

Quiz Answer:

Quiz: Faraday's Law: Optimus Prime

You are given that $\int_{loop} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_{surface} \vec{B} \cdot \hat{n} dA$ (Faraday's Law)

You are also given that the magnetic field inside a solenoid $B \approx \frac{\mu_0 NI}{L}$. You should assume that this is the case for the entire problem.

Ignore time-retardation effects for this problem.

Optimus Prime has two solenoids of the same length and radii a and b , so that $L \gg a > b$, one inside of the other. Both solenoids have N coils in them. The outermost solenoid carries a maximum current I_0 , varying at 60Hz e.g. $I = I_0 \cos\left(\frac{60}{2\pi}t\right)$.

****NOTE:** There is an error in the problem above: the function at 60Hz should be $I = I_0 \cos(2\pi(60)t)$. This change of the constant does not affect the outcome of the problem or the validity of your proof—except that the frequency given corresponds to roughly 1.5Hz. ******

Part 1) 3 Points: What is the frequency of the induced current in the inside solenoid? Prove this using Faraday's Law. An answer without a proof is worth zero points.

$$\begin{aligned} IR = V &= \int_{loop} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_{surface} \vec{B} \cdot \hat{n} dA = -\frac{d}{dt} \left(\frac{\mu_0 NI}{L} \cdot \pi b^2 \right) = -\frac{d}{dt} \left(\frac{\mu_0 N}{L} \cdot \pi b^2 I_0 \cos\left(\frac{60}{2\pi}t\right) \right) \\ &= \frac{\mu_0 N}{L} \cdot \pi b^2 I_0 \left[\frac{60}{2\pi} \sin\left(\frac{60}{2\pi}t\right) \right] = \frac{\mu_0 N I_0 b^2 (60)}{2L} \sin\left(\frac{60}{2\pi}t\right) \end{aligned}$$

Now that I have $I = \frac{\mu_0 N I_0 b^2 (60)}{2LR} \sin\left(\frac{60}{2\pi}t\right)$, I see that the only time dependence is in the sine and that the frequency has not changed at all.

Part 2) 2 Points: If Optimus places a tap on either end of the solenoid, what will the total voltage reading be as a function of time? You may use your result from part 1.

$$V_{\text{one coil}} = \frac{\mu_0 N I_0 b^2 (60)}{2L} \sin\left(\frac{60}{2\pi} t\right)$$

$$V_{\text{total}} = N V_{\text{one coil}} = \frac{\mu_0 N^2 I_0 b^2 (60)}{2L} \sin\left(\frac{60}{2\pi} t\right)$$

This formula gives the relationship between length, coil density, and radii in a transformer to the secondary voltage.

How to do well on the final: General Strategy:

To get 60%:

Go over all of your mid-term tests, and ensure that on every test you fully understand the solution to every problem—available online.

To get 85%:

Do some practice problems on Faraday's Law, and check the solutions to your recent homework against the solutions available online. If you did well on today's quiz, you should be fine.

To get 100%:

You need a strong conceptual understanding of every concept—these problems are not difficult with proper understanding. For each chapter covered in Physics 2, look at the section in your book Review Questions with the problems labeled "RQ". You should be able to read over these and quickly answer every review question. If you can't do this for any question, you should re-read the corresponding part of the chapter.

Comments on studying and test-taking for specific topics:

Coulomb's Law:

This is the first topic we studied. I'm sure that by now you've got it completely memorized. The important things to know are:

The superposition principle—electric fields for sub-distributions are additive.
Vectors vs. Scalars—electric field is a vector. Remember that by the superposition principle we may break up a charge distribution into smaller sections, and then integrate over these sections to find the electric field anywhere (Exam 1 question 3).

For the exam: ensure that every time you write a quantity you specify whether the quantity is a vector or a scalar, and either specify the direction with an appropriate unit vector or with words defining the direction.

Conductors and Insulators:

Conductors and insulators are similar in some ways—both have a degree of charge mobility that affects the fields around them. Conductors are “perfect” in that they cancel all electric fields inside them, while insulators only try to.

For the exam: Be completely certain that you draw net induced charge only on the surface of the conductors and insulators. Insulators gain a polarization from the fields present around them. Remember that electric fields are always perpendicular to the surface of the conductor, and beware that while electric field is zero inside a conductor (and NOT necessarily zero inside an insulator), there may be some polarization inside the insulator since electric fields may penetrate them.

Circuits:

I am certain you will see a circuit problem on the exam. You need to know the node rule and the loop rule (and more importantly how to use them to derive an independent set of equations), understand how the voltmeter works, and know the derivation of the charging capacitor formula.

For the exam: Be certain that you know the difference between independent and dependent equations. It would be a safe bet to draw the loops on the equation. Remember! Electric potential is path-independent.

Electric Potential:

You will certainly see an electric potential problem on the exam. In all probability, you'll be given a charge distribution, asked to find the electric field with Gauss's Law, and then be asked to find the electric potential based on that. Remember that it is measured with respect to a reference point.

For the exam: Remember, potential is not a vector. The easiest way to find it is often to use superposition with respect to a known charge distribution, but can also be found with $-\int \vec{E} \cdot d\vec{l}$. A number of people on the first exam used a bad explanation: here is the reality: the potential at a known point (possibly infinity) is given. Then,

$$\Delta V(\vec{s}, \vec{f}) - \int_{\vec{s}}^{\vec{f}} \vec{E} \cdot d\vec{l}. \text{ Now, } V(\vec{f}) = \Delta V(\vec{s}, \vec{f}) + V(\vec{s}).$$

Current:

If you're asked a current problem on the final, it will probably include either the Lorentz force or Faraday's Law as part of the problem. I am almost certain that you won't be asked to integrate over a cross product, and if you are you should use the symmetries of the problem along with $|\vec{a} \times \vec{b}| = |a||b|\sin\theta$. In general, most problems use the long

straight wire formula which should be given, along with superposition. The other easy way to handle these is by applying Ampere's law along with the symmetries of the problem.

For the exam: Be certain to include a magnitude and direction with your magnetic field, and don't try to integrate over the cross product! Use Ampere's Law! You will use the microscopic theory of current wherever you need to discuss the actual moving particles (as in the Hall effect), and the macroscopic theory when there is no need. When using Ampere's law, be sure to draw your surface.

Hall Effect:

This is unlikely to appear on the final, but if it does don't bother trying to remember the formula. You will use the microscopic current and then set the forces $qE = qvB$ equal to solve for the equilibrium electric field, having invoked the microscopic current theory to determine q and v .

Gauss's Law:

This will certainly appear on the final. Most people have the general concept correct, but here is what you need to remember for the final:

For the exam:

-Set up any Gaussian surface so that either the surface flux vanishes or is constant over the surface. In the last exam, the surface flux did not necessarily vanish on the ends, but you were to justify why these were small and choose a corresponding surface where the contributions are indeed small.

-Draw your Gaussian surface.

Lorentz Force:

This is not a difficult equation, and typically makes its appearance in the moving bar problem, discussed in a previous week's recitation notes.

For the exam: Be certain that the force has the proper direction and that you use the right hand rule with the appropriate hand!

Faraday's Law:

We covered several examples of Faraday's Law recently.

For the exam: Be certain to draw your loop and corresponding surface. Remember the forms of Faraday's Law that I gave you in recent classes and use this to make the connection between flux, induced voltage and current.

Good luck! It's been fun—and have a great summer!