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Nuclear and Particle Physics Homework 6

1) **The Yukawa potential is of the form** $V \propto \frac{e^{-mr}}{mr}$**.**

a) **Show that** $\vec{\nabla}^2 V = m^2 V$**, i.e., show that it satisfies the time-independent Klein-Gordon equation.**

$$\vec{\nabla}^2 V(r) = \frac{1}{r}\frac{\partial^2}{\partial r^2}\left[rV(r)\right]$$

$$\vec{\nabla}^2 \frac{e^{-mr}}{mr} = \frac{1}{r}\frac{\partial^2}{\partial r^2}\left[\frac{e^{-mr}}{m}\right] = \frac{1}{r}\left[m^2 \frac{e^{-mr}}{m}\right] = m^2\left[\frac{e^{-mr}}{mr}\right]$$

just as expected.

b) **Calculate the Fourier transform of V in 3-D. This means** $\int e^{i\vec{q}\cdot\vec{r}} V(r) d^3 r$**.**

Since this is a spherically symmetric distribution, I may make the simplification:

$$\int e^{i\vec{q}\cdot\vec{r}} V(r) d^3 r = 4\pi \int_0^\infty r^2 dr \frac{\sin(qr)}{qr}\frac{e^{-mr}}{mr} = \frac{4\pi}{mq}\int_0^\infty dr \sin(qr) e^{-mr} = \frac{4\pi}{mq}\left[\frac{q}{m^2+q^2}\right]$$

$$= \frac{4\pi}{m}\left[\frac{1}{m^2+q^2}\right]$$

I have used Mathematica to perform the integral.

2) **Consider the interaction** $\bar{c}\gamma_\mu s W^{+,\mu}$**. Write down all eight vertices implied by this interaction. Please write them in the form of a transition like** $s \to cW^-$**; this is one of the eight.**

Acting like a chemist, I merely indicate each possible array creations and destructions that would result in this net effect.

$$s \to cW^- \quad \bar{c} \to \bar{s}W^- \quad W^+ \to \bar{s}c \quad s\bar{c}W^+ \to 0$$

$$sW^+ \to c \quad s\bar{c} \to W^- \quad \bar{c}W^+ \to \bar{s} \quad 0 \to \bar{s}cW^-$$

3) **Convert the mean lifetime (886 s) of the neutron to its decay width** Γ **in GeV. Also calculate the combination** ${G_F}^2\Delta^5$ **where** $G_F = 1.166\times 10^{-5}\, GeV^{-2}$ **and** $\Delta = m_n - (m_p + m_e)$ **is the Q-value, or energy release, of neutron beta decay. Are the two numbers somewhat similar?**

$\Gamma \tau = \hbar$

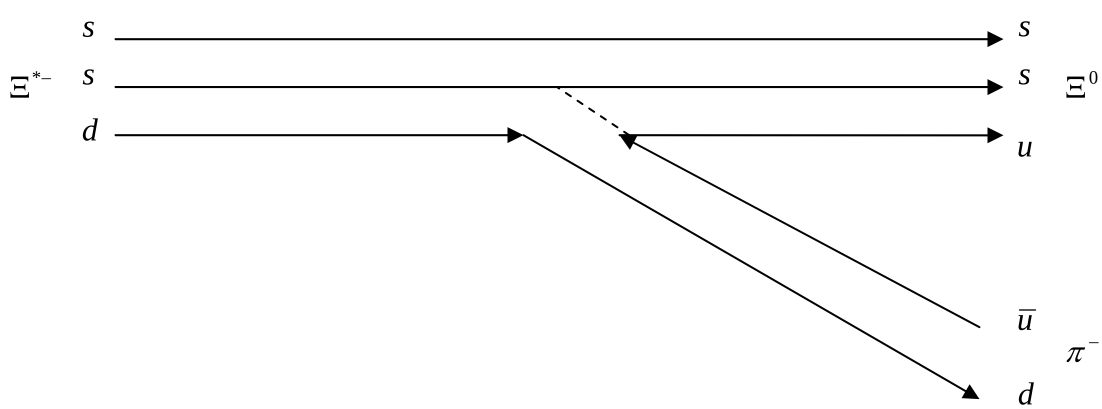
$$\Gamma = \frac{\hbar}{886s} = \frac{6.58 \cdot 10^{-25} GeV \cdot s}{886s} = 7.42 \cdot 10^{-28} GeV$$

$$G_F{}^2 \Delta^5 = \left(1.166 \times 10^{-5}\right)^2 \cdot \left(0.939565 - \left(0.93827231 + 0.000511\right)\right)^4 GeV$$
$$= 4.0 \cdot 10^{-26} GeV$$

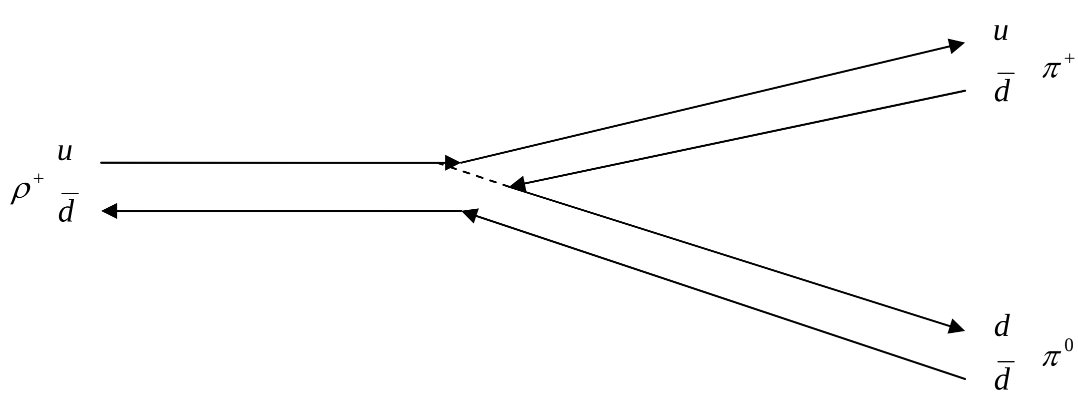
These numbers disagree by about one order of magnitude; not too bad.

4) **Draw the simplest decay diagrams for the following processes, showing the quarks inside the hadrons (recall that we omit binding gluons).**

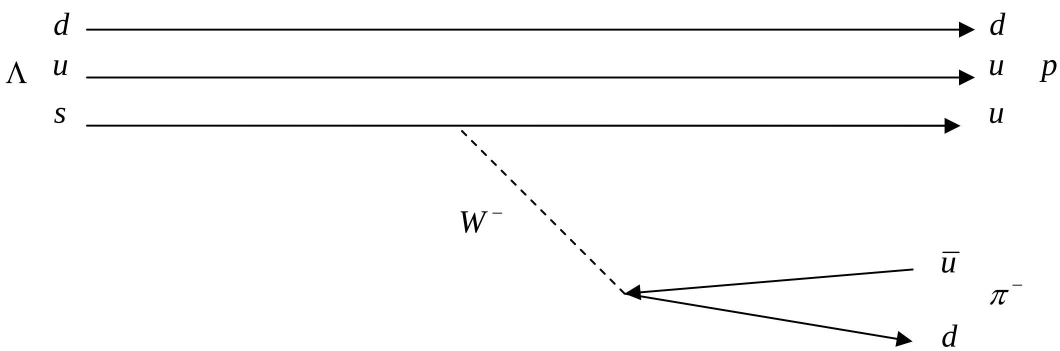
$\Xi^{*-} \rightarrow \Xi^0 \pi^-$



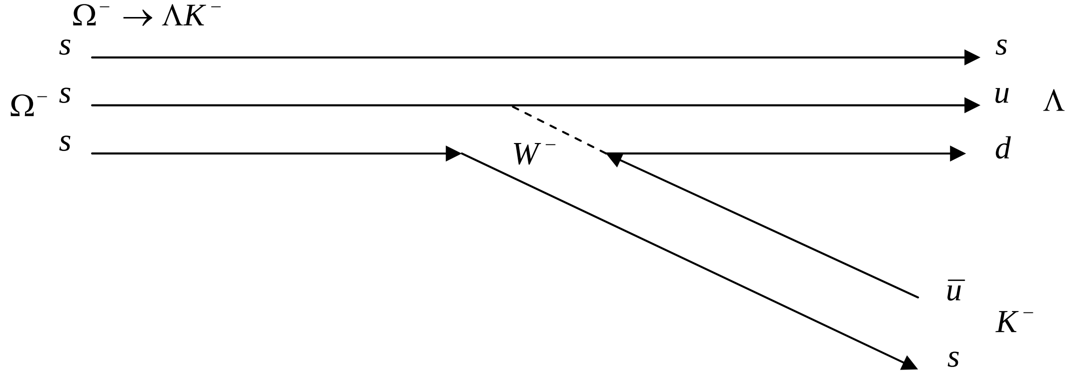
$\rho^+ \rightarrow \pi^+ \pi^0$



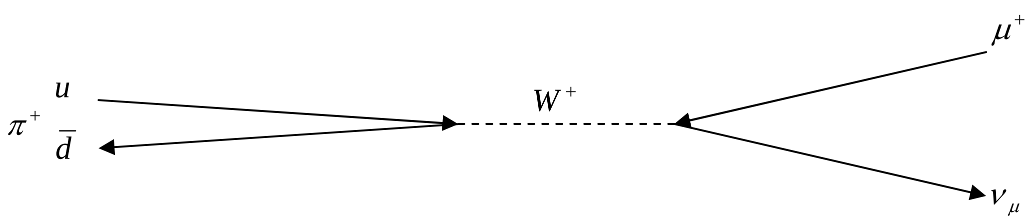
$\Lambda \rightarrow p \pi^-$



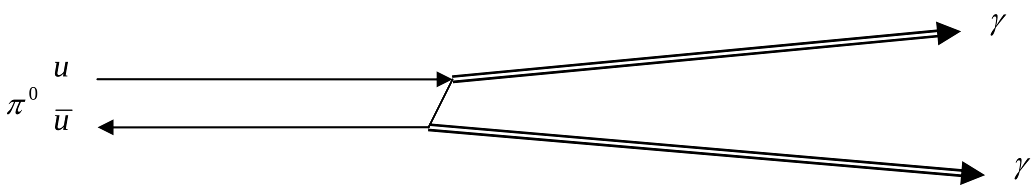
$\Omega^- \rightarrow \Lambda K^-$



$\pi^+ \rightarrow \mu^+ \nu_\mu$

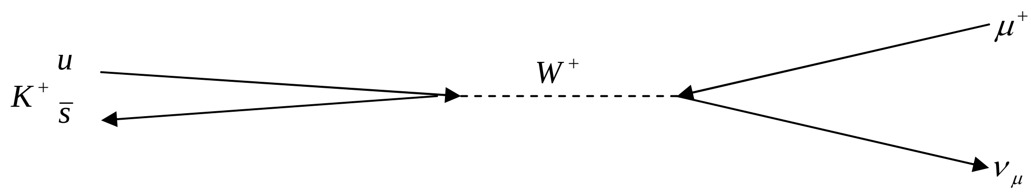


$\pi^0 \rightarrow \gamma\gamma$

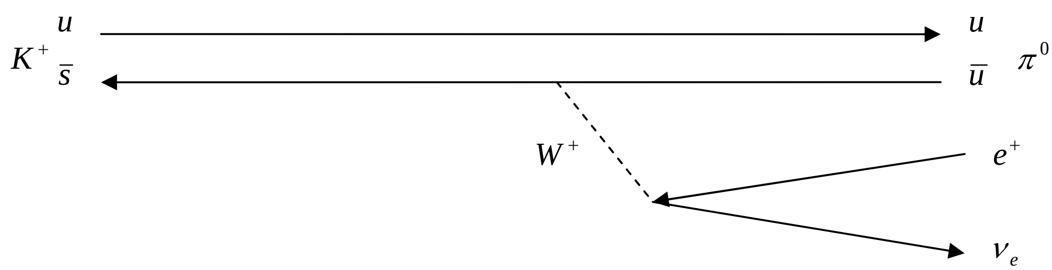


Two photons must be emitted due to the conservation of momentum during annihilation in the quarks' rest frame.

$K^+ \rightarrow \mu^+ \nu_\mu$



$K^+ \to \pi^0 e^+ \nu_e$



5) **Imagine we use a high-energy (800 GeV) proton beam to produce a secondary beam of some new particle, like a K meson, in a fixed target collision with a metal target. We want as many kaons as possible to exit the target; since they are highly Lorentz boosted, the kaons must travel through the remaining thickness of the target after being produced at some internal point.**

a) **Explain in words what is bad about very thin or very thick targets.**

At this energy, protons will likely not interact much or at all with a very thin target, meaning I will have very little kaon production. In a very thick target, the kaons are likely to interact with the target and degrade their energy to a point where their lifetime will be shortened so that they decay before they can be recovered or convert to other particles, or will possibly simply be absorbed.

b) **Calculate how long, in interaction lengths, such a target should be in order to produce the most kaons. For simplicity, assume that the nuclear interaction lengths of protons and kaons are about the same.**

I need to first understand each possible interaction route. If no route can result in more production of kaons (e.g., a result particle to more kaons reaction) than the initial proton-nucleus interaction, the answer will be 1 interaction length.

To justify this, I first notice that there does not appear to be a simple way for kaons to propagate more kaons. Looking up some papers (Phenomenological Analysis of K-Meson Production in Proton-Nucleus Collisions, Buscher et al), I see further that the cross-sections the interactions $pp \to ppK^+K^-$ (Buscher) and $K^- + p \to \Omega^- + K^+ + K^0$

(Perkins p.113) are extraordinarily small, indicating that these are very unlikely to contribute substantially compared to the processes $pA \rightarrow KX$. Thus, it seems probable that one interaction length should suffice if the interaction length for kaons is the same as that for protons. If the interaction length were not the same, then I would expect the optimum length would be the maximum number of interactions until the proton can produce no more kaons.

Mathematically, one can write the rate R as:

$$R = \int_0^L e^{-\frac{x}{L_0}} (k\sigma n dx) e^{-\frac{L-x}{L_0}}$$

Or, if you will, the rate is the integral of the chance that the proton has reacted by this position the chance that the result particle will not react again and be destroyed. Setting $\frac{dR}{dL} = 0$ gives the maximum at $L = L_0$, one interaction length, as expected.