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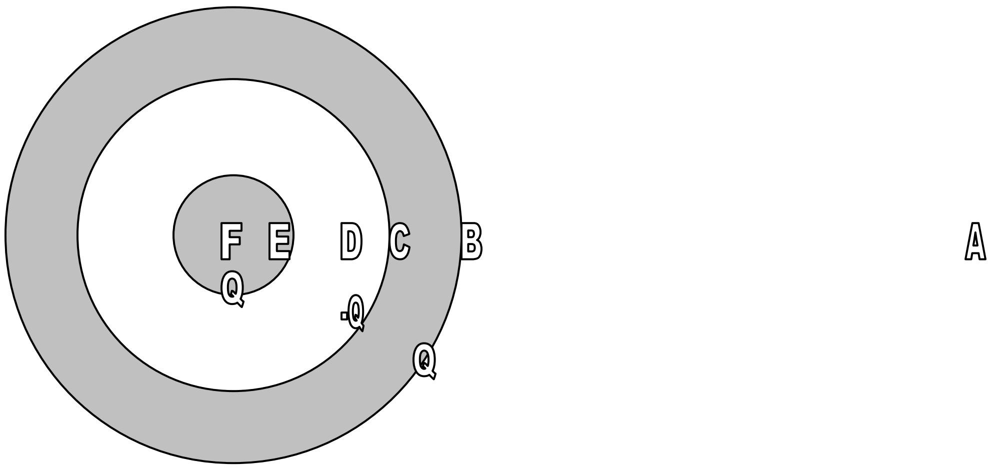
Problem 16.30 from "Matter and Interactions", modified:

Write the voltage at each point in terms of Q and the given distances.



All spheres are conducting. The innermost sphere has radius r_1, the next has inner radius r_2 and outer r_3, and A is at a distance r_4.

First, let me find the electric field everywhere. Inside the inner conductor it will be $\vec{E} = \vec{0}$. Between the inner and the outer conductors, it will be $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$. Inside the outer conductor, it will again be $\vec{E} = \vec{0}$. Outside the outer conductor, I have again $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$.

$$\Delta V_{outside} = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}[-dr] = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\bigg|_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$\Delta V_{outer\quad shell} = -\int_{\infty}^{r_3} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_3}$$

$$\Delta V_{inside} = \frac{Q}{4\pi\varepsilon_0 r_3} - \int_{r_2}^{r} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}dr = \frac{Q}{4\pi\varepsilon_0 r_3} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\bigg|_{r_2}^{r} = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r_3} + \frac{1}{r} - \frac{1}{r_2}\right]$$

$$\Delta V_{inner\quad shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_3} + \frac{1}{r_1} - \frac{1}{r_2}\right]$$

Write the voltage V at which point the air will break down and go conducting, or "Corona Discharge" will occur (this is very cool, and very dangerous: you should read about it at Wikipedia). First find Q, and then substitute this Q into the formula you found earlier. Recall that this occurs when the electric field strength is $3\times10^{6}\frac{V}{m}$, or in this case when $-\frac{\partial V}{\partial x}=3\times10^{6}\frac{N}{C}$. Evaluate for

$r_1=1cm, r_2=2cm, r_3=3cm$

Consider where the voltage is most likely to break down: it will break down at the highest-field point first: the closest to the center. So, I check the field at the center

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1^{\,2}}=3\times10^{6} \qquad Q=4\pi\varepsilon_0 r_1^{\,2}\left[3\times10^{6}\right]$$

Now I substitute this:

$$\Delta V_{inner \quad shell}=\left[3\times10^{6}\right]r_1^{\,2}\left[\frac{1}{r_3}+\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Substituting, I get $25000V$. This seems roughly on the scale of high-voltage power lines, which you often hear hissing and cracking at about $130000V$, especially given the small scale. However, coronas are often observed on apparatus as low as $2000V$! This is a function of surface irregularities where charge may build up, which our ideal model has none of.

If a proton from some ionized Hydrogen were to find itself at rest just outside the inner surface, at what velocity would it hit the outer surface neglecting interactions on the way out? (Ignore relativistic effects)

$$\frac{1}{2}mv^2=qV_{inner}-qV_{outer}=q\left[\frac{1}{r_3}+\frac{1}{r_1}-\frac{1}{r_2}\right]-q\left[3\times10^{6}\right]r_1^{\,2}\left[\frac{1}{r_3}\right]$$

$$v^2=\frac{2q}{m}\left[3\times10^{6}\right]r_1^{\,2}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]=\frac{2\times1.6\times10^{-19}}{1.7\times10^{-27}}\left[3\times10^{6}\right]\left[0.01\right]^2\left[\frac{1}{0.01}-\frac{1}{0.02}\right]$$

$$v=1.6\times10^{6}\frac{m}{s}$$

This is roughly half the speed of light, and of course I have ignored the relativistic kinetic energy.