Ben Sauerwine
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Gessel-Viennot Algorithm for the Weakly Interacting Case

I would like to investigate the behavior of the Gessel-Viennot algorithm, particularly in the case "weakly interacting" case where D is relatively large compared to N and P, where I use the problem definition from the last several progress reports.

First, I'd like to both implement the solutions given in "Phase Diagram of a Random Tiling Quasicrystal", (Li, Park, Widom; Journal of Statistical Physics Vol.66) in the context of the approach I've been taking.

A low-order expansion given there is:

$$f_Q(d_H, d_V, \mu_H, \mu_V) \approx \begin{array}{c} -(\mu_H + \log 2)d_H + \dfrac{\pi^2}{24} d_H^3 - (\mu_H + \log 2)d_V + \dfrac{\pi^2}{24} d_V^3 \\ -\dfrac{1}{2}(\delta - a)d_H d_V + b d_H d_V (d_H + d_V) \end{array}$$

where a and b are unknown constants arising from the interaction between domain walls, so I will let them be zero. $\delta = \mu_{02} - \mu_H - \mu_V$.

d_H is the horizontal rhombus density.
d_V is the vertical rhombus density.

The variables mu are related to the chemical potential of these horizontal and vertical rhombi, which is a function of the derivative of entropy with respect to the number of these present. In this case, I'm not clear on how to find these: here are my thoughts:

Suppose I set my array of starting points and ending points, so that each starting point was offset uniformly from its corresponding end point by a rightward offset H and a downward offset V. It seems, then, that the derivative of entropy with respect to either an increase in H or an increase in V is a function of the other parameter, being held constant. For example, if the ending points are directly below the starting points (H = 0), the increase in entropy per downward step (V changing) is zero. If they are one step to the right (H = 1), the increase in entropy per downward step (V changing) is linear, etc. Thus, I'm not clear on the role of these variables.

I'm not clear on how to translate this into my language with path length N, path count P, slope α, and separation D between paths.

Let me examine the behavior of my system from previous weeks as D grows to the weakly interacting case, then consider how to find an expansion to give the behavior in this region. (See GVLongterm9.nb)

Now consider a modification on my old approximation that might work very well for the weakly-interacting case: I recall that my old approximation was (N>>P)

$$\frac{1}{N} Log\left[PathCount(\alpha, N, D, P)\right] \approx$$

$$\frac{P}{N}\log\binom{N}{\alpha N} + \frac{1}{N}\sum_{p=2}^{P}\log\left(1+\sum_{i=2}^{p}(-1)^{i-1}\frac{\binom{N}{\alpha N+(i-1)D}\binom{N}{\alpha N-(i-1)D}}{\binom{N}{\alpha N}\binom{N}{\alpha N}}\right)$$

And it is abundantly clear that for N>>P and large D, this becomes

$$\frac{1}{N} Log\left[PathCount(\alpha, N, D, P)\right] \approx \frac{P}{N}\log\binom{N}{\alpha N} + \frac{P-1}{N}\log\left(1-\frac{\binom{N}{\alpha N+D}\binom{N}{\alpha N-D}}{\binom{N}{\alpha N}\binom{N}{\alpha N}}\right)$$

$$\approx \frac{P}{N}\log\binom{N}{\alpha N} - \frac{P-1}{N}\frac{\binom{N}{\alpha N+D}\binom{N}{\alpha N-D}}{\binom{N}{\alpha N}\binom{N}{\alpha N}}$$

On comparison of this with the actual behavior, one sees that the agreement is very, very good in all regions: N>P, N=P and N<P. This arises from the observation that as D becomes relatively large compared to N, the truncation inside the logarithm becomes more valid. Further, when I recall the argument from which this was constructed originally, I notice that the nearest-neighbor effects only came into play in this first element of the expansion and that the determinant for this effect was totally accurate (For levels 1 and 2 of the determinant, this matched exactly. For later levels, I had to assume that the resultant terms were similar to those in the determinant.)