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Nuclear and Particle Physics Homework 1

1a) Calculate both the gravitational and electrical force between two protons separated (center to center) by 2 Fermi. What is the ratio of these forces?

$$F_G = G\frac{m_p m_e}{d^2} = \left(6.672\times10^{-11}Nm^2kg^{-2}\right)\frac{\left(1.6726\times10^{-27}kg\right)^2}{\left(2\times10^{-15}m\right)^2} = 4.67\times10^{-35}N$$

$$F_E = -\frac{1}{4\pi\varepsilon_0}\frac{q_p q_e}{d^2} = \left(8.9876\times10^9 Nm^2C^{-2}\right)\frac{\left(1.602\times10^{-19}C\right)^2}{\left(2\times10^{-15}m\right)^2} = 57.665N$$

$$\frac{F_G}{F_E} = 8.1\times10^{-37}$$

1b) Calculate the fractional decrease in the (rest) mass of a free proton and electron, compared to a ground-state hydrogen atom.

$$m_p = 938.3MeVc^{-2}$$
$$m_e = 0.511MeVc^{-2}$$

$$V_H = -13.6eV$$
$$E_{frac} = \frac{m_pc^2 + m_ec^2 + V_H}{m_pc^2 + m_ec^2} = 0.999\quad 999\quad 986$$
$$1.4\times10^{-6}\%\quad decrease\quad in\quad rest\quad mass$$

1c) How different, in terms of order of magnitude, is the answer to part b compared to the fractional decrease due to a typical chemical reaction?

As a benchmark chemical reaction, consider the combination of hydrogen and oxygen to make water.

$$\Delta E = -57.80\frac{kcal}{mol} = -57.80\frac{kcal}{mol}\left(\frac{4185J}{kcal}\right)\left(\frac{1mol}{6.022\times10^{23}reactions}\right)\left(\frac{1.6\cdot10^{19}eV}{1J}\right) = 0.001\frac{eV}{reaction}$$

$$\frac{9m_pc^2 + 9m_ec^2 + 8m_nc^2 - 0.001eV}{9m_pc^2 + 9m_ec^2 + 8m_nc^2} \approx \frac{0.001}{26\cdot1000\cdot10^6}\%\quad decrease \approx 0.5\times10^{-11}\%$$

So I see that the percent decrease in rest mass in Hydrogen production is roughly 10^5 times that of my benchmark chemical reaction of water production.

1d) What happens to mass in an endothermic chemical reaction?

In an endothermic chemical reaction, the mass of the precipitants is slightly higher than that of the reactants since some energy is absorbed into the reaction.

2) **Calculate the typical distance between atoms of lead in sold form. Calculate in terms of basic quantities like Avogadro's number, atomic number, density, etc. (i.e., do NOT use any lattice data. You can assume a simple cubic lattice for simplicity since effects of different packings of the atoms are small.**

$$density: 11.34 \cdot g \cdot cm^{-3}$$
$$atomic \quad mass: 207.2 \cdot g \cdot mol^{-1}$$
$$spacing(cm) \approx \left(\frac{atoms}{cm^3}\right)^{\frac{-1}{3}} = \left(\frac{density}{atomic \quad mass} \cdot Avogadro\right)^{\frac{-1}{3}} = \left(3.2958 \cdot 10^{22} \frac{atoms}{cm^3}\right)^{\frac{-1}{3}} = 3.1 \cdot 10^{-8} cm = 3.1 \dot{A}$$

3) **Consider a non-relativistic, fully elastic collision between a 'projectile' of mass m and a stationary 'target' of mass M. The projectile scatters by an angle θ and the target by an angle ϕ relative to the initial projectile direction.**

a) **Find an expression for the fraction of the projectile energy transferred to the target as a function of ϕ and $r \equiv \frac{m}{M}$ only.**

Since momentum is conserved dimensionally, certainly:

$$mv_{f,m} \sin\theta = Mv_M \sin\phi$$
$$mv_{f,m} \cos\theta + Mv_M \cos\phi = mv_{i,m}$$

also, in this totally elastic collision,

$$E_{f,m} + E_M = E_{i,m}$$
$$with$$
$$E_{f,m} = \frac{1}{2} mv_{f,m}{}^2 \quad E_M = \frac{1}{2} Mv_M{}^2 \quad E_{i,m} = \frac{1}{2} mv_{i,m}{}^2$$
$$\sqrt{\frac{2E_{f,m}}{m}} = v_{f,m} \quad \sqrt{\frac{2E_M}{M}} = v_M \quad \sqrt{\frac{2E_{i,m}}{m}} = v_{i,m}$$

Now writing:

$$\frac{E_M}{E_{i,m}} = \frac{M}{m}\frac{v_M^2}{v_{i,m}^2} = \frac{1}{r}\frac{v_M^2}{v_{i,m}^2}$$

I take

$$r^2 v_{f,m}{}^2 \sin^2\theta = v_M{}^2 \sin^2\phi$$
$$r^2 v_{f,m}{}^2 \cos^2\theta = r^2 v_{i,m}{}^2 - 2r v_{i,m} v_M \cos\phi + v_M{}^2 \cos^2\phi$$
$$adding,$$
$$r^2 v_{f,m}{}^2 = v_M{}^2 + r^2 v_{i,m}{}^2 - 2r v_{i,m} v_M \cos\phi$$
$$from \quad energy,$$
$$r v_{f,m}{}^2 + v_M{}^2 = r v_{i,m}{}^2$$

Now combining the last two equations,

$$\frac{1}{r}\left[v_M{}^2 + r^2 v_{i,m}{}^2 - 2r v_{i,m} v_M \cos\phi\right] + v_M{}^2 = r v_{i,m}{}^2$$
$$\frac{1}{r} + r\frac{v_{i,m}{}^2}{v_M{}^2} - 2\frac{v_{i,m}}{v_M}\cos\phi + 1 = \frac{r v_{i,m}{}^2}{v_M{}^2}$$
$$Note \quad E_{ratio}^{-1} = r\frac{v_{i,m}{}^2}{v_M{}^2}$$
$$-2\sqrt{\frac{E_{ratio}^{-1}}{r}}\cos\phi + 1 + \frac{1}{r} = 0$$
$$E_{ratio} = \frac{4r\cos^2\phi}{(r+1)^2}$$

3b) What is the relation between r, ϕ and the maximum possible energy transfer, and how large can this transfer be?

The larger E_{ratio} is, the more energy is transferred, or when the cosine term is largest being inverse-squared is maximal, e.g. $\phi \in \{0, \pi\}$.

At maximum, then, $E_{ratio,\max} = \frac{4r}{(r+1)^2}$.

3c) Discuss the allowed range of θ for $r << 1, r \approx 1, r >> 1$.

Take:

$$\frac{v_{f,m}}{v_{i,m}}\sin\theta = \frac{1}{r}\frac{v_M}{v_{i,m}}\sin\phi$$

$$\frac{v_{f,m}}{v_{i,m}}\cos\theta + \frac{1}{r}\frac{v_M}{v_{i,m}}\cos\phi = 1$$

$$\frac{v_M}{v_{i,m}} = \sqrt{rE_{ratio}} \quad \frac{v_{f,m}}{v_{i,m}} = \sqrt{1-E_{ratio}}$$

$$(1-E_{ratio})\sin^2\theta = \left(\frac{E_{ratio}}{r}\right)\sin^2\phi \quad (1-E_{ratio})\cos^2\theta - 2\sqrt{1-E_{ratio}}\cos\theta + 1 = \left(\frac{E_{ratio}}{r}\right)\cos^2\phi$$

$$(1-E_{ratio}) - 2\sqrt{1-E_{ratio}}\cos\theta + 1 = \left(\frac{E_{ratio}}{r}\right)$$

$$\cos\theta = \frac{2r+(r-1)E_{ratio}}{2r\sqrt{1-E_{ratio}}} \to \cos\theta_{\max} = \frac{2r+(r-1)\frac{4r}{(r+1)^2}}{2r\sqrt{1-\frac{4r}{(r+1)^2}}} = \frac{1+\frac{2(r-1)}{(r+1)^2}}{\sqrt{1-\frac{4r}{(r+1)^2}}}$$

For r<<1, I have:

$$\cos\theta_{\max} = \frac{1-2}{1} = -1$$

so that any angle of scattering is possible, including backscatter straight at the source.

If $r >> 1$, then

$$\cos\theta_{\max} = \frac{1+0}{1} = 1$$

so that the only possible scattering angle is in the same direction as the incident particle, or very close to it.

If $r \approx 1$, then $E_{ratio,\max} = 1$ (corresponding to a total transfer of momentum to the target), but anything less than this is allowed:

$$\cos\theta = \frac{2r+(r-1)E_{ratio}}{2r\sqrt{1-E_{ratio}}} \to \frac{2}{2\sqrt{1-E_{ratio}}}$$

$$E_{ratio} : (0,1)$$

$$\theta \in \left[0, \frac{\pi}{2}\right)$$

I have excluded anything else, since at the ninety-degree scatter, this particle doesn't move at all!

3d) Consider a neutron with a 1 MeV kinetic energy. How much is this in units of kT? About how many p-n collisions are needed to decrease the neutron energy to near kT?

$k_B = 8.617 \times 10^{-5} \frac{eV}{K}$.
$T_{Background} = 300K$ (in the lab)

$1MeV = 3.86832 \times 10^7 kT$

The particle will lose roughly half (the masses are practically equal) of its kinetic energy in each non-relativistic collision (this result will differ in the relativistic case), and I have assumed that the targets are practically stationary.

$Log_2\left(4.2 \cdot 10^9\right) = 25.2052$, for about 25 collisions.

4) **Look up and tabulate the binding energies of all A = 8 nuclei. You should derive this from mass-excess data at the "Nuclear Wallet Cards" link on the course web page. Be sure you understand the relationship between what they list and the binding energy! Use the results to explain why there are no stable A = 8 nuclei in nature.**

http://www.nndc.bnl.gov/wallet/wallet05.pdf

Sample Calculation:

Note that $B = Z \cdot M\left({}^1H_1\right) + \left(A - Z\right)M_n - M\left(A, Z\right)$ and $Mass \quad Excess = M - A$.

Further, $\Delta n^0 \rightarrow 8.071 MeV \quad \Delta {}^1H_1 \rightarrow 7.289 MeV$

Element (A = 8)	Mass Excess (MeV)	Binding Energy (MeV)
Helium	31.598	31.406
Lithium	20.947	41.275
Beryllium	4.942	56.498
Boron	22.921	37.737
Carbon	35.09	24.786

$$
\begin{aligned}
B &= Z \cdot M\left({}^{1}H_{1}\right) + (A - Z)M_{n} - M(A, Z) \\
&= Z \cdot M\left({}^{1}H_{1} - 1\right) + (A - Z)(M_{n} - 1) - M(A, Z) + Z + (A - Z) \\
&= Z \cdot M\left({}^{1}H_{1} - 1\right) + (A - Z)(M_{n} - 1) - \left(M(A, Z) - A\right) \\
&= Z \cdot M\Delta^{1}H_{1} + (A - Z)\Delta M_{n} - \Delta M
\end{aligned}
$$

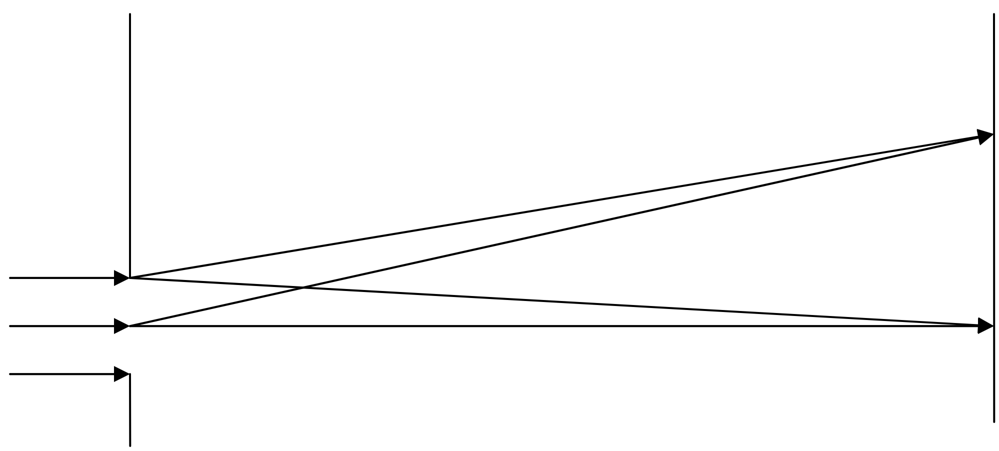
These are unstable since the binding energy per nucleon is so low compared to other available options. For example, consider the binding energy for an alpha particle:

$\Delta\alpha = 28.30MeV$, for $7.075MeV / nucleon$.

In order for decay to alpha particles to be unattractive, then, the binding energy would have to be at least 56.6 MeV, and none of these qualify. It is important to note, however, that an entirely "desirable" (more bound) decay path must exist to any state it decays to, and so a stable state might well exist if such a path didn't exist.

5) **Calculate the separation between the main image and the first-order diffraction peak for a 1.0 angstrom X-ray imaged 1.0 m away from a grating with 1.0 micron spacing. Do you think conventional diffraction gratings work for X-rays? What is used instead?**

In a diffraction grating, I have:



A diffraction peak occurs where $d_{edge} - d_{center} = n\lambda$. Then, assuming that by "main image" the question means the center of the diffraction pattern, this distance will be given by: $d_{edge} - d_{center} = \pm 1\lambda = \pm\lambda$

$$
\sqrt{\left(\frac{10^{-6}}{2}m - d_{peak}\right)^{2} + (1.0m)^{2}} - \sqrt{d_{peak}^{2} + (1.0m)^{2}} = \pm 10^{-10}m
$$

Solving numerically with Mathematica, I have:

$d_{peak} \approx 0.2mm$

This is identical to the result (in the $\sin\theta \approx \theta \quad \theta \approx 0$ approximation) given by

$$a\sin\theta = m\lambda$$
$$a = \frac{10^{-6}}{2} meters$$
$$m = 1$$
$$\lambda = 10^{-10} m$$
$$D = 1m$$
$$D\frac{\lambda}{a} = 0.2mm$$

This is very small. The probably works a lot better with a finer-spaced grating, closer to the x-ray wavelength so as to space out the peaks, perhaps a crystal or thin sheet of foil.