Ben Sauerwine
Qualifying Exam Study Sheet

Mechanics

(In this section, K indicates the kinetic energy of the system in terms of the variables and V indicates the potential.)

Lagrangian: $L = K - V$

Hamiltonian: $H = K + V = \sum_i q_i p_i - L$

Above, the "conjugate momentum" p of a coordinate is $p = \left(\frac{\partial L}{\partial \dot{q}}\right)$. Thus note that the equations of motion take a form like $\frac{d}{dt} p = \left(\frac{\partial L}{\partial q}\right)$ which is essentially, "The time derivative of the momentum is the force."

Electricity and Magnetism

Radiation pressure: $P = \frac{\Phi}{c}$.

Maxwell's Equations:

Gauss: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

Gauss's Law (magnetism): $\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \cdot \vec{H} = 0$

Faraday's Law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampere's Law: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

The integral forms are related to these such that the cross products become loop integrals and the dot products become surface integrals. Note that in these may vary in matter. For example, in a linear dielectric:

$\vec{H} = \frac{\mu_0}{\mu} \vec{B} \quad \vec{D} = \frac{\varepsilon}{\varepsilon_0} \vec{E}$

or in a conductor, $\vec{D} = 0$.

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \vec{\nabla}^2 \vec{F}$$

Mathematical Methods

Be able to solve a general n-th order differential equation using the matrix method, evaluate a contour integral, and find eigenvalues and eigenvectors.

Quantum Mechanics

Time-Dependent Schrodinger Equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\vec{r}) \Psi$$

Time-Independent Schrodinger Equation:

$$E\Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\vec{r}) \Psi$$

Selection Rules:

Electric Dipole:

$$\Delta l = \pm 1 \quad \Delta m \in \pm 1, 0$$

Clebsch-Gordan:

Consider $\langle L \quad m_L \quad S \quad m_s | F \quad M \rangle$:

$$F \in |L + S|, |L + S - 1|, \ldots, |L - S|$$
$$M = m_L + m_S$$

Heisenberg Uncertainty Principles:

$$\Delta x \Delta t \geq \frac{\hbar}{2} \quad \Delta E \Delta t \geq \frac{\hbar}{2} \quad \Delta J_i \Delta J_j \geq \frac{\hbar}{2} |J_k| \quad (angular \quad momentum)$$

Perturbation Theory:

The perturbations on the Hydrogen atom:

Fine Structure:

Caused by: Relativistic motion of the electron and electron Spin-Orbit Coupling

$W_{rel} \propto P^4$

$W_{SO} \propto \frac{1}{R^3}\left(\vec{L} \cdot \vec{S}\right)$

$W_D \propto \delta^3(\vec{r})$

Hyperfine Structure

Caused by: Nuclear spin interacting with electron's magnetic moment.

$$W_{hf} \propto \left(\vec{L} \cdot \vec{I}\right), \frac{1}{R^3}\left[3(\vec{S} \cdot \hat{n})(\vec{I} \cdot \hat{n}) - (\vec{S} \cdot \vec{I})\right], (\vec{S} \cdot \vec{I})\delta(\vec{r})$$

First-Order:

$|\psi_i\rangle$: deg*enerate eigenstates of Hamiltonian*

W *perturbation*

eigenvalues / eigenstates of $\langle\psi_i|W|\psi_j\rangle$

e.g., the energy shifts are given by the eigenvalues of this matrix and the eigenvectors give the perturbed states.

Second-Order:

Eigenvalues: $\Delta E_i^{(2)} = \sum_{i \neq j} \frac{\left|\langle\psi_i|W|\psi_j\rangle\right|^2}{E_i - E_j}$

Time-Development:

States that are not eigenstates fluctuate over time as $e^{-i\frac{E}{\hbar}t}$.

Statistical Mechanics

Helmholtz Free Energy:

$$dF = -SdT - pdV + \mu dN$$

The above equation gives the formulae for entropy, volume and chemical potential.

$$F = -kT\ln Z = U - TS$$

Stirling Approximation:

$$\ln N! \approx N\ln N - N$$

Classical Partition Function:

$$Z_c = \frac{1}{h^{\dim\cdot N} N!} \int\int e^{-\beta H} d^{3N} p d^{3N} q$$

(N particles in dim dimensions)

(from this arises the quantum concentration and thermal wavelength, see 2006FebQualSTAT)

$$n_Q = \frac{1}{\lambda_T^{\dim}} = \left(\frac{mk_B T}{2\pi\hbar^2}\right)^{\frac{\dim}{2}}$$ (in "dim" dimensions)

At equilibrium of classical systems, $\mu_I = \mu_{II} = ...$

Black Body:

$$W = \sigma T^4 \quad \sigma = 5.67 \times 10^{-8} \frac{J}{K^4 m^2 s}$$

Heat Capacities:

$$C_x = \left(\frac{\partial U}{\partial T}\right)_X$$

Canonical Ensemble:

$$Z = \sum_i e^{-\beta\varepsilon_i}$$

Note that in the Canonical Ensemble, $S = -k\sum_i \Pr_i \ln \Pr_i$, where $\Pr_i = \frac{e^{-\beta\varepsilon_i}}{Z}$.

Grand Canonical Ensemble:

$$Z = \sum_i e^{-\beta(\varepsilon_i - \mu)}$$

Fermions:

For a particular orbital, $Z = \frac{(2S+1)}{e^{\beta(\varepsilon-\mu)}+1}$

When all Fermions are in the lowest available state, this is the energy of the highest-energy particle.

Bosons:

For a particular orbital, $Z = \frac{(2S+1)}{e^{\beta(\varepsilon-\mu)}-1}$

Bose-Einstein Condensation: This is the temperature at which, if the temperature were any lower, some particles would be forced into the ground state. This is not the same as the expectation value of particles in the ground state being at least one—the absolute minimum number of particles in the ground state must be one.

General Physics

Ground state energy of the Hydrogen atom (derived from the Bohr model of a de Broglie electron orbiting a central Coulomb potential):

$= -13.6eV$