

Condensed Matter  
February 16 Progress Report

**Consider three questions concerning the entropy of two paths:**

**Holding initial and final separation constant, how does the entropy of two paths approach the long-term value as path length increases?**

**Holding path length constant, how does the entropy of two paths approach the infinite-separation value as separation increases?**

**How does this extend to larger quantities of paths?**

Taking results from an earlier progress report, I have

$$A = \binom{N}{N\alpha_1} \binom{N}{N\alpha_2} - \binom{N-d_x-d_y}{N\alpha_1-d_x} \binom{N+d_x+d_y}{N\alpha_2+d_x}$$

$$\text{let } d_x = -d_y = d \quad \alpha_1 = \alpha_2 = \alpha$$

$$A = \binom{N}{N\alpha} \binom{N}{N\alpha} - \binom{N}{N\alpha-d} \binom{N}{N\alpha+d}$$

Now let me make an assumption: Namely, that

$$\frac{\ln A(N, d)}{N} \rightarrow \frac{2}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \text{ which is supported by the January 31 progress report.}$$

Now let me take

$$\frac{\ln A(N, d)}{N} \rightarrow C, \quad \frac{\partial}{\partial N} \frac{\ln A(N, D)}{N} \rightarrow 0. \text{ By determining this derivative, I should be able}$$

to determine how quickly the parent function dies through integration.

$$\frac{\partial}{\partial N} \frac{\ln A(N, d)}{N} = -\frac{\ln A(N, d)}{N^2} + \frac{1}{NA(N, d)} \frac{\partial A(N, d)}{\partial N}$$

These functions are still extraordinarily difficult to work with symbolically. Certainly, the simple one-path case has been done already in an earlier progress report. Expanding the natural logarithm is not plausible with the d terms intact. Clearly, then, I have to go to the function itself for answers.

I see that in the long term, the function A's behavior is controlled primarily by the similarity of the similarity of  $(N\alpha - d)!(N - (N\alpha - d))!(N\alpha + d)!(N - (N\alpha + d))!$  to  $(N\alpha)^2 (N(1-\alpha))^2$ , which ultimately dictates how I can put these fractions together in such a way that the natural logarithm could be meaningfully dealt with. Unfortunately, even this is a very difficult function to work with. It appears, then, that the only real hope for comparing these symbolically is to attack the function directly. I will go to Mathematica and employ a classic computational technique to try to determine the rate of convergence.

The strategy is described in GVLlongterm4.nb.

It is rather complex. The idea is as follows:

I find entropy per unit length given any slope, starting separation, and path length in the 2-path case. I then divide this entropy by the [path length, initial separation] raised to a real power. I use a simple heuristic to evaluate the “flatness” of the resulting function. The next step is to binary search on the real powers described earlier to make the resulting function most flat. I plot these as a function of alpha, with either initial separation or slope allowed to vary and approach equilibrium in order to evaluate its relationship shown in the y coordinate.