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Recitation Notes and Quiz Answer March 22, 2006

Capacitors

I've done some example problems on capacitors here. They're pretty quick, but no one sent me any requests concerning what problems I should do. Maybe I didn't get to them all in class.

19.7)

(a) Putting capacitors in series adds their corresponding potentials, just like putting batteries in series. However, it decreases their aggregate capacitance. You can think of it this way: if you add the gap between two capacitors (which is essentially the model at work), you get a larger gap for a larger potential difference. However, $C = \frac{Q}{V}$, and since they hold the same amount of charge now but at a higher voltage, capacitance drops. Now in this light I consider the problem: the final charge will be $C \cdot emf = Q$.

(b) Now when I put the glass in the capacitor, the electric field inside will be reduced and you will see an increase in capacitance: namely now, $C = \frac{Q}{\frac{V}{K}} \quad \frac{CV}{K} = Q$ (since the electric field inside is reduced by a factor of $\frac{1}{K}$, the amount of voltage my capacitor needs to see for the same effect is KV). Now more current will flow through my light bulb to the charge plates of the capacitor. When I added the glass to the capacitor, the apparent voltage decreased and the battery moves to fill the capacitor up until the field inside exactly counters the applied electromotive force (voltage).

19.8)

When you remove the slab of glass from the capacitor, charge will flow from the plates of the capacitor back into the battery and will "recharge" it a little bit. Interestingly, if you had removed the battery and tried to remove the slab of glass, the slab would actually have resisted your removing it! In fact, you'd have to apply enough energy to remove the glass to make up for the boost in energy given by the same amount of charge at the same distance but now at a higher voltage!

19.9)

(a) To be fair, air is a dielectric, but I will assume for now that it's just empty space. Again, $C \cdot emf = Q$.

(b) Again, using my explanation from 19.7, $C = \frac{KQ}{(emf)} \quad \frac{C(emf)}{K} = Q$.

(c) Immediately after inserting the plastic, my voltage has changed from emf across the plates to suddenly $\frac{emf}{K}$. Now using $V = IR$ $\quad \frac{1}{R}\left(emf - \frac{emf}{K}\right) = I$.

(d) The final charge on the positive plate, again using the explanation from 19.7, is $C = \frac{Q}{\frac{V}{K}} \quad \frac{CV}{K} = Q$.

19.11)

First I need to know the capacitance of this capacitor. For the parallel-plate capacitor, from page 764 I have $C = \frac{K\varepsilon_0 A}{s} = \frac{1 \cdot \varepsilon_0 \cdot (0.02) \cdot (0.1)}{0.001} = 2\varepsilon_0 \quad F$.

Now certainly I have $V = IR$, where $V(t) = 100 - \frac{Q(t)}{C} \quad Volts$, and certainly at time zero $V(0) = 100 \quad Volts \quad Q(0) = 0 \quad Coulombs$. Now simply writing $V = IR$, noting that $I = \frac{\partial Q(t)}{\partial t} = \frac{Coulombs}{Second}$. Finally, using $R(t) = 1000 \quad ohms$ I can write this all out in terms of Q:

$$100 - \frac{Q(t)}{C} = 1000\frac{\partial Q(t)}{\partial t}$$

$$Q(t) = 100C - 1000C\frac{\partial Q(t)}{\partial t}$$

This is what's known as a "first order differential equation". Solutions of any such equations you might see are of the form Ae^{kt}, so taking $Q(t) = Ae^{kt}$, I see that then I have implied that

$$\begin{aligned} Ae^{kt} &= 100C - 1000CAke^{kt} \\ (1 - 1000Ck)Ae^{kt} &= 100C \end{aligned}.$$

For now, however, suppose I was applying zero voltage, so that 100C hasn't yet come into play. In equilibrium, then, I have $(1 - 1000Ck)Ae^{kt} = 0$, constraining $k = \frac{-1}{1000C}$: this RC appearing in the denominator is known as the "time constant" of the circuit. In this light, it's now a trivial matter to solve for A using my initial equation $t = 0$: I have

$$Ae^{kt} = 100C - 1000CAke^{kt}$$

$$A = 100C - 1000CAk$$

$$A = \frac{100C}{1 + 1000Ck}$$

. Now if I substitute everything I've gotten so far in, including C, I should be able to see when I reach 95 volts.

$$95 = V = 100 - \frac{Q(t)}{C} = 100 - \frac{Ae^{kt}}{C}$$

$$\frac{5C}{A} = e^{kt}$$

$$\frac{1}{k}\ln\left(\frac{5C}{A}\right) = t$$

Making the substitutions, then, I have

$$C = 2\varepsilon_0 = 1.77\times10^{-11}$$

$$A = 1.59\times10^{7}$$

$$k = -5.64\times10^{7}$$

$$t = 7.03\times10^{-7}\ s$$

This isn't very long, but that's not really surprising. There's a lot of voltage and not much capacitance. At this speed, in fact, the inhibiting factor for charging would not be resistance: it would be inductance (which you haven't learned about yet).

Quiz: Die Hard

Have you seen the "Die Hard" movie where the protagonists had a water supply and water jugs, and had to make the water jugs fill to just the perfect size in gallons? Today you'll do the same thing with capacitors.

Important: If you don't know how to do this, explain what you know about charging capacitors and the properties of capacitance and potential for capacitors in series and parallel for partial credit. If you can solve the problem, I will take it as sufficient evidence that you know what you're doing.

(Section B)

Given a three-volt battery and four capacitors of equal capacitance, how could you produce a potential of exactly 7.5 volts?

Take three capacitors, and immediately charge them to 3 volts (be sure to make note of the positive and negative sides!). Take one capacitor and discharge it totally. Now take the discharged one and put it in parallel with a charged one: each will now hold half the charge, and therefore half the voltage for 1.5 volts apiece. All that's left is to throw out one of the 1.5 volt capacitors and put the remaining ones in series, with the positive sides all in the same direction.

(Section C)

Given a three-volt battery and four capacitors of equal capacitance, how could you produce a potential of exactly 10 volts?

Take one capacitor, and immediately charge it to 3 volts (be sure to make note of the positive and negative sides!). Now take two others and put them in parallel with this one so that each gets 1/3 of the charge and so 1/3 of the voltage, for 1 volt. Now put one of these aside and charge the other three up to the full three volts. Put all four of these in series with the positive sides all in the same direction for the 10 volt total.

(Extra Credit: 2 Points)

What is the highest instantaneous potential that you could produce with this equipment?

15 volts: charge up each capacitor, then put them in series with your battery pack!

A Quick Electric Potential Review:

16.26)

(a) The potential difference between point 2 (just barely outside the sphere) and point 1 is not a function of the charge on the sphere: the fact is, there's just not enough space between the sphere and point 2 to amount to much potential (recall our formula $\Delta V = -\int_i^f \vec{E} \cdot d\vec{l}$). In the region between the two disks, I may approximate the disks as infinite planes of charge: notice that d and the radius of the sphere are so small compared to the radius of the disks. Then, using the formula for field close to the disk from page 590 I have $\vec{E} = \frac{\frac{Q}{A}}{2\varepsilon_o}\hat{z}$, and so between the plates I have $\vec{E} = \frac{Q}{A\varepsilon_o}\hat{z}$. Now integrating, I have

$$\Delta V = -\frac{Q_2}{A\varepsilon_0}\int_{R_2}^{R_1} \hat{z}\cdot\hat{z}dz = -\frac{Q_2(R_1 - R_2)}{A\varepsilon_0}.$$

(b) In this case, between point 3 and point 2 I have two overlaid fields: that from the plates, which is $\Delta V_C = -\frac{Q}{A\varepsilon_0}\int_{R_3}^{R_2} \hat{z}\cdot\hat{z}dz = -\frac{Q(R_3 - R_2)}{A\varepsilon_0}$. I also have the potential from a point charge in the center of the sphere, which is

$$\Delta V_S = -\frac{Q_1}{4\pi\varepsilon_0}\int_{R_3}^{R_2} \frac{\hat{z}}{(z - R_1)^2}\cdot\hat{z}dz = -\frac{Q_1}{4\pi\varepsilon_0}\int_{R_3}^{R_2} \frac{1}{(z - R_1)^2}dz = -\frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_3 - R_1} - \frac{1}{R_2 - R_1}\right)$$

By superposition, I may simply add these two up.

$$\Delta V = \Delta V_C + \Delta V_S = -\frac{Q(R_3 - R_2)}{A\varepsilon_0} - \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_3 - R_1} - \frac{1}{R_2 - R_1}\right)$$

(c) If a metal sphere were placed where the plastic sphere is, things would be almost the same except for one new potential that would appear: that of a large dipole opposing the capacitor field where the sphere is (that does not take effect inside the sphere). The potential from part a would become almost zero, since the distance between point 2 and the surface of the sphere is negligible and there is no electric field inside the sphere. The magnitude of the potential between point 3 and point 2 would become smaller since the electric field due to the surface charge would remain the same (V_S from part b) but the electric field due to the capacitor would be opposed by the induced charge on the conductor.