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Physics 2 Final Exam Review

- General Tips:
  - $e = 1.6 \cdot 10^{-19} C$
  - $\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{Nm^2}{C^2}$
  - $\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$
  - $\frac{\mu_0}{4\pi} = 1 \times 10^7 \frac{Tm}{A}$
  - Of three introductory physics courses I have taught, all three have had final exam problems corresponding closely or exactly to previous recitation, homework or exam problems.
  - Do not forget to write units on your solutions. If you can't write anything else, at least write down the units you'd expect.
  - Try doing some 'sample problems' on your cheat sheet! These will help you more than formulas as they are probably easier to reverse-engineer.
  - **All fields in this course work under the superposition principle. You may superpose total potentials, electric, magnetic and force fields!**
  - Remember that for Ampere's Law and Gauss's Law, if you have a piecewise function defining your current or charge sources, a correct answer will have a piecewise definition of $q_{enc}$ or $I_{enc}$ giving a piecewise electric and magnetic field. This piecewise electric field will then yield a piecewise function for electric potential.
  - The best way to study for this exam is not to memorize formulas, but to read this sheet and mark what you don't know—if you know it now, you'll know it for the final. You should then do problems that correspond to things you don't know so that you are prepared.
- Electric Potential
  - Electric Potential is always path-independent, in both the static and dynamic case. Electric Potential may manifest itself as EMF, but not all EMF is attributable to Electric Potential. Thus, there is no contradiction when you consider motional EMF.
  - $V_{AB} = \Delta V_{B \to A} = -\int_B^A \vec{E} \cdot d\vec{l}$
    - Remember, above A and B correspond to coordinate points.
    - Remember, there are two components to $d\vec{l}$, a vector and a differential. The differential is the coordinate direction you are going, the vector is the vectorial way you are facing. An example would be $d\vec{l} = (-dx)(-\hat{i})$ if I am facing in the –x direction and traveling in the same direction.
  - Work-Energy Theorem: $\Delta U_{particle} = q_{particle} \Delta V_{particle} + \Delta K.E._{particle}$.

- Since electric field has units N/C = V/m, if you have reason to believe that electric fields in a region must be constant you can simply divide $\Delta V$ across the region by the length to find electric field.
- Total potential at point A: $V_A = \sum_{all\ sources} \frac{q}{4\pi\varepsilon_0 r}$.

- Resistors and Capacitors
  - Resistors
    - In Series: $R_{eq} = \sum R_{individual}$
    - In Parallel: $(R_{eq})^{-1} = \sum (R_{individual})^{-1}$
    - $\Delta V = IR$
    - $P_{dissipated} = |I\Delta V|$
    - The sign of $\Delta V$ is negative if you travel across the resistor in the direction of the current and positive if you travel across the resistor opposite the direction of the current.
    - Resistance of a wire: $R = \rho \frac{L}{A_{cross-sectional}}$
  - Capacitors:
    - In Parallel: $C_{eq} = \sum C_{individual}$
    - In Series: $(C_{eq})^{-1} = \sum (C_{individual})^{-1}$
    - $\Delta V = |Q_{1-surface}| C^{-1}$
    - The sign of $\Delta V$ is positive if you travel from the negative surface to the positive surface, negative if you travel from the positive surface to the negative surface.
    - $E_{stored} = \frac{1}{2} C (\Delta V)^2$
    - Please see the Gauss's Law section to find how dielectrics affect electric field in a region.
  - Batteries:
    - $P_{supplied} = I\Delta V$. Note that here, I is the current flowing off of the positive terminal, so that if current is flowing into the positive terminal the battery is supplying negative power, or being charged.
    - The sign of $\Delta V$ is positive if you travel from the negative surface to the positive surface, negative if you travel from the positive surface to the negative surface.
- Kirchoff's Laws
  - Procedure:
    1. Label all regions with distinct currents $I_1, I_2, \ldots$ and directions. This step is extremely important, as no part of the problem can be correct until this step is complete. It is OK if you pick a wrong direction, as this choice will manifest itself if you find that you get

a negative current in your final result. You simply have a positive current in the opposite direction.

2. Use the node rule and loop rule to write as many independent equations as you have currents. This means that you want to make sure that you do not include redundant information in your set of equations. Choose a mix of loops and nodes being sure to include each current from step 1.
3. Solve the equations and interpret the results.

- Loop Rule: Derived from the path-independence of electric potential, this is based on $\Delta V_{closed\ \ loop} = 0$. Choose any closed path about the circuit, and add the $\Delta V$ contribution for each item, setting the result equal to zero. Use the $\Delta V$ formulas from the section above.
- Node Rule: At any particular point in the circuit, as much current is flowing in as out. Thus, at any intersection of wires, $\sum I_{in} = \sum I_{out}$.
- Remember that in the uncharged limit, capacitors have $\Delta V = 0$ and can be ignored. In the fully charged "long time" limit, no current flows through the capacitor.
- The time constant for a simple circuit composed of a resistor, capacitor and possibly battery in series has a time constant $\tau = RC$. This means that the capacitor will have $Q_C(t) - Q_C(0) \propto e^{-\frac{t}{RC}}$.

- Electric Fields
  - Conductors
    - Electric field is zero in conductors only in the static (no currents) case. Because of this, all charge in the static case lies on the surface of the conductor.
    - In Ohmic conductors with currents, with respect to the current density we have $\vec{E} = \rho\vec{J}$ with $J = \dfrac{I}{A_{cross-sectional}}$.
  - Direct Integration (Coulomb's Law):
    - $d\vec{E} = \dfrac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$
    - Usually in this case you are integrating over a line of charge. In this case, you are typically given the line charge density $\lambda = \dfrac{dq}{dl}$ so that you have $d\vec{E} = \dfrac{\lambda dl}{4\pi\varepsilon_0 r^2}\hat{r}$.
    - As usual, try to calculate $\hat{r}$ first and simplify. You can always be explicit and take $\vec{r} = \langle destination\rangle - \langle source\rangle$ $\quad \hat{r} = \dfrac{\vec{r}}{\|r\|}$.
  - Gauss's Law:
    - $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0}$

- In dielectrics or other materials, let $\varepsilon_0 \to \varepsilon = K\varepsilon_0$.
- Procedure
  1. Determine symmetries of the problem
  2. Select an appropriate Gaussian surface
  3. Simplify the surface integral in terms of a constant electric field over the surface
  4. Simplify the charge enclosed by integrating over the volume inside your Gaussian surface.
  5. Solve for the electric field over the surface.
- Symmetries for Electric Fields
  - Rotational and Translational: These symmetries eliminate their respective degrees of freedom from the parameters of electric field: For example, translational symmetry on z and rotational symmetry on $\phi$ would indicate that $\vec{E}(r,\phi,z) = \vec{E}(r)$.
  - Reflectional: These symmetries eliminate the out-of-plane components fields in planes of mirror symmetry. For example, for an infinite sheet of charge in the xy-plane, planes of symmetry include any plane parallel to the z-axis. Thus, the planes with normals in the $\hat{i}$ direction eliminate $E_x(x,y,z)\hat{i}$ and similarly for planes with normals in the $\hat{j}$ direction so I would get:
    $\vec{E}(x,y,z) = E_x(x,y,z)\hat{i} + E_y(x,y,z)\hat{j} + E_z(x,y,z)\hat{k} = E_z(x,y,z)\hat{k}$
  - 180° Rotation: This symmetry indicates that if I may rotate a system 180° about an axis without changing its appearance, then for fields perpendicular to this axis I have (assuming here the axis of the rotation is z) $\vec{E}(-x,-y,z) = -\vec{E}(x,y,z)$. This becomes important in problems with planar symmetry.
- Three Possible Cases:
  - Gaussian Pillbox: In a problem with planar symmetry, e.g. the symmetries give $\vec{E}(z) = E(z)\hat{k}$, you will get $\oint \vec{E} \cdot d\vec{A} = AE(z_1) - AE(z_2)$ for 'top' and 'bottom' surfaces. Clearly, you must eliminate one or the other surface by either placing it inside a conductor or using 180° Rotation symmetry. Further, A will cancel out against the integral which gives $q_{enc}$.
  - Gaussian Cylinder: In a problem with cylindrical symmetry, e.g. the symmetries give $\vec{E}(r) = E(r)\hat{r}$ (cylindrical coordinates), you will get $\oint \vec{E} \cdot d\vec{A} = 2\pi r l E(r)$.

Note that the length variable l will cancel against the integral which gives $q_{enc}$.

- Gaussian Sphere: In a problem with spherical symmetry, e.g. the symmetries give $\vec{E}(r) = E(r)\hat{r}$ (spherical coordinates), you will get $\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E(r)$.

- Magnetic Fields
  - Biot-Savart Law
    - $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$
    - See $d\vec{l}$ explanation from Electric Potential section.
    - Hint: Calculate the direction of $d\vec{l} \times \hat{r}$ first, before doing the integral, using the right hand rule.
    - The cake is a lie.
  - Ampere's Law
    - $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt}\oint \vec{E} \cdot d\vec{A}$
    - $\varepsilon_0 \frac{d}{dt}\oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \frac{d}{dt}\Phi_E$ is "displacement current" and can be ignored unless the problems specifically asks for it. Typically, electric fields will be given in the region in question for tractability.
    - Symmetries for Magnetic Fields:
      - Beware! When you decide symmetries with magnetic fields, you must consider both magnitude and direction. In reflectional symmetry, are you sure that if you placed a mirror in the plane you claim and watched the charges go by, that none are going 'into' the mirror?
      - Rotational and Translational symmetries work the same way as with Electric fields.
      - Reflectional: These symmetries eliminate the in-plane components fields in planes of mirror symmetry. For example, for an infinite line of current going along planes of symmetry include any plane containing the z-axis. Thus, the planes with normals in the $\hat{\phi}$ direction eliminate all magnetic fields except those in the $\hat{\phi}$ direction, $E_\phi(r)\hat{\phi}$.
      - 180° Rotation symmetry works the same way as with Electric fields.
    - The procedure for Ampere's Law is very similar to Gauss's Law. Determine the symmetries, simplify the loop integral in terms of a constant magnetic field, and integrate the enclosed current over the area inside the loop to solve.
    - Simplified:

        - In the case where symmetries give $\vec{B} = B\hat{\phi}$, as with a line of current or a torus of current, you will use an circular path. In this case, $\oint \vec{B} \cdot d\vec{l} = 2\pi r B(r)$.
        - In the case where symmetries give $\vec{B} = B\hat{k}$ (or any non-curved coordinate), $\oint \vec{B} \cdot d\vec{l} = lB(x_1) - lB(x_2)$ for two non-vanishing sides of an Amperian square. You will clearly have to eliminate one side by either knowing that the magnetic field there is zero or by using the $180^\circ$ rotation symmetry. The loop height parameter l should vanish against your integral to find the current enclosed by the box.
    - Law of No Magnetic Monopoles
        - $\oint \vec{B} \cdot d\vec{A} = 0$
        - The procedure for use of this law uses the same symmetry rules as with Ampere's law, and the same procedure for calculation as with Gauss's law.
        - For your purposes, this law will typically be used for the purpose of verifying that a result is plausible.
- Faraday's Law
    - $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\oint \vec{B} \cdot d\vec{A}$
    - Procedure:
        1. Parameterize magnetic flux as a function of time. This may involve finding the area as a function of time, the direction of the normal changing in time, the direction of the magnetic field changing in time, or the magnitude of the magnetic field changing in time.
        2. Take negative the time derivative.
        3. Interpret positive EMF corresponding to a battery with its positive terminal pointing in the direction of your fingers, if your thumb was in the direction of the surface normal. You may also use Lenz's law here, described below.
    - Simplified form: $EMF = -\frac{d}{dt}\Phi_B$
        - Changing magnetic field (in direction of normal): $EMF = -A\frac{dB}{dt}$
        - Changing area (field in direction of normal): $EMF = -B\frac{dA}{dt}$
    - The positive EMF acts like a battery with its positive terminal in the direction indicated by the right-hand rule if your thumb is along your selected normal.
    - Lenz's Law:
        - The induced EMF acts to oppose a change in magnetic flux.

        - If magnetic flux changes, then put your right fingers in the direction opposite that change. Then, the current flows in the direction of your thumb.
    - Magnetic forces never do work. Faraday's Law is no exception.
- Lorentz Force and Motional EMF
    - Lorentz Force
        - $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$
        - Wire: $d\vec{F}_B = Id\vec{l} \times \vec{B}$, or typically $F_B = ILB$ if $\vec{L} \perp \vec{B}$.
        - It is possible that you will see a problem where you will be expected to use the magnetic force as a centripetal force $a_c = \frac{v^2}{r}$.
        - For helical motion of a charged particle in uniform parallel electric and magnetic fields, project the particle's velocity into the plane of the electric field and the plane perpendicular to the magnetic field and treat the accelerations due to each as separate problems:

          $$\vec{v}_{\parallel E} = \vec{v} \cdot \frac{\vec{B}}{\|B\|}$$

          $$\vec{v}_{\perp E} = \vec{v} - \vec{v} \cdot \frac{\vec{B}}{\|B\|}$$
    - Motional EMF
        - This procedure will give the same results as Faraday's Law.
        - $EMF = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$
        - Simplified, this typically becomes $EMF = vBL$ for a moving bar of length L in uniform magnetic field with $\vec{v} \perp \vec{B}$.
        - Use the right-hand rule with a positive test charge to find the 'positive' end. You may model the EMF as a battery on the rod with the positive end in this direction.
- Waves
    - Designing a sinusoidal wave function:
        - $y(x,t) = A\sin(kx \pm \omega t)$
        - A: Amplitude (same units as wave)
        - k: wave number $k = \frac{2\pi}{\lambda}$
        - $\omega$: angular frequency $\omega = 2\pi f$
        - $\pm$: + indicates propagation in the –x direction, - in the +x direction.
        - $\lambda$: wavelength $v = \lambda f$
        - f: frequency $v = \lambda f$
        - $v$: propagation velocity.
    - Transverse Velocity: $v_y = \frac{\partial y(x,t)}{\partial t}$.

- All classical waves satisfy: $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$, and can be parameterized in terms of $y(x - vt)$.
- On a wire: $v_{wire} = \sqrt{\frac{T}{mass / length}}$
- Normal Modes: The n-th harmonic in a string of length L satisfies $\lambda = \frac{2L}{n}$, with n an integer.
- Electromagnetic Waves
  - Speed: $c = 3 \cdot 10^8 m/s$
  - Power in this wave: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$
  - Force and momentum in light: $F = \Delta p = \frac{Intensity}{c}$
  - The electric and magnetic contributions are perpendicular.

- Recommended Problems:

Potential:
23.57/23.61 Recitation 9
* 24.12/24.12 Recitation 10
Recitation 11, Last Problem

Kirchoff's Laws:
26.65/26.69 Recitation 14
(Charging Capacitor) Recitation 15
* Assignment 7, Special problem

Electric Fields
(Direct Integration):
* Problem 3 (2007 Final)
Recitation 4, Last Problem

(Gauss's Law):
Recitation 5, Problem 1
* Recitation 6, Problem 1
Recitation 7, Problem 2

Magnetic Fields
(Biot Savart):
* Problem 3 (2007 Final)
28.64/28.66 Recitation 19
Assignment 9, Special Problem

Magnetic Fields
(Ampere's Law):
Assignment 11, Special problem
28.72/28.74 Recitation 20
* (SLAB of uniform cur.) Recitation 21
29.35/29.36 Recitation 25
(Law of No Magnetic Monopoles):
27.13/- Recitation 15
27.12/27.12 Assignment 8

Faraday's Law:
* 29.9/29.9 Recitation 22
(Coil moving through nonuniform field) Recitation 22
(Coaxial Cable) Recitation 24

Lorentz Force:
* 27.31/27.31 Recitation 17
27.25/27.27 Recitation 17

Motional EMF:
* Problem 7 (2007 Final)
(Rotating Rod) Recitation 23
Assignment 12, Special Problem

Waves:
* Problem 8 (2007 Final)
(Is it a Wave?) Recitation 26
32.3/32.7 Recitation 28
Assignment 13, Special problem part d