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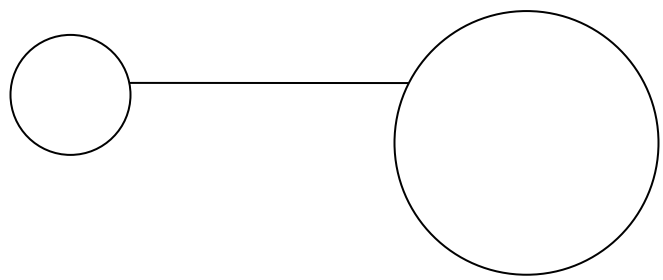
Consider two metal spheres connected by a negligibly small metal wire, used to keep them at the same potential. The wire's surface is so small that almost no charge is present there. The spheres are present in free space far from all other charges and the system is in equilibrium. The smaller sphere has radius r and the larger sphere has radius R, and the entire system contains total charge Q. The wire is long enough that "image charge" from one sphere onto the other is negligible. What is the ratio of charge densities on the spheres?



This system actually demonstrates mathematically how charge "builds up" on sharp regions with a very small radius of curvature. Let me calculate the amount of charge on each sphere, neglecting image charge and the charge on the thin wire.

First, I know that this conductor constitutes an equipotential body. Thus, the potential at the surface of the small one equals the potential at the surface of the big one:

$$\frac{q_{small}}{4\pi\varepsilon_0 r} = \frac{q_{big}}{4\pi\varepsilon_0 R}$$
$$q_{small} + q_{big} = Q$$

Now I can write concerning the respective charges:

$$\frac{q_{small}}{q_{big}} = \frac{r}{R}$$

However, I can write concerning their areas:

$$\frac{A_{small}}{A_{big}} = \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2}$$

Concerning the charge densities, then:

$$\frac{\sigma_{small}}{\sigma_{big}} = \frac{\frac{q_{small}}{A_{small}}}{\frac{q_{big}}{A_{big}}} = \frac{R}{r} > 1$$

So I certainly see that charge has 'concentrated' on the smaller sphere with the smaller radius of curvature. In particular, I can from this model infer that for a solid conductor with no internal cavities in equilibrium in free space I can expect that charge densities on separate regions of a surface will relate *roughly* by , e.g. $\frac{\sigma_{small}}{\sigma_{big}} = \frac{R}{r}$.

To get some physical intuition as to why this should be the case, consider this:

For a conductor in the static case, all electric fields at the surface must point perpendicular to the surface, along the surface normal. In regions where the radius of curvature is small, the surface normals and so the directions of the electric fields are changing a lot in a small distance. However, to change the direction of electric fields more dramatically over a smaller distance it requires more charge or closer charge, corresponding to a higher local charge density.

Suggested problems:

Chabay and Sherwood, Matter and Interactions II, Second Edition, 16.P.45
Griffiths, Introduction to Electrodynamics, Third Edition, Problem 2.52