Financial Networks and Contagion
Elliot, Golub and Jackson (2013)

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Carlos’s Networks @ Tepper
Network Integration and Diversification

- Integration (level of exposure of each organization to each other)

\[
\begin{align*}
1 & \quad 1 \\
1/2 & \quad 1/2 \\
1-c & \quad 1-c \\
\end{align*}
\]

\[
\begin{align*}
1 & \quad 1 \\
1/2 & \quad 1/2 \\
c & \quad c \\
\end{align*}
\]

\[
\begin{align*}
1 & \quad 1 \\
1/2 & \quad 1/2 \\
c & \quad c \\
\end{align*}
\]

\[
c = \text{Fraction of each organization held by others}
\]
Network Integration and Diversification

- Diversification (How spread out cross-holdings are)

\[ d_i = \text{Number of organizations that hold the } i\text{th organization} \]

\[ 0.5/d_1 = \frac{1}{4} \]

\[ d_1 = 2 \]
Network Integration and Diversification

- Diversification (How spread out cross-holdings are)

(a) Low diversification
(b) Medium diversification
(c) High diversification
Essential Ingredients of a Cascades

I. A First Failure: One organization must be susceptible enough to shocks in some assets that it fails.

II. Contagion: Some other organizations are sufficiently sensitive to the first organization’s failure that they also fail.

III. Interconnection: The network of cross-holdings is sufficiently connected so that the failures can continue to propagate.
Essential Ingredients of a Cascades

Increase in *Integration*

-> Less sensitive to its own investments but more sensitive to other organizations’ values

-> First failures can become less likely while contagion is more likely conditional on a failure

I. A First Failure is *less likely* the higher the integration.

II. Contagion is *more likely* the higher the integration.

III. Interconnection is not changed by changes in integration.
Essential Ingredients of a Cascades

Increase in *Diversification*

- Less dependence on a particular neighbor but the network becomes more connected
- Contagions are harder to start but the extent of a contagion broadens

I. A First Failure is not affected by diversification.

II. Contagion is *less likely* the higher the diversification.

III. Interconnection is *more likely* the higher the diversification.
Results

Diversification

When diversification is low any contagion is limited to a small component. As diversification increases, the interconnection increases and more cascades spread. However, after some point, contagion is harder to start and hence less organizations fail in a cascade.
Results

Integration

(b) Five levels of integration and the percentage of organizations failing as a function of expected degree ($\theta = 0.96, n = 100$).

Percentage of simulation with at least one organization failing as a function of diversification

Increasing integration makes shocks more likely to propagate to neighbors and increase contagion (II.). For high values of integration first failures become less likely.
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Example

- There are 100 organizations.
- Cross-holdings from adjacency matrix \( G \) with entries in \( \{0,1\} \)
- A fraction \( c \) of each organization is held by other organizations, spread evenly among the \( d_i = \sum_j G_{ji} \) organizations that hold it.
- For \( i \neq j \)
  \[
  C_{ij} = \frac{c G_{ij}}{d_j},
  \]
  The remaining 1-c of the organization is held by external shareholders.

- Each organization has one asset.
- \( p_i = 1 \)
- \( \bar{v}_i = \theta v_i \) and \( \beta_i = \bar{v}_i \)
Example

The book value of organization $i$:

$$V_i = \sum_{j \neq i} C_{ij} V_j + \sum_k D_{ik} p_k - \beta_i I_{v_i < v_i}$$

Or in matrix notation:

$$V = (I - C)^{-1}(Dp - b(v)),$$

Equity value held by outside investors:

$$v = \hat{C}(I - C)^{-1}(Dp - b(v)) = A(Dp - b(v)).$$
Network Integration and Diversification

- Diversification (How spread out cross-holdings are)