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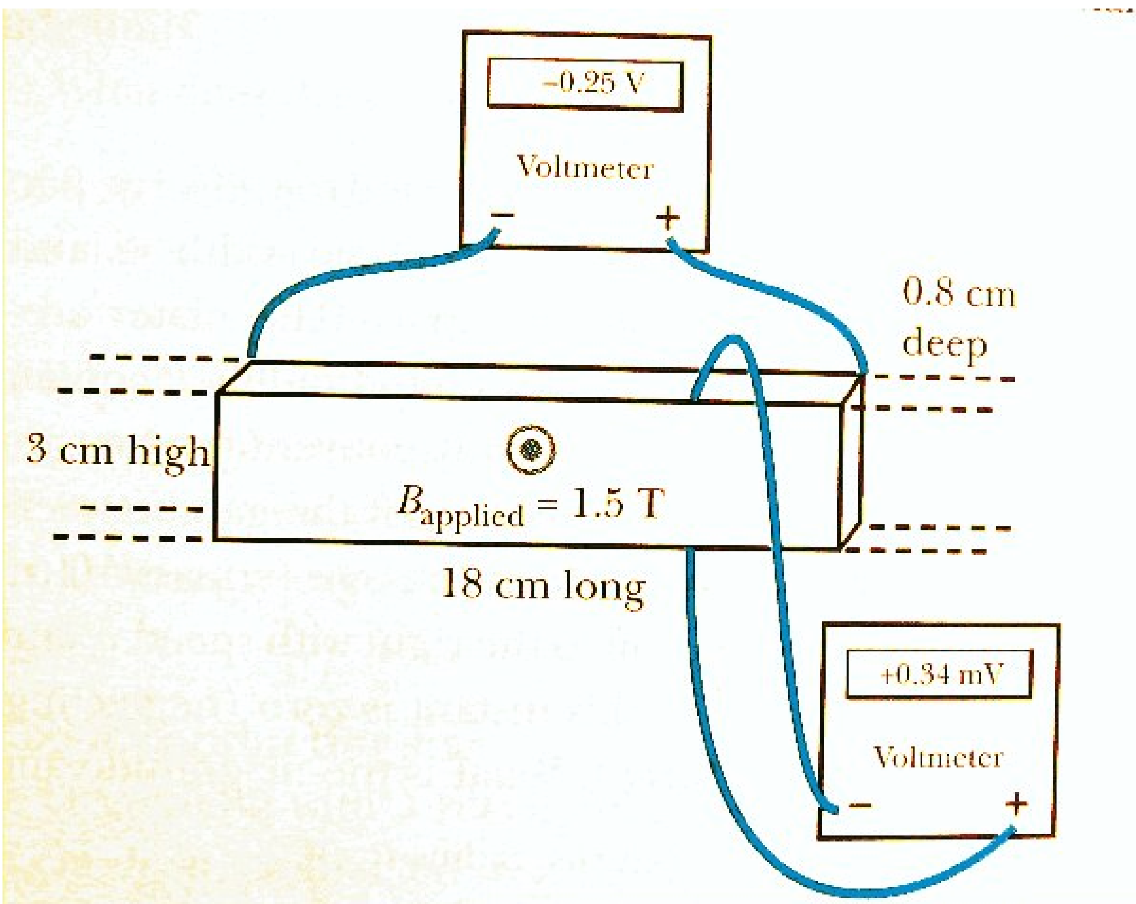
Physics 2 Spring 2007--Hall Effect

Chabay and Sherwood, Problem 20.P.60

An experiment was carried out to determine the electrical properties of a new conducting material. A bar was made out of the material, 18cm long with a rectangular cross section 3 cm high and 0.8cm deep. The bar was part of a circuit and carried a steady current (the figure shows only part of the circuit). A uniform magnetic field of 1.5 tesla was applied perpendicular to the bar, coming out of the page (using some coils that are not shown). Two voltmeters were connected along and across the bar and read steady voltages as shown (mV = millivolt = 1×10^{-3} volt). The connections across the bar were carefully placed directly across from each other to eliminate false readings corresponding to the much larger voltage along the bar. Assume that there is only one kind of mobile charge in this material.

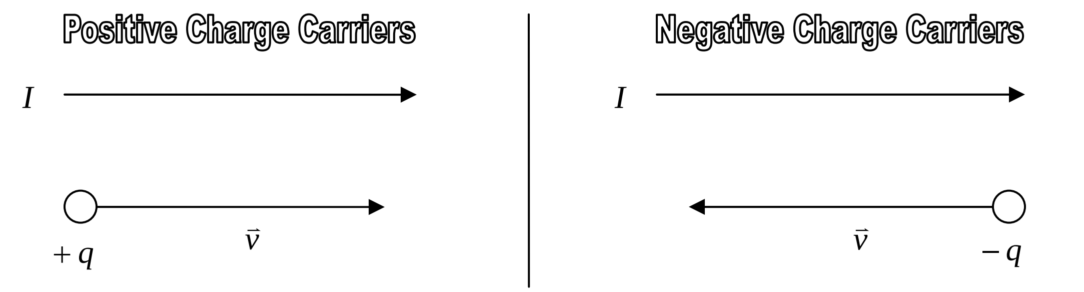


(a) **What is the sign of the mobile charges, and which way do they move? Explain carefully, using diagrams to support your explanation.**

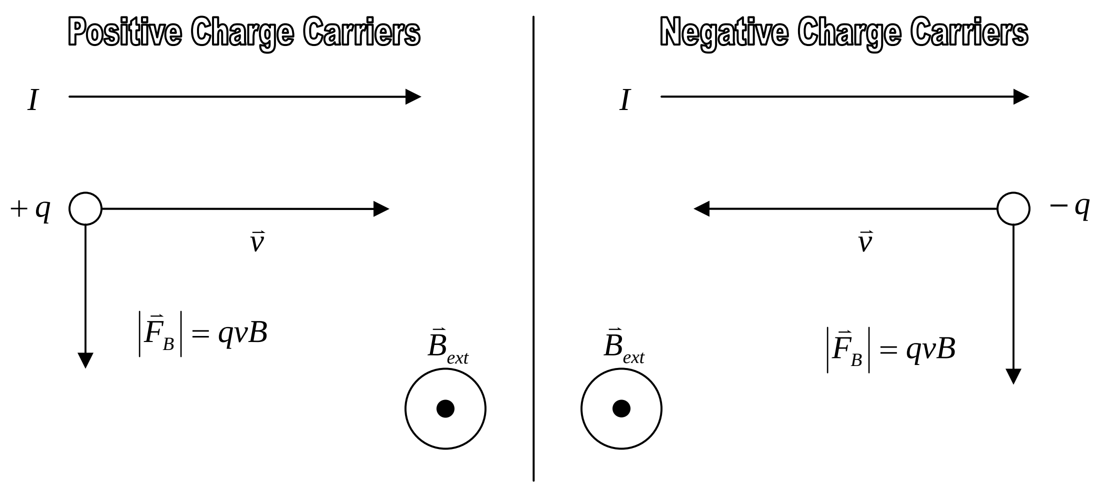
The important feature of the Hall effect that you have to understand is that this gives us a classical way to observe a quantum-mechanical phenomenon: namely, that electron holes can act as a charge carrier rather than the electrons themselves.

First, I need to know which direction the current is flowing in the bar pictured above. Note that for real materials, resistance is, except in the case of superconductors, positive. Using $\Delta V = IR$ in resistors, then, I notice that the measured ΔV is negative and that the resistance of the bar is positive, indicating that with respect to the partial loop going from the negative terminal of the voltmeter to the positive terminal that the current must be in the same direction. That is, when the loop is in the same direction as the current, ΔV for the resistor is negative. When the loop is in the opposite direction from the current, ΔV for the resistor is positive.

Now I know that the conventional current is flowing to the right, this can mean one of two things for a charge carrier: Either a positive charge carrier is flowing to the right, or a negative charge carrier is to the left.



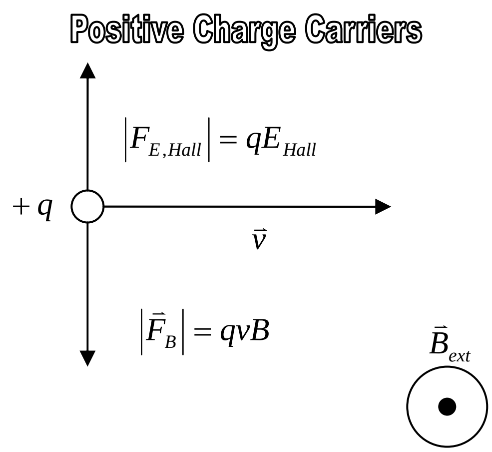
Next, consider what this means as a function of the external magnetic field:



In other words, I see from calculating the force on the positive and negative charge carrier case that the charge carrier, be it positive or negative, is going to be pushed to the bottom face of the bar. However, the sign of the voltmeter placed top-to-bottom of the bar in the picture above indicates that the electric field points from the bottom face to the top face in order to produce a 0.34 mV potential difference. In this case, then, thinking of my top and bottom faces as plates of a large capacitor, I need the positive charges on the bottom and the negative charges on the top, indicating that I need the picture above where the positive charge is being pushed down. This indicates that I have positive charge carriers in this material.

(b) What is the drift speed v of the mobile charges? Explain your reasoning.

In steady state, I expect no more charge to be accumulating on the top and bottom of the bar. Considering a free body diagram, then:



Since this is a steady state and no more charges are moving towards the top or bottom, then I expect the forces to cancel so that:

$$|F_E| = |F_B| = qE_{Hall} = qvB.$$

However, thinking of my top and bottom faces as plates of a large parallel-plate capacitor, I may take

$$E_{Hall} = \frac{\Delta V_{top-bottom}}{height}$$

and so

$$E_{Hall} = vB$$

$$\frac{\Delta V_{top-bottom}}{height \cdot B} = v = \frac{3.4 \cdot 10^{-4} V}{0.03m \cdot 1.5T} = 7.6 \cdot 10^{-3} \frac{m}{s}$$

(c) What is the mobility u of the charges?

The mobility of charges is the velocity response to a force per coulomb: Namely,

$$u = \frac{m/s}{N/C} = \frac{v}{E}$$

The electric field, length-wise, is

$$\left|E_{left-right}\right| = \frac{\left|\Delta V_{left-right}\right|}{length} = \frac{0.25V}{0.18m} = 1.38\frac{N}{C}$$

Using the velocity from part (b), I have

$$u = \frac{v}{E} = \frac{7.6\cdot 10^{-3}\, m/s}{1.38N/C} = 5.4\cdot 10^{-3}\,\frac{Cm}{Ns}$$

(d) The current running through the bar was measured to be 0.6 ampere. If each mobile charge is singly charged, $|q| = e$, how many mobile charges are there in $1\ m^3$ of this material?

Using my microscopic current formulae, I have:

$$i = \frac{I}{q}$$

and

$$i = nAv$$

Then,

$$\frac{I}{q} = nAv$$

$$n = \frac{I}{qAv} = \frac{I}{q(depth\cdot height)v} = \frac{0.6A}{\left(1.6\cdot 10^{-19}C\right)(0.03m)(0.008m)\left(7.6\cdot 10^{-3}\,\frac{m}{s}\right)} = 2\cdot 10^{24}\,\frac{carriers}{m^3}$$

(e) What is the resistance in ohms of this length of bar?

$$|\Delta V| = IR$$

$$0.25V = 0.6A\cdot R$$

$$R = \frac{0.25V}{0.6A} = 0.42\Omega$$