hence inefficient liquidations occur less frequently, which is value increasing.

Figure 5 shows that as the lag in disclosure increases, the equilibrium volatility $\sigma_s$ increases and the cutoff $y_s$ decreases, making unclear whether the bank value increases or decreases. This is exactly the trade-off that this paper is presenting. Both extremes (no disclosure or immediate disclosure) present some cost to the bank and hence there is room to propose the lag in disclosure as a policy tool. Which effect dominates (reduction in the cutoff, $y_s$, or increase in volatility, $\sigma_s$) will depend on the slopes of the curves.

### 4.5 Disclosure lag and the probability of a run

A disclosure lag causes the bank value to change. As the lag in disclosure changes, the equilibrium run-cutoff and project riskiness change affecting the probability of a run. A higher probability of a run decreases bank value because asset liquidation is costly. Given $(\sigma_s, y_s)$, the probability of a run can be computed by using

$$P_{y_0} \left( \inf_{0 \leq s \leq \tau_6} y_s \leq y_s \right) = \left( \frac{y_s}{y_0} \right) \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2 \sigma^2}{\sigma_s^2} + \left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)}.$$

(12)
Figure 4: Equilibrium for two lags, $I = 0$ and $I = 2.5$ years

Figure 5:

Equation (12) is increasing in $y_*$ and it is also increasing in $\sigma^2$ provided that $\mu \geq \sigma^2/2$. That is, higher run cutoffs and riskier projects increase the probability of a run.

A lower probability of a run increases the bank value. For the disclosure lag to increase the value of the bank, the reduction of the run cutoff should compensate the increase in project risk. Alternatively, the reduction in risk should be enough to compensate for an increase in the cutoff.

Figure 6 adds contour plots to the equilibrium analysis with two lags. As expected from equation (12), the probability of a run increases with the riskiness of the project and with the run cutoff. The equilibrium with no delay and a delay of 2.5 years are located in a zone where the probability of a run is very low. Since the run probability stays almost equal in the presence of a lag, the bank value should not change.

Figure 7 plots the bank value as a function of the disclosure lag. As expected, the bank value is flat for low values of the lag. For a disclosure lag of three or more years, the bank value decreases because the increase in risk is not compensated by the change in the run cutoff.
Figure 6: Probability of a run and delay

Figure 7: Bank value and run probability
5 Optimal Disclosure Lag

With the parameters from He and Xiong (2012) and introducing a disclosure lag with endogenous volatility, the equilibrium has a very low probability of default. According to Figure 7 it increases from 0 up to 0.2% when the disclosure lag increases from 0 to 6 years. In essence, the equilibrium with no information delay has no real probability of default, and hence the changes in bank value are small.

We modify the parameters to study how the bank value changes in response to a lag in information disclosure when the probability of a run is significant. Keeping the value of the asset constant, we let \( \mu \) decrease so that \( \mu = 0.5\% \) and \( r = 8.42\% \). A lower drift (\( \mu \)) has effects on both the choices of the bank manager and creditors. The bank manager selects a higher volatility when the drift (\( \mu \)) is lower, keeping the rollover threshold fixed. The reason is that a lower drift decreases the final asset payoff and hence the manager optimally increases her risk bet. For creditors, as lower drift (\( \mu \)) in the final payoff increases their incentives to run as they see lower quality in the assets.

Increasing the coupon rate (\( r \)) affects the optimal response by creditors. With a higher coupon rate, creditors incentives to run decreases they see a better prospect for the asset of the bank. The bank manager’s behavior is mildly affected by a change in the coupon rate.

A lower drift (\( \mu \)) of the final payoff and a higher coupon rate (\( r \)) make the equilibrium to shift to higher volatilities, which allows for more action in the bank value as the disclosure lag is introduced.

Figure 8 plots the bank value and run probability as the disclosure lag changes from 0 to 6 years. As the delay increases from 0 to 2 years, the probability of a run decreases and the bank value increases. The bank value attains a maximum and then starts decreasing for values higher than 3 years. The probability of a run increases from 3 years onwards. The probability of a run decreases because the equilibrium risk reduction effect dominates the increase in the run cutoff. After 2 years, the increase in the run cutoff dominates and hence the probability of a run increases, which means that bank value decreases.

There are three forces that cause the bank value to change when the lag changes. First, the
Figure 8: Value and Run Probability

![Graph showing the relationship between bank value and run probability over different lags.]

Figure 9: Run probability and Disclosure Lag

![Graph showing the relationship between run probability and asset risk with different lag settings for managers and creditors.]
optimal roll over cutoff for creditors changes. In particular, a decrease in the cutoff (everything else equal), increases both the creditors and manager’s position and hence it increases the bank value. The second force at play is the optimal volatility selected by the manager. An increase in optimal volatility decreases bank value because it makes more likely for the fundamental to cross the roll over threshold. The third force is the extra volatility that one gets from the delay. Today’s estimate of the fundamental value \( y_t, E_t[y_t] = x_t e^{\mu t} \) is kept constant by setting, \( x_t = 1.4e^{-\mu t} \). However, the extra volatility decreases the bank value as the lag increases according to equation (13),

\[
\rho A(x_t; y_*, \sigma) = \mu x_t A_x + \frac{\sigma^2}{2} x_t^2 A_{xx} + r + \phi [x_t e^{\mu t} - A] + \theta \delta 1_{\{x_t < y_*\}} [L + l x_t e^{\mu t} - A].
\]

where \( A(x_t; y_*, \sigma) \) is the value of the bank computed with public information \( x_t \).

6 Optimal delay and bank characteristics

Using the parameters in He and Xiong (2012), we have seen that the run probability increases as the disclosure lag increases. This translates into a lower bank value for higher lags. The reason for this result is that the decrease in the run cutoff effect is dominated by the increase in the risk of the project.

In the previous section, we decreased \( \mu \) and increased \( r \) while keeping constant the initial value of the assets. An increase in \( r \) decreases the run cutoff keeping other parameters constant. The result is the equilibrium occurs at higher values of \( \sigma \) where the probability of a run is higher. As the disclosure lag increases, the equilibrium has lower asset risk \( (\sigma_*) \) and higher run cutoff \( (y_*) \). The effect of lower asset risk dominates, decreasing the run probability and increasing the bank value. The delay of information is beneficial. Moreover, there is an optimal delay that maximizes the bank value.
Increasing the coupon rate $r$ and decreasing the growth rate $\mu$ (Keeping the firm value constant) is changing value from the final payoff to the continuous coupons. The asset is becoming more liquid as it produces more cash from the coupons and not by the final payoff that is subject to random fluctuations. Another interpretation is that increasing $r$ and decreasing $\mu$ is decreasing the average cash-flows maturity of the asset. Yet another interpretation is that a higher coupon $r$ and lower $\mu$ is decreasing the growth opportunities of the asset.

Figure 10 plots how the equilibrium changes when there is an increase in $r$ and a decrease in $\mu$ while keeping the asset value constant. The black line with circles plots how the equilibrium changes from a high drift ($\mu$) and low coupon rate ($r$) (Benchmark parameters, bottom left) to a low drift and high coupon rate. The equilibrium moves from a low asset risk with low run threshold ($\sigma = 4\%, \ y_\text{r} = 0.88$) to a higher asset risk and higher run threshold ($\sigma = 12\%, \ y_\text{r} = 0.95$).

The black line in Figure 10 plots how the equilibrium changes when assets become more liquid (high $r$ and low $\mu$). The equilibrium riskiness of the assets, $\sigma_\text{r}$, increases because the manager compensates the low growth opportunities by increasing the bet on volatility. As the asset becomes more risky, creditors’ optimal response is to increase the run threshold ($y_\text{r}$). A project with low growth opportunities generates high probabilities of a run and lower bank values.
We can state the previous result in a different way. Relatively illiquid assets decrease the manager’s risk appetite and this reduces the bank run probability. Keeping the value of the assets constant, the bank manager selects low risk projects when the project has relatively higher growth opportunities (illiquid assets). This occurs because a high growth rate increases the manager’s payoff and hence she depends less on risk-shifting to increase her utility. With low risk, creditors’ aggregate response is to run less frequently. Hence, as growth opportunities increase we see banks that have lower probability of default.

The grey line with circles in Figure 10 plots how the equilibrium changes when there is an increase in $r$ and a decrease in $\mu$ while keeping the asset value constant for a disclosure lag of 2 years. In the bottom left corner, when the asset has relative low coupon rate and high drift, we are back the situation presented in Figure 5. Increasing the lag produces a more risky project and lower run cutoff. The probability of a run stays practically the same and the bank value does not change. The lag in disclosure is not beneficial.

A disclosure lag is not beneficial when the asset is illiquid (Low $r$ and high $\mu$) because the equilibrium asset risk and run threshold already guarantee a low run probability. Even more, if the disclosure lag is big, the bank value decreases as the manager can take more risk helped by the lower run cutoff. The disclosure lag decreases the equilibrium run-threshold but the manager’s optimal response is to take advantage of this situation by increasing risk. Even when the lag initially decreases the probability of a run, the endogenous choice of the manager offsets this benefit and increases the project risk, which in turn increases the default probability.

When the asset are more liquid, (they have relative higher coupon rate, $r$, and lower drift, $\mu$) the equilibrium moves to the top right in Figure 10. We are back to the situation presented in Figure 9. Increasing the lag in disclosure produces an equilibrium outcome that has lower risk and higher run cutoff. The reduction in risk effect dominates and there is an increase in the bank value as the lag increases.

A disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank holds liquid assets. Liquid assets have low growth opportunities that motivate the bank man-
ager to compensate by taking on a risky project. This produces an asset with a low growth-to-risk ratio. As the disclosure lag increases, creditors become nervous and tend to run more frequently because of the bad characteristics of the project. The manager optimally reduces risk to compensate the increased probability a bank run. The risk reduction effect dominates and the bank value increases with a disclosure lag.

7 Conclusion

This paper studies how lags in asset information disclosure affect the benefits and costs associated with disclosure. In particular, we study the effects of disclosure lags on runs and incentive provision for a bank financed with short-term debt. Runs occur because there is an intertemporal coordination problem among creditors as in He and Xiong (2012). A risk-shifting problem occurs because bank management has discretion over the asset riskiness. Creditors receive lagged information about the asset’s value. Our contribution is threefold. First, we show that a disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank holds liquid assets. A disclosure lag is beneficial because the bank manager reduces risk as creditors get nervous with opacity. The risk reduction increases the bank value.

Second, when the bank’s assets are illiquid, a disclosure lag increases the occurrence of bank runs and hence decreases the value of the bank. In the presence of a disclosure lag, creditors run less frequently but the manager increases the project risk. The increase in risk makes the bank value to decrease. Third, illiquid assets decrease the manager’s appetite for risk and this translates into a reduced run probability. Illiquid assets have relatively high growth opportunities that decrease the need of risky bets by the manager.

References


8 Appendix

8.1 Creditor’s HJB Equation

In this section we review the creditor’s problem and some of its properties. Recall that the creditor’s problem can be written as the solution of the following value function equation.

\[
V(x_t) = E_t\left[\int_t^\tau e^{-\rho(s-t)}rd{s} + e^{-\rho(\tau-t)}\{\min (1, y_\tau) 1_{\{\tau=\tau_\phi\}} + \min (1, L(y_\tau)) 1_{\{\tau=\tau_\phi\}} + \max_{\text{run or rollover}} (1, V(x_\tau; y_\tau)) 1_{\{\tau=\tau_\delta\}}\}\right]
\]

Now, we rewrite the problem in a different way. Fix a threshold \(y_\phi\). At each point in time \(u \geq t\), the creditor receives interest payments \(r\) and when the project ends he receives \(\min (1, x_{u+1})\) or \(\min (1, L(x_{u+1}))\) when the bank is forced into bankruptcy. Therefore, the creditor’s expected payoff at time \(u \geq t\) is given by

\[
r + \frac{\phi}{\phi + \theta\delta 1_{\{x_u < y_\phi\}}} E_u[\min (1, x_{u+1})] \\
+ \frac{\theta\delta 1_{\{x_u < y_\phi\}}}{\phi + \theta\delta 1_{\{x_u < y_\phi\}}} E_u[\min (1, L(x_{u+1}))]
\]
where \( E_u[\min (1, x_{u+I})] \) and \( E_u[\min (1, \mathcal{L}(x_{u+I}))] \) have Black-Scholes like formulas. Let \( p_1(x, u + I) = E[\min (1, x_{u+I}) | x_u = x] \) and \( p_2(x, u + I) = E[\min (1, \mathcal{L}(x_{u+I})) | x_u = x] \) and integrate the discounted payoff across possible values of \( x \) to get

\[
p(t, x_t) = E_{t,x_t} \left[ e^{-\rho(u-t)} \left( r + \frac{\phi}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_s\}} p_1(x_u, u + I) + \frac{\theta \delta \mathbf{1}_{\{x_u < y_s\}}}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_s\}}} p_2(x_u, u + I) \right) \right].
\]

We show that \( e^{-\rho t} p(x_t, t) \) is a martingale under \( P \).

Let

\[
E \left[ e^{-\rho u} \left( r + \frac{\phi}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_s\}} p_1(x_u, u + I) + \frac{\theta \delta \mathbf{1}_{\{x_u < y_s\}}}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_s\}}} p_2(x_u, u + I) \right) | F(t) \right] | F(s) = p(s, x_s).
\]

by the tower property. Hence, \( p(x_t, t) \) is a martingale and it satisfies the Black-Scholes formula.

The creditor’s value can be written as the Laplace transform of \( p(t, x_t) \), or

\[
V(x_t; y_s) = \int_0^\infty \left( \phi + \theta \delta \mathbf{1}_{\{x_u < y_s\}} \right) e^{-\phi + \theta \delta \mathbf{1}_{\{x_u < y_s\}}} u p(t, x_t) \, du
\]

and hence we can apply the Laplace transform to both sides of the Black-Scholes PDE to get equation (5).

### 8.2 Manager’s HJB Equation

In this section we show that the second part in the manager’s problem, \( Q^2(y_t; y_s, \Gamma) \), satisfies equation (9). Recall that \( Q^2(\cdot) \) represents the value for the manager after \( t + I \). Mathematically,

\[
Q^2(y_t; y_s, \Gamma) = E[e^{-\rho(t-I)} \{ (y_r - 1)^+ \mathbf{1}_{\{\tau = \tau_0\}} + (\mathcal{L}(y_r) - 1)^+ \mathbf{1}_{\{\tau = \tau_0\}} \} | A_2].
\]

Fix a time \( u \geq t + I \). At time \( u \), the project could expire with payoff \((y_u - 1)^+\), the bank could
be forced into bankruptcy with payoff \((\mathcal{L}(y_u) - 1)^+\) or it could continue.

Creditors’ receive information with a lag and hence the intensity relevant at time \(u\) depends on the value of the fundamental at time \(u - I\). The expected payoff at time \(u\) can be therefore written as

\[
\left(\phi(y_u - 1)^+ + \theta \delta I_{y_u < y^*}(\mathcal{L}(y_u) - 1)^+ \right)
\]

where \(\phi\) represents the intensity of the project expiring at time \(u\) and \(\theta \delta I_{y_u < y^*}\) represents the intensity of the bank failing at time \(u\).

Consider the discounted expected payoff at time \(u - I\)

\[
payoff(u - I, y_{u-I}) = E_{u-I}[e^{-\rho I}((\phi(y_u - 1)^+ + \theta \delta I_{y_u < y^*}(\mathcal{L}(y_u) - 1)^+))]
\]

which can be written as

\[
payoff(u - I, y_{u-I}) = \phi E_{u-I}[e^{-\rho I}(y_u - 1)^+] \\
+ \theta \delta I_{y_u < y^*} E_{u-I}[e^{-\rho I}(\mathcal{L}(y_u) - 1)^+]]
\]

The expected discounted payoff at time \(t\) is therefore

\[
c(t, y_t) = E_t[e^{-\rho(u-I-t)}\text{payoff}(u - I, y_{u-I})].
\]

We show that \(e^{-\rho t}c(t, y_t)\) is a martingale under \(P\). Let \(s \leq t \leq u - I\) and consider

\[
E \left[ e^{-\rho t}c(t, y_t) | F(s) \right] \\
= E \left[ E[e^{-\rho(u-I-t)}\text{payoff}(u - I, y_{u-I}) | F(t)] | F(s) \right] \\
= E \left[ e^{-\rho(u-I-t)}\text{payoff}(u - I, y_{u-I}) | F(s) \right] \\
= e^{-\rho s}c(s, y_s)
\]
which indeed means that \( c(\cdot) \) is a martingale. Therefore \( c(s,x) \) satisfies the Black-Scholes formula

\[
-\rho c(s,x) + c_t(s,x) + \mu x c_x(s,x) + \frac{1}{2} \sigma^2 x^2 c_{xx}(s,x) = 0
\]

with boundary conditions

\[
c(u - I, x) = \text{payoff}(u - I, x) \quad \forall x \in [0, \infty) \\
c(s,0) = 0 \quad \forall s \in [t, u - I] \\
\lim_{x \to \infty} c(s,x) = \phi(x e^{(\mu - \rho)(u-s)} - e^{-\rho(u-s)} \forall s \in [0, u - I].
\]

It is convenient to change the time variable to the time to expiration \( \tau = u - I - t \) so that \( g(\tau, x) = c(u - I - t, x) \) satisfies the PDE

\[
-\rho g(\tau, x) - g_\tau(\tau, x) + \mu x g_x(\tau, x) + \frac{1}{2} \sigma^2 x^2 g_{xx}(\tau, x) = 0
\]

with boundary conditions

\[
g(0, x) = \text{payoff}(u - I, x) \quad \forall x \in [0, \infty) \\
g(s,0) = 0 \quad \forall s \in [0, u - I] \\
\lim_{x \to \infty} g(s,x) = \phi(x e^{(\mu - \rho)(s+I)} - e^{-\rho(s+I)} \forall s \in [0, u - I].
\]

Since the maturity or bank failure are exponentially distributed, the manager’s value is the Laplace transform of \( g(\tau, x) \), or

\[
V(y, y_s) = \int_0^\infty e^{-\phi(T)(u-I)} g(\tau, x) d(u-I)
\]

and hence we can apply the Laplace transform to both sides of the Black-Scholes PDE (equation
\(- \rho G(x) - \left( \left( \phi + \theta \delta_{\{y_u \leq y_u \}} \right) G(x) - g(0, x) \right) + \mu x G_x(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}(x) = 0 \)

where \( G(\cdot) \) is the Laplace transform of \( g(\tau, x) \).

Finally, replace the boundary condition for \( g(0, x) \) to get equation (9)

\[
\rho G(x) = \mu x G_x(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}(x) + \phi \left( E_{\omega \sim i\mathcal{L}}[e^{-\rho \omega} (y_u - 1) \mathcal{L} - G(x)] \right) + \theta \delta_{\{y_u \leq y_u \}} \left( E_{\omega \sim i\mathcal{L}}[e^{-\rho \omega} (\mathcal{L} (y_u) - 1) \mathcal{L} - G(x)] \right)
\]

### 8.3 Asset Duration

The average time when cashflows are received (duration) can be computed as

\[
E \left[ \int_0^\tau se^{-\rho s} ds + \tau e^{-\rho \tau} y_\phi \right] = \frac{r}{(\rho + \phi)^2} + \frac{\phi}{(\rho + \phi - \mu)^2} y_0
\]

which is also the derivative of the asset value with respect to \( \rho \).
If we divide by the value of the assets we get the average maturity

\[
\text{Duration} = \frac{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0}{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0}
\]

\[
= \frac{\frac{r}{(\rho + \phi)} + \frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0}{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0}
\]

\[
= \frac{\frac{r}{(\rho + \phi)} + \frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0}{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0}
\]

\[
= \frac{r (\rho + \phi)(\rho + \phi - \mu)}{(r (\rho + \phi - \mu) + (\rho + \phi) \phi y_0) (\rho + \phi) + \phi y_0 (\rho + \phi)} + \phi y_0 (\rho + \phi) (\rho + \phi - \mu)^2
\]

\[
= \frac{r (\rho + \phi)(\rho + \phi - \mu)}{(r (\rho + \phi - \mu) + (\rho + \phi) \phi y_0) (\rho + \phi) + \phi y_0 (\rho + \phi)} + \phi y_0 (\rho + \phi) (\rho + \phi - \mu)^2
\]

\[
= \frac{r (\rho + \phi)(\rho + \phi - \mu)}{(r (\rho + \phi - \mu) + (\rho + \phi) \phi y_0) (\rho + \phi) + \phi y_0 (\rho + \phi)} + \phi y_0 (\rho + \phi) (\rho + \phi - \mu)^2
\]

\[
= \frac{r (\rho + \phi)(\rho + \phi - \mu)}{(r (\rho + \phi - \mu) + (\rho + \phi) \phi y_0) (\rho + \phi) + \phi y_0 (\rho + \phi)} + \phi y_0 (\rho + \phi) (\rho + \phi - \mu)^2
\]