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Problem Source: **<u>CMU August 2006 Qualifying Exam</u>** (General Physics)

(a) (i) The intensity of electromagnetic radiation from the Sun arriving at the Earth is $1.4\frac{kW}{m^2}$**. Compute the radiation pressure on a "solar sail" placed at a distance from the Sun equal to the Earth's orbit radius.**

$$W = \frac{N\cdot m}{s} \therefore \frac{W}{c} = N$$

Proposing that this solar sail is black and absorbs all incident photons,

$$1.4\frac{kW}{m^2} \to \frac{1400}{c}\frac{N}{m^2} = 4.6\cdot 10^{-6}\frac{N}{m^2}$$

A shiny sail reflecting all incident photons would feel double this pressure.

(ii) The cloud of particles known as "solar wind" which is continually ejected from the Sun has an average density of 7 protons per cubic centimeter and moves at an average speed of 400 km/s. Compute the pressure on the solar sail due to that solar wind.

Relativistic momentum is given by $p = \gamma m v$. Considering a relatively stationary solar sail, and proposing that these protons don't shoot straight through the sail but instead hit it and remain stationary, I need the "volume" of wind passing through this square-meter times the number of protons in that volume times the momentum of each proton:

$$\frac{\frac{\Delta p}{\Delta t}}{m^2} = proton\quad velocity\cdot\frac{protons}{m^3}\cdot\gamma(proton\quad velocity, proton\quad mass)$$

$$= 400{,}000\frac{m}{s}\cdot 7\frac{protons}{cm^3}\cdot\frac{100^3 cm^3}{1m^3}\cdot\frac{400{,}000\frac{m}{s}\cdot 1.672\cdot 10^{-27}kg}{\sqrt{1-\left(\frac{400{,}000}{3\cdot 10^8}\right)^2}} = 1.87\cdot 10^{-9}\frac{N}{m^2}$$

(b) The famous supernova SN1987A occurred at a distance of about $5\cdot 10^{12}$ **light-seconds from Earth. For the first time in history this supernova was also observed via the burst of neutrinos it emitted. Within that burst, one neutrino of** $20MeV$ **energy and another neutrino of** $10MeV$ **energy arrived on Earth within ten seconds of each other. Assuming that both were emitted**

simultaneously at the same location, derive an approximate upper limit on the neutrino mass in units of $\frac{eV}{c^2}$.

It is easy to see that:

$$E = mc^2\gamma$$

$$10MeV = mc^2\gamma_{10} = \frac{mc^2}{\sqrt{1-\frac{v_{10}^2}{c^2}}}$$

$$20MeV = mc^2\gamma_{20} = \frac{mc^2}{\sqrt{1-\frac{v_{20}^2}{c^2}}}$$

$$\left(5\cdot 10^{12} c\right)\left(\frac{1}{v_{10}} - \frac{1}{v_{20}}\right) = 10 \quad \sec$$

Then,

$$\frac{v_{10}^2}{c^2} = 1 - \left[\frac{mc^2}{10MeV}\right]^2$$

$$\frac{v_{20}^2}{c^2} = 1 - \left[\frac{mc^2}{20MeV}\right]^2$$

$$\frac{c}{v_{10}} + \frac{c}{v_{20}} \approx 2$$

$$\left(\frac{c}{v_{10}} - \frac{c}{v_{20}}\right)\left(\frac{c}{v_{10}} + \frac{c}{v_{20}}\right) = \frac{2\cdot 10}{5\cdot 10^{12}}$$

$$\frac{c^2}{{v_{10}}^2} - \frac{c^2}{{v_{20}}^2} = \frac{1}{1-\left[\frac{mc^2}{10MeV}\right]^2} - \frac{1}{1-\left[\frac{mc^2}{20MeV}\right]^2} = \frac{2\cdot 10}{5\cdot 10^{12}}$$

$$1-\left[\frac{mc^2}{20MeV}\right]^2 - \left(1-\left[\frac{mc^2}{10MeV}\right]^2\right) = \frac{2\cdot 10}{5\cdot 10^{12}}\left(1-\left[\frac{mc^2}{10MeV}\right]^2\right)\left(1-\left[\frac{mc^2}{20MeV}\right]^2\right)$$

$$\left[\frac{mc^2}{10MeV}\right]^2 - \left[\frac{mc^2}{20MeV}\right]^2 = \frac{2\cdot 10}{5\cdot 10^{12}}\left(1-\left[\frac{mc^2}{10MeV}\right]^2\right)\left(1-\left[\frac{mc^2}{20MeV}\right]^2\right)$$

Now define:

$$a = \left[\frac{mc^2}{10MeV} \right]^2$$

Then,

$$a - \frac{1}{4}a = \frac{2 \cdot 10}{5 \cdot 10^{12}}(1-a)\left(1 - \frac{1}{4}a\right)$$

$$0 = \left(\frac{2 \cdot 10}{5 \cdot 10^{12}} + \left(-\frac{5}{4}\left(\frac{2 \cdot 10}{5 \cdot 10^{12}} \right) - \frac{3}{4} \right) a + \frac{1}{4}\left(\frac{2 \cdot 10}{5 \cdot 10^{12}} \right) a^2 \right)$$

$$a = \{7.5 \cdot 10^{11}, 5.3 \cdot 10^{-12}\}$$

$$mc^2 = \sqrt{a} \cdot 10MeV = 23.09eV$$

$$m \leq 23.09 \frac{eV}{c^2}$$

(c) One of the mechanisms of energy production in stars is the "proton-proton cycle", which eventually fuses four protons into an alpha particle:

-proton-proton fusion, $^1H + ^1H \rightarrow ^2H + e^+ + \nu_e + \gamma$

-deuterium-proton fusion, $^2H + ^1H \rightarrow ^3He + \gamma$

-helium-3 fusion, $^3He + ^3He \rightarrow ^4He + ^1H + ^1H$

The masses of hydrogen, deuterium, helium-3 and helium-4 are, respectively, $m(^1H) = 1.008amu \quad m(^2H) = 2.014amu \quad m(^1H) = 3.016amu \quad m(^1H) = 4.003amu$ **.**

(i) Estimate the total amount of energy in MeV released in one cycle.

$$6(^1H + ^1H + ^1H) \rightarrow 3(^2H + ^1H) \rightarrow 2(^3He) \rightarrow ^4He + 2(^1H)$$

$$6(1.008amu) \rightarrow 4.003amu + 2(1.008amu)$$

$$\Delta m = 0.029amu$$

$$\Delta mc^2 = 0.029 \cdot 938.3MeV = 27.2107MeV$$

$$\frac{27.21MeV}{6^1H} = 4.535 \frac{MeV}{^1H}$$

So that for each Hydrogen participant, 4.535 MeV of energy is released.

(ii) Which of the above reactions would you expect to have the slowest rate, and why? Can you think of an important consequence of this on the evolution of the solar system?

Proton-Proton fusion is the slowest—unlike the other steps, it requires energy. A Hydrogen atom would have to hit the other Hydrogen incredibly hard for this to work. This long timescale is going to dominate in determining the lifetime of stars.

(d) Astronomical data are often based on absorption lines seen in light originating in stars but passing through interstellar material. Consider the simple case of a cold cloud of atomic hydrogen illuminated by a star behind it whose spectrum has uniform intensity from 1000 to 10000 Angstroms and is zero at all other wavelengths. For parts (i) and (ii) assume the cloud to be at rest relative to the star.

(i) What absorption lines can be observed? Give wavelengths and Quantum number changes.

$$E_n = -\frac{13.6eV}{n^2}$$

$$\lambda(\dot{A}) = \left(\frac{10\dot{A}}{nm}\right) \cdot \frac{\hbar c}{(E_1 - E_m)} = 10\frac{1240}{13.6\left(1 - \frac{1}{n^2}\right)}$$

$$2 \to 1 : 1215\dot{A}$$

$$3 \to 1 : 1025\dot{A}$$

$$4 \to 1 : 972.5\dot{A}$$

So the absorption lines from $E_1 \to E_2$ and $E_1 \to E_3$ are visible. None other will be apparent since this is a cold gas, assumed entirely at the ground state.

(ii) Studying the lowest energy absorption line, an observer with a high-resolution spectrometer notices that the line is split into a series of closely spaced lines. Assuming that this is due to the cloud being situated in a weak magnetic field, write down the perturbing Hamiltonian that gives rise to the splitting. Derive a "calibration" formula which gives the magnetic field in terms of the smallest observed magnetic splitting, $\Delta\lambda$. Sketch a level diagram and label the quantum numbers.

$$H_1 = \mu_B \vec{B} \cdot (\vec{L} + \vec{S}) = \mu_B B m_{L+S} .$$

In the lowest splitting, then, $\Delta E = \mu_B B, \Delta\lambda = \frac{\hbar c}{\Delta E}$, corresponding to a single step in total spin magnetic moment.

(iii) Actually, the gas cloud may be moving relative to the star. What is the relative velocity between the cloud and the star if the lowest energy absorption line is red-shifted by 14% of the wavelength given off by the star?

Assuming a non-relativistic shift, if $z = \frac{v}{c} = 0.14 \quad v = 0.14c$.