

**Ben Sauerwine**  
**Intro to Research II**  
**Progress Report – January 24, 2006**

Proof:

$$\Omega(a, b) = \frac{(a+b)!}{a!b!}$$

$$\lim_{N \rightarrow \infty} \frac{1}{N} \ln \Omega(\alpha N, (1-\alpha)N)$$

Use the Stirling approximation  $\ln n! \approx n \ln n - n$ . Now I have

$$\begin{aligned} \ln \Omega(\alpha N, (1-\alpha)N) &= \ln N! - \ln(\alpha N)! - \ln((1-\alpha)N)! \\ &\approx N \ln N - N - \alpha N \ln \alpha N + \alpha N - (1-\alpha)N \ln(1-\alpha)N + (1-\alpha)N \\ &= N \ln N - \alpha N \ln \alpha N - (1-\alpha)N \ln(1-\alpha)N \end{aligned}$$

$$\begin{aligned} \lim_{N \rightarrow \infty} \frac{1}{N} \ln \Omega(\alpha N, (1-\alpha)N) &= \lim_{N \rightarrow \infty} \frac{1}{N} [N \ln N - \alpha N \ln \alpha N - (1-\alpha)N \ln(1-\alpha)N] \\ &= \lim_{N \rightarrow \infty} [\ln N - \alpha \ln \alpha N - (1-\alpha) \ln(1-\alpha)N] \\ &= \lim_{N \rightarrow \infty} \left[ \ln \frac{N}{(\alpha N)^\alpha ((1-\alpha)N)^{1-\alpha}} \right] = \ln \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \end{aligned}$$

Reading Summary:

Questions from what I've read are listed below.

Much of what I've read looks at random tilings and Monte Carlo simulation of tilings in order to calculate functions thereof, e.g. entropy. My research so far has focused on exact calculation of the number of valid tilings. Elser's paper, "Solution of the dimer problem on a hexagonal lattice with boundary" might be useful for this, but I haven't yet been able to find a copy for free download. I did, however, find the Gessel-Viennot method which does give a matrix determinant form for the total number of allowed paths from arbitrary starting and ending points in  $O(n^3)$  time. This is a good algorithm, lacking only the ability to "limit" the streak size which will be necessary on the torus. The algorithm is described here:

[http://www.arxiv.org/PS\\_cache/math/pdf/0302/0302105.pdf](http://www.arxiv.org/PS_cache/math/pdf/0302/0302105.pdf)

and is implemented in a companion Mathematica notebook.

This algorithm is actually the same as the one shown in figure 3 in "Random Tilings of High Symmetry II" Fig. 3

[http://euler.phys.cmu.edu/widom/pubs/PDF/jsp120\\_2005\\_p837.pdf](http://euler.phys.cmu.edu/widom/pubs/PDF/jsp120_2005_p837.pdf). The major issue with adapting this algorithm to the torus, as described in last week's progress report, is

the inability to limit the maximum up-down and left-right streak size, and the paper with its initial derivation might provide clues as to how one might impose this additional property.

**Questions:**

In the coordinate-system language, what is the difference between perp-space and phason space?

When using a projection in this coordinate-system language, what is a “sufficient” number of projections to take to ensure that no overlaid tiles exist?

What is the difference between calculating entropy from the number of possible arrangements versus the number of “possible paths” between them?

Much of the papers I’ve been reading talk about random tiling and Monte Carlo simulation