

Problem Source: CMU August 2003 Qualifying Exam

Consider a supernova explosion occurring at a known distance  $D$  from Earth. The energy spectrum of the photons received on Earth is measured and photons of energies ranging from one electron volt (eV) to a million electron volts (MeV) are detected. Suppose further that all of these photons arrive within a time  $\Delta T$  of each other. We can use this to put upper bounds on the rest mass  $m_\gamma$  of the photon.

- (a) What is the relativistic relation  $E(v)$  between energy  $E$  and speed  $v$  for a particle with rest mass  $m_\gamma$  ?

$$E = mc^2 \gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- (b) If we assume that the rest energy of the photon is much less than one eV, use part (a) to argue that to lowest order,

$$\frac{v(E)}{c} \approx 1 - \frac{1}{2} \left( \frac{m_\gamma c^2}{E} \right)^2$$

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$1 - \frac{v^2}{c^2} = \left( \frac{mc^2}{E} \right)^2$$

$$\frac{v}{c} = \sqrt{1 - \left( \frac{mc^2}{E} \right)^2}$$

Taylor Expand In  $x = \frac{mc^2}{E} \rightarrow 0$

$$\sqrt{1-x^2} \Big|_{x \rightarrow 0} \approx 1 + \left[ \frac{1}{2\sqrt{1-x^2}} (-2x) \right] \Big|_{x=0} x + \frac{1}{2} \left( \frac{-1}{\sqrt{1-x^2}} - \frac{x^2}{(1-x^2)^{\frac{3}{2}}} \right) \Big|_{x=0} x^2 + \dots$$

$$\sqrt{1-x^2} \Big|_{x \rightarrow 0} \approx 1 - \frac{1}{2} x^2 + \dots$$

giving the expected relation.

- (c) Use part (b) to compute  $\Delta T(E, E')$ , the difference in arrival times between photons of energy  $E$  and  $E'$ .

$$\Delta T = \frac{D}{v(E)} - \frac{D}{v(E')} = D \left( \frac{1}{1 - \frac{1}{2} \left( \frac{mc^2}{E} \right)^2} - \frac{1}{1 - \frac{1}{2} \left( \frac{mc^2}{E'} \right)^2} \right)$$

- (d) Suppose a supernova explodes at a distance of 300 Mpc. Furthermore, suppose that the photons in the above energy range all arrive within 10 seconds of one another. What is the upper bound on the rest energy of the photon?

$$10s = \left( 300 \text{Mpc} \left( \frac{3.086 \times 10^{22} \text{m}}{1 \text{Mpc}} \right) \right) \left( \frac{1}{1 - \frac{1}{2} \left( \frac{m \left( 3.0 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2}{1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}} \cdot 1 \text{eV}} \right)^2} - \frac{1}{1 - \frac{1}{2} \left( \frac{m \left( 3.0 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2}{1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}} \cdot 1,000,000 \text{eV}} \right)^2} \right)$$

$m \rightarrow kg$

To solve this algebraically would be a chore: the result actually has four roots, two of which are negative. The largest of these is  $m = 2.39 \times 10^{-18}$ .