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Nuclear and Particle Physics Homework 9

1) **A real W^+ boson may decay into the following final states: $e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau, u\bar{d}, c\bar{s}$. The couplings to each of these are of the same strength (ignore the CKM mixing matrix)**

a) **Calculate the branching ratio for W^+ to each of these 5 final states:**

There is then one way to produce each of the lepton pairs $e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$, and chromatically speaking three ways to produce each of the quark pairs $u\bar{d}, c\bar{s}$ corresponding to red-antired, green-antigreen, blue-antiblue. Now I have ratios

$$e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau, u\bar{d}, c\bar{s} \rightarrow 1:1:1:3:3.$$

Then, each of the lepton pairs $e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$ gets a branching ratio of $\frac{1}{9}$ and each of the quark pairs $u\bar{d}, c\bar{s}$ get a ratio of $\frac{1}{3}$.

b) **How much would the branching factor for $W^+ \rightarrow e^+\nu_e$ change if the top quark was very light such that $W^+ \rightarrow t\bar{b}$ were allowed equally with the other quark final states?**

In this case, sharing $e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau, u\bar{d}, c\bar{s}, t\bar{b} \rightarrow 1:1:1:3:3:3$ would leave a mere $\frac{1}{12}$ for the branching ratio $W^+ \rightarrow e^+\nu_e$.

2) **The vertex factor for coupling between a fermion and the Z^0 is proportional to $\frac{1}{2}\left(c_V - c_A\gamma^5\right)$. This can be written as:**

$$\frac{1}{2}\left(c_V - c_A\gamma^5\right) = \frac{1}{2}\left(c_V + c_A\right)\frac{1}{2}\left(1-\gamma^5\right) + \frac{1}{2}\left(c_V - c_A\right)\frac{1}{2}\left(1+\gamma^5\right)$$

The first term leads to the decay $Z^0 \rightarrow e_R^+ e_L^-$ and the second to $Z^0 \rightarrow e_L^+ e_R^-$, etc. Thus, the total rate is proportional to the squares of $\frac{1}{2}\left(c_V \pm c_A\right)$; that is, to $c_V{}^2 + c_A{}^2$.

Use this fact to predict the branching ratios of real Z of the $Z \to f\bar{f}$ where $f = \{\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau, u, d, s, c, b\}$; these are all of the possible final states. The answer is thus a list of 11 percentages for each of the 11 possible f.

Recall that $c_V = T_{3,L} - 2Q\sin^2\theta_W \quad c_A = T_{3,L} \quad \sin^2\theta_W \approx 0.23$

Then substituting, I have (for the final percentage, the result is normalized as:

$$\% = \frac{color \cdot (c_V^2 + c_A^2)}{\sum_{all} color \cdot (c_V^2 + c_A^2)}$$

Particle	Q	$T_{3,L}$	c_A	c_V	$c_V^2 + c_A^2$	*Color*	%
ν_e	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	6.8%
ν_μ	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	6.8%
ν_τ	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	6.8%
e	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	≈ -0.04	≈ 0.25	1	3.4%
μ	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	≈ -0.04	≈ 0.25	1	3.4%
τ	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	≈ -0.04	≈ 0.25	1	3.4%
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	≈ 0.19	≈ 0.29	3	12%
d	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	≈ -0.34	≈ 0.37	3	15%
s	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	≈ -0.34	≈ 0.37	3	15%
c	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	≈ 0.19	≈ 0.29	3	12%
b	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	≈ -0.34	≈ 0.37	3	15%

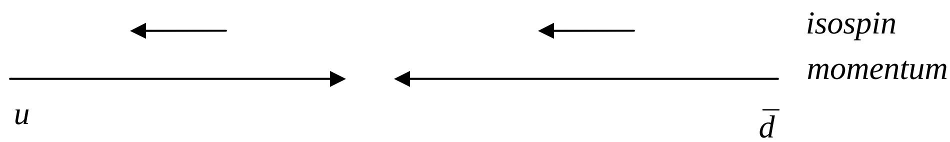
3) **W bosons are produced in $p\bar{p}$ collisions by annihilation of a quark-anti-quark pair. We will treat quarks as massless and explore the effects of chirality on this process. Let's assume the W is always detected via the decay $W \to l\nu_l$.**

In each case below, what is the polarization of the $W^\pm$ produced, and what is the angular distribution of e from the decay, in the lab frame? Be

quantitative when possible and discuss the W^+, W^-, e^+ and e^- separately when and if they differ.

a) **Assume we could make beams of u and $\bar{d}$ quarks and collide them with the lab frame being the center of mass, and the u beam in the $+z$ direction.**

Charge conservation indicates that this interaction will produce only W^+, which will decay only to e^+.



The weak interaction uses a left-handed particle and a right-handed antiparticle. This configuration indicates that the resulting distribution will be anisotropic with a differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi}(1-\cos\theta)^2$$

The positron will tend to travel in the –z direction and the neutrino in the +z direction, so that the angle theta in the cross section gives the offset from this favored direction for each—e.g., the interaction releases a right-handed positron and left-handed neutrino.

b) **Now assume we have $p\bar{p}$, with the lab frame being the $p\bar{p}$ center of mass and the p beam being in the $+z$ direction. Assume that only valence quarks contribute.**

First, I see that I have two possible interactions to produce W bosons: $u\bar{d} \to W^+ \to e^+\nu_e$ and $d\bar{u} \to W^- \to e^-\bar{\nu}_e$, with both of these occurring with equal probability. Now all varieties of particles are possible. Further, since the W bosons are produced in the center of mass frame for the partons, there will be smearing of the distributions longitudinally.

I see that these interactions are ultimately the same as the one from part (a), and so their differential cross sections will be overlaid and anisotropic with the same distribution as part (a), except with the $d\bar{u} \to W^- \to e^-\bar{\nu}_e$ cross-section pointing in the opposite direction. Thus, I see that particles released from this interaction tend to travel in the direction of particles, and antiparticles tend to be released in the direction of the original antiparticle travel.

c) **Same as (b), but discuss the changes when we account for virtual sea quarks producing W bosons.**

If the sea quarks popped from the vacuum at the site of interaction, their properties would be essentially random in terms of how they interact with the quark gas that momentarily exists and so this would change the parity of the resultant W bosons. In fact, then, at higher energies, weaker sea quarks are allowed to contribute and I would gradually see the resultant distribution of leptons released become more and more symmetric, appearing as $\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{8\pi}\left[(1-\cos\theta)^2 + (1+\cos\theta)^2\right]$.