Ben Sauerwine
Physics 2 Recitation Notes Feb 23 2006

Your Questions from Tuesday Answered (I hope!)

Relevant Formulas for Today:

$$\vec{E}_{point} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

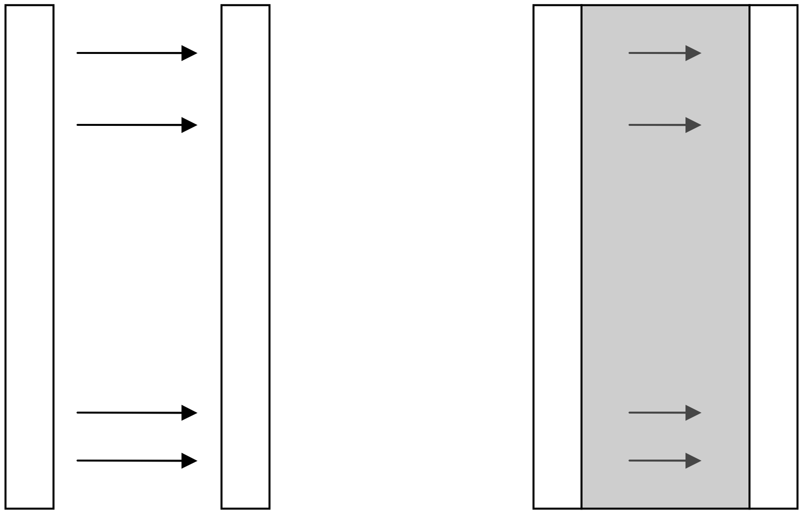
$$\Delta V = -\int_{start}^{finish} \vec{E} \cdot d\vec{l}$$

Important things to remember:

In the static case, electric field inside a conductor is always zero. This does not necessarily mean that potential inside is always zero, but it does mean that it's constant.

We typically assign the potential at infinity to be zero, but this isn't always possible, for example in the case of an infinitely long charged rod. The potential at infinity does not converge! Also note that electric field has units $\frac{N}{C} = \frac{V}{m}$, so that if you have a slowly varying potential, you may take its average over a length.

The dielectric constant of a linear dielectric is typically expressed by taking your standard electric field equations, like $\vec{E}_{point} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$, and replacing the ε_0 with ε, which is a constant of the material. It has the same units as ε_0. You may wonder: since Maxwell's laws are valid everywhere, how is it possible that I am seeing less electric field in this region? What has happened? One way of looking at it is as such: consider two charged plates before,



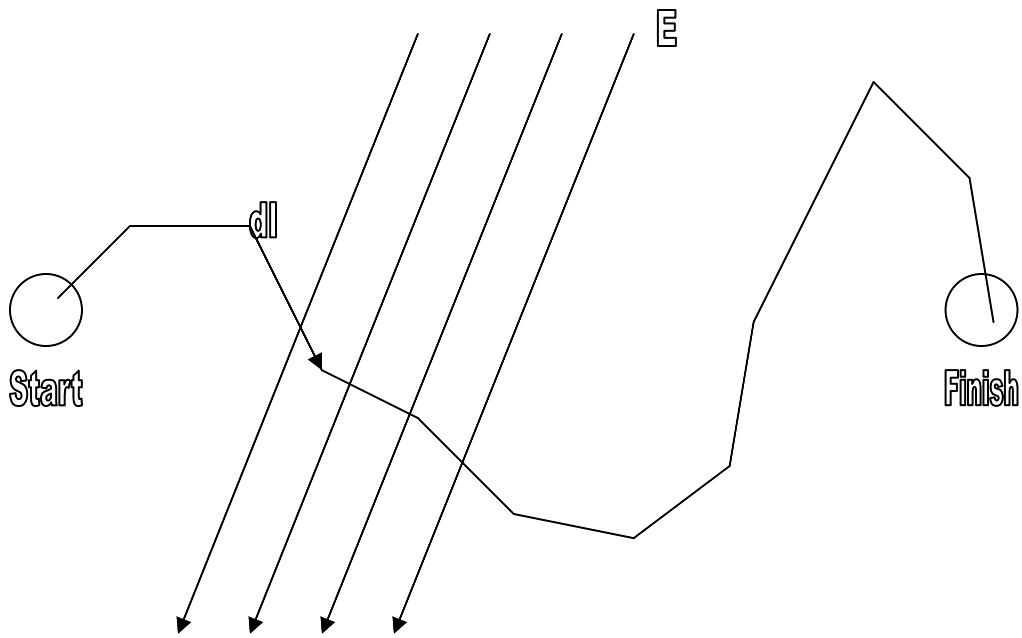
What has happened is as follows: the induced dipoles in the dielectric have rested their endpoints against the slabs, "canceling" some of the charge present there. This is essentially how the dielectric works to decrease the voltage but increase the net charge present in the conductors!

Dielectric breakdown is the critical electric field at which your dielectric stops behaving linearly and becomes ions. It's no different than pulling on a slinky until it becomes broken and will no longer pull itself back together.

Two Explanations of Potential Energy, How and Why it Works:

The electric potential, as I've said before, has units Joules per Coulomb. The formula for it is effectively just the work-energy theorem, multiplied by $\frac{1}{Coulombs}$. You may visualize its meaning as such: if you were to put a test charge with charge q at a point with a certain voltage V, the electric potential *energy* will be $U = qV$.

Let me visualize the formula, and what's happening: $\Delta V = -\int_{start}^{finish} \vec{E} \cdot d\vec{l}$. This formula gives a change in voltage with respect to the reference potential of the starting point, so that if we want the overall potential we must integrate in from infinity, where the reference potential is zero.

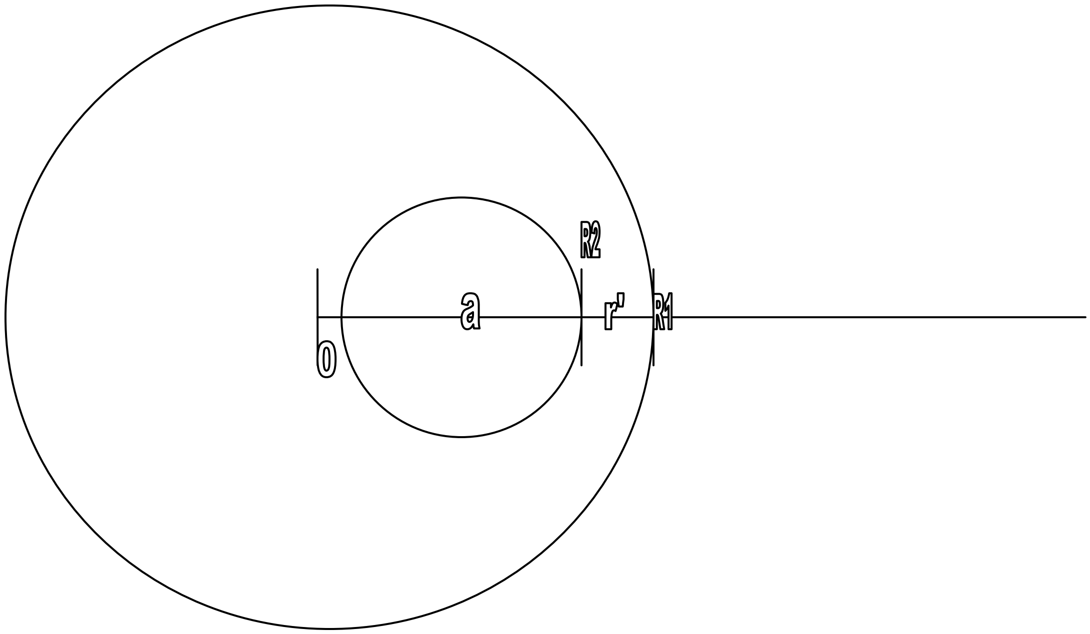


so consider I take the $d\vec{l}$ shown. Now $d(\Delta V) = -\vec{E}\cdot d\vec{l}$. My positive test charge sitting at the beginning will happily ride the electric field along its path. This means that my potential energy must be decreasing, since the field is working with me. Thus, since $\vec{E}\cdot d\vec{l}$ is positive when the vectors are going in the same direction, I must include the negative sign. In fact, a more logical way to write this might be $\Delta V = \int_{start}^{finish} \vec{E}\cdot\left(-d\vec{l}\right)$. The "finish" point may be a new variable itself. For example, when you write the potential from a point charge, you take $\Delta V(r') = \int_{\infty}^{r'} \frac{Q}{4\pi\varepsilon_0 r^2}(-dr) = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{-1}{r'} - \frac{1}{\infty}\right] = \frac{Q}{4\pi\varepsilon_0 r'}$.
I note that potential energy is path-independent, meaning that any route I decide to take from start to finish will result in the same potential energy reading in physical systems.

This formula is valid wherever the integrand (the $\vec{E}\cdot d\vec{l}$) is, which is why we must integrate over some regions separately, as when we pass through a conductor.

Quiz Solution

Consider two spherical shells, one inside another. The larger has radius R_1, the smaller has radius R_2, and their centers are a distance a apart. The larger has a charge Q_1, the smaller Q_2. If their shells do not overlap, find the potential energy in the region between the two shells (shown below) explicitly.



Section B: Use the principle of superposition to find the potential due to the big sphere at the boundary, and add the potential due to the small sphere.

$$-\int_{\infty}^{R_1} \frac{Q_1}{4\pi\varepsilon_0 r^2} dr - \int_{\infty}^{r'} \frac{Q_2}{4\pi\varepsilon_0 (r-a)^2} dr = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 (r'-a)}$$

Section C: I put the small sphere on the other side. You'll get instead:

$\frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 (r'+a)}$ by the same logic.

Magnetic Field Introduction

The Biot-Savart Law: $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$. This indicates the magnetic field caused by a moving charge. Let me do a couple of sample problems instead of actually trying to explain it.

RQ 17.2: Applying the formula to the electron with the velocity $v = 4 \times 10^6 \frac{m}{s}$, I have

$B = 10^{-7} \frac{(1.6 \cdot 10^{-19})(4 \cdot 10^6)\hat{x} \times \hat{r}}{(0.05)^2}$. I recall that the cross product has magnitude

$\left|\vec{a} \times \vec{b}\right| = |a||b| \sin\theta$, and so at each point around the circle I get

$B = 10^{-7} \frac{(1.6 \cdot 10^{-19})(4 \cdot 10^6)\sin\theta}{(0.05)^2}$, so directly in front of and behind the particle there must be no magnetic field at all. At the thirty-degree angles, I will get

$B = \pm 10^{-7} \frac{(1.6 \cdot 10^{-19})(4 \cdot 10^{6}) \sin 30^{o}}{(0.05)^2} = \pm 1.28 \cdot 10^{-17} \, Tesla$, taking the positive value above and the negative value below.

RQ 17.3: The direction of the magnetic field corresponds to the right-hand rule, so placing my thumb in the direction of positive current flow, I see that the direction of the field is up, out of the page outside the loop and down, into the page on the inside. I'll demonstrate this via experiment!