Recitation Notes March 30
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Since the other portions of Ch. 19 have been covered in recitation and are in my notes, here's an example that may help you with your homework.

19.13) Using the node rule, my six equations correspond to these loops:

$$A \to C \to E \to F \to G \to A$$

$$A \to C \to E \to D \to F \to G \to A$$

$$A \to C \to D \to E \to F \to G \to A$$

$$A \to C \to D \to F \to G \to A$$

$$D \to F \to E$$

$$C \to D \to E$$

corresponding, respectively, to

$$20 - 10I_1 - 15I_4 - 12I_6 - 20I_1 = 0$$

$$20 - 10I_1 - 15I_4 - 5 - 30I_3 - 20I_1 = 0$$

$$20 - 10I_1 - 20I_2 + 5 - 12I_6 - 20I_1 = 0$$

$$20 - 10I_1 - 20I_2 - 30I_3 - 20I_1 = 0$$

$$-5 - 30I_3 + 12I_6 = 0$$

$$5 + 15I_4 - 20I_2 = 0$$

Let me put these into Mathematica (the preferred way to do this would be row reduction, using linear algebra, but this is an example, not an exercise).

To solve this, I need six equations in six unknowns. I took three of the ones above:

$$20 - 10I_1 - 15I_4 - 12I_6 - 20I_1 = 0$$

$$20 - 10I_1 - 15I_4 - 5 - 30I_3 - 20I_1 = 0$$

$$20 - 10I_1 - 20I_2 + 5 - 12I_6 - 20I_1 = 0$$

and supplemented them with three node rule equations:

$$I_1 = I_4 + I_2 \quad I_1 = I_3 + I_6 \quad I_6 = I_5 + I_4 .$$

Mathematica gives me as my results:

```
In[1]:= N[Solve[{20 - 10 I1 - 15 I4 - 12 I6 - 20 I1 == 0,
          20 - 10 I1 - 15 I4 - 5 - 30 I3 - 20 I1 == 0,
          20 - 10 I1 - 20 I2 + 5 - 12 I6 - 20 I1 == 0,
          I1 == I4 + I2, I1 == I3 + I6, I6 == I5 + I4},
        {I1, I2, I3, I4, I5, I6}]]

Out[1]= {{I1 → 0.439394, I2 → 0.331169, I3 → 0.00649351,
        I4 → 0.108225, I5 → 0.324675, I6 → 0.4329}}
```

b) It is easy to verify that these work as solutions:

```
In[2]:=
Clear[I1, I2, I3, I4, I5, I6];
I1 = 0.440;
I2 = 0.331;
I3 = 0.006;
I4 = 0.108;
I5 = 0.325;
I6 = 0.432;
20 - 10 I1 - 15 I4 - 12 I6 - 20 I1
20 - 10 I1 - 15 I4 - 5 - 30 I3 - 20 I1
20 - 10 I1 - 20 I2 + 5 - 12 I6 - 20 I1
20 - 10 I1 - 20 I2 - 30 I3 - 20 I1
-5 - 30 I3 + 12 I6
5 + 15 I4 - 20 I2

Out[9]= -0.004

Out[10]= 0.

Out[11]= -0.004

Out[12]= 0.

Out[13]= 0.004

Out[14]= 0.
```

just as expected (notice that these results should equal the right-hand sides from my equations in part a).

c) With the negative lead on G and a positive lead on C, I use the loop rule, partially: The voltmeter should read $-10I_1 + 20 - 20I_1 = 6.8$ volts (I have used the loop rule following the path from negative to positive, with a positive sign since the meter is properly connected with negative to low potential and positive to high.) Since potential is path independent, there's no reason that I couldn't also follow the loop around backwards from G: I could take $15I_4 + 12I_6 = 20I_2 + 30I_e = 6.8$ also.

d) The power output is simply $I\Delta V = 5I_5 = 1.62$ Watts. The battery is being charged This is not a very good way to charge a modern battery. Pulse-charging is recommended—but this does work, and interestingly an immense amount of research goes into determining how charged a battery is. Its voltage will quickly jump from zero to the charging potential, but then will level out for a long period of time. In this region, it is hard to tell how charged the battery is. Charging it too much will destroy it!

$$\frac{I}{A} = \sigma E \quad R = \frac{L}{\sigma A}$$

e) $$E = \frac{I}{A\sigma} \quad \sigma = \frac{L}{RA}$$

$$E = \frac{I_6 R}{L} = \frac{0.432 \cdot 12}{0.003} = 1728 \frac{N}{C}$$

Interestingly, area dropped out! It seems paradoxical at first until you realize that the resistance includes it as a factor subtly.