Physics 2 for Engineers Final Exam Review
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Coulomb's Law

- $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$
- $\vec{E} = \int d\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$
  1. Rewrite the integral in terms of a representative dq with its r and $\hat{r}$.
  2. Use the chain rule to rewrite dq in terms of a coordinate differential.
  3. Apply limits corresponding to where this coordinate begins and ends and integrate.
- **Examples:**
  - Recitation 3 (Field of Semicircular Line Charge)
  - Recitation 3 (Field of Straight Line Charge)

Gauss's Law

- $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$
  - Gaussian Sphere: $4\pi r^2 E(r) = \frac{q_{enc}(r)}{\varepsilon_0}$ where $\vec{E}(r) = E(r)\hat{r}$
  - Gaussian Cylinder: $2\pi r E(r) = \frac{q_{enc}(r)}{\varepsilon_0}$ where $\vec{E}(r) = E(r)\hat{r}$
  - Gaussian Pillbox: $E_{top}(z) - E_{bot} = \frac{q_{enc}(z)}{\varepsilon_0}$ where $\vec{E}(z) = E(z)\hat{k}$
    - Conductors Present: Place one surface inside the conductor so that you may set $E_{bot} = 0$.
    - 180-degree rotation symmetry present: Place one surface so that you may set $E_{bot} = -E_{top}(z)$.
  - Inside a Dielectric: $\varepsilon_0 \to K\varepsilon_0$.
- **Algorithm:**
  1. Draw your Gaussian Surface based on the symmetry present in the problem.
  2. Select the appropriate simplified Gauss's Law. Use $\varepsilon_0 \to K\varepsilon_0$ in a region occupied by dielectric.
  3. Write q_{enc} as a possibly piecewise function of the free parameter of your Gaussian Surface.
  4. Simplify and solve for electric field.
- **Examples:**
  - Spherical Symmetry: Recitation 7 (Two Spherical Shells)
  - Cylindrical Symmetry: Assignment 3 (Special Problem)

- Planar Symmetry (Conductors): Recitation 7 (Charged Slab Beside Conducting Slab)
- Planar Symmetry (180-degree reflection): Recitation 6 (Electric Field of Uniformly Charged Infinite Slab)
- Dieletrics Present: Recitation 11 (Dielectrics in Coaxial Cable)

Electric Potential

- **By Superposition:**
  - $V = \int dV = \int \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$
    1. Rewrite in terms of a representative dq with its r.
    2. Use the chain rule to rewrite dq in terms of a coordinate differential.
    3. Apply limits corresponding to where the coordinate begins and ends and integrate.
  - Useful in point or line distributions where electric field cannot be found easily with Gauss's Law.
  - **Example:** Recitation 9 (23.79)
- **By Integration:**
  - $\Delta V_{start \to finish} = -\int_{start}^{finish} \vec{E} \cdot d\vec{\ell}$
    1. Determine $\vec{E}$ everywhere.
    2. Write $d\vec{\ell}$ as the product of the vector direction you are traveling and the way your coordinate differential is changing. *In most logically chosen coordinate systems, this will be positive.* Ensure that it has units of length.
    3. Apply limits corresponding to where your path begins and ends and integrate.
    4. You may have to apply a reference potential at the starting point.
  - Useful where electric field is given, in infinite distributions of charge, where a reference potential is given, or where electric field is highly uniform.
  - **Example:** Recitation 11 (Dielectrics in Coaxial Cable)
- **Energy**
  - External Potential: $U = QV$
  - Of a Distribution: $U = \frac{1}{2} QV$

Capacitors

- **Capacitance**
  - $Q = C\Delta V$
  - **Algorithm:**
    1. If one is not given, assume a charge on each of two surfaces.
    2. Use Gauss's Law to find electric field between these surfaces.
    3. Use Electric Potential by Integration with the electric field to find the potential between these surfaces.

4. Use $Q = C\Delta V$ to solve for the capacitance. In all cases covered in this course, the final result should not depend on the charge Q assumed.
   - **Challenge:** Can you think of a case that would?
     - I wouldn't want to get you Hooked.
     - Maybe you should think about this in Spring.
- Energy $U = \frac{1}{2}CV^2 = \frac{1}{2}QV$
- **Examples:**
  - Recitation 10 (Spherical Capacitor)
  - Recitation 10 (Cylindrical Capacitor)

- **In Circuits**
  - **In Series:**
    - $C_{eq}^{-1} = \sum C_i^{-1}$.
    - Charge is equal across all capacitors.
  - **In Parallel:**
    - $C_{eq} = \sum C_i$
    - Voltage is equal across all capacitors.
  - **Examples:**
    - Homework 5 (24.60)
    - Homework 5 (24.16)

Resistors

- **In Series:**
  - $R_{eq} = \sum R_i$
  - Current is equal across all resistors.
- **In Parallel:**
  - $R_{eq}^{-1} = \sum R_i^{-1}$
  - Voltage drop is equal across all resistors.
- **Ohmic Resistors:**
  - $\vec{J} = \rho\vec{E}$
  - $\Delta V = EL$
  - $R = \frac{\rho L}{A}$
- **Example:** Recitation 12 (25.58)
- **Challenge:** http://www.xkcd.com/356/
  - If you can solve this with pen and paper, you can probably just stop studying now.

Kirchoff's Laws:

- **Circuit Elements**
  - **Resistors**
    - $\Delta V = IR$
  - **Capacitors**
    - $\Delta V = \frac{Q}{C}$
    - Instantaneously, you may think of these as batteries.
    - Long term DC limit: Current through these should go to zero.

- To get the appropriate sign on your voltage, let electric field guide you. If you travel against the electric field (against the current in a resistor, or from negative to positive plates in a resistor or battery), ΔV should be positive. Otherwise, ΔV should be negative.

- **Power in a Circuit Element:** $P = I\Delta V$.
  - Always dissipative for a resistor.
  - For batteries or capacitors, this is provided when current flows from the positive side and stored when current flows into the positive side.
- **Loop Rule:** $\Delta V = 0$ for a closed loop.
- **Node Rule:** $\sum I_{in} = \sum I_{out}$ for any point on the circuit.
- **Time Constant:** $\tau = RC$ for a resistor and capacitor in series with a battery. This is the time necessary for the capacitor's charge to become $\left(1 - e^{-1}\right)\left(Q_f - Q_i\right)$.
- **Algorithm:**
  1. Label all currents in the circuit, being certain to include a direction.
  2. Write a linearly independent set of loop and node rules.
  3. Solve the set of equations given for currents.
  4. Interpret the currents: you chose the incorrect direction in part (1) if your result is negative.
  5. Solve for any remaining required quantities with these currents.
- **Examples:**
  - Solving a Circuit: Recitation 14 (26.22)
  - Power: Assignment 7 (26.84)
  - Time Constant: Recitation 15 (Charging Capacitor)

Gauss's Law for Magnetism

- $\oint \vec{B} \cdot d\vec{A} = 0$
- All procedures are the same as for Gauss's Law for Electric Fields, but magnetic field is used instead. Magnetic charge enclosed should always be zero.
- **Example:** Recitation 15 (27.13)
- **Challenge:** Why is magnetic charge enclosed always zero?
  - Please claim your Nobel Prize in Physics for your satisfactory answer.

Magnetic Force

- **On a point charge:** $\vec{F} = q\vec{v} \times \vec{B}$
- **On a wire segment:** $\vec{F} = I\vec{\ell} \times \vec{B}$
- **Torque on a wire loop:** $\vec{\tau} = \vec{\mu} \times \vec{B}$ where $\vec{\mu} = IA\hat{n}$
- **Examples:**
  - On a point charge: Recitation 17 (27.21)
  - Net force on a wire: Recitation 17 (27.39)
  - Torque on a wire: Recitation 18 (27.42)
  - On a wire, integral: Recitation 18 (27.81)

Current and Magnetic Fields

- **Biot-Savart Law**
  - For a wire: $\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{\ell} \times \hat{r}}{r^2}$

- For a point charge: $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$
- Useful when you have a wire with a complicated shape or a lone point charge creating a magnetic field.
- **Algorithm:**
    1. Write $d\vec{\ell}$, r and $\hat{r}$ for a sample piece of the wire. If the wire is composed of multiple regimes, you may have to break the wire up into several integrals and use superposition. $d\vec{\ell}$ should point in the direction that the positive current travels.
    2. Use the chain rule to write $d\vec{\ell}$ in terms of a coordinate differential and a vector.
    3. Simplify the cross product.
    4. Perform the integral, using as your limits where your coordinate differential begins and ends.
- **Examples:**
    - For a point charge: Recitation 19 (Forces between Two Particles)
    - For a wire: Recitation 21 (Field of a Current Loop)

- **Ampere's Law**
    - $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$
        - Long straight wire (circular path):
            - $2\pi r B(r) = \mu_0 I_{enc}(r)$
            - $\vec{B}(r) = B(r)\hat{\theta}$
        - Solenoid or Sheet of current (rectangular path):
            - $B_{top}(z) - B_{bot} = \mu_0 I_{enc}(z)$
            - Solenoid: Use the fact that magnetic field outside a long solenoid equals zero to set magnetic field outside equal to zero.
            - Sheet: Use the fact that you have 180-degree rotational symmetry to set $\vec{B}(z) = -\vec{B}(-z)$.
        - Displacement Current:
            - $\oint \vec{B}\cdot d\vec{\ell} = \mu_0\left(I_{enc} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$
            - The term $\varepsilon_0 \frac{d\Phi_E}{dt}$ is the displacement current, in Amps.
            - To use this in Ampere's Law, simply find the flux through your Amperian Loop as a function of time and differentiate, then add it to your Ampere's Law in step 3 of the algorithm below.
    - **Algorithm:**
        1. Draw an Amperian loop appropriate to the symmetry of your problem. You must include direction and a right-handed normal.
        2. Simplify Ampere's Law with the appropriate symmetries as above.

3. Rewrite I_{enc} for your family of loops. Discontinuous regimes of current may mean that this is a piecewise function. Remember that positive current flows in the direction of the normal you chose in part (1).
4. Solve for $B(r)$ and interpret the direction.

- **Examples:**
  - Long Straight Wire: Recitation 20 (28.74)
  - Solenoid: Recitation 20 (Rotating Charged Cylinder)
  - Sheet of Current: Recitation 21 (Slab of Uniform Current)
  - Displacement Current:
    - Recitation 25 (29.34)
    - Recitation 25 (Solenoid)

Faraday's Law

- $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{A}$
- **Lenz's Law:**
  - Determine the initial "direction" of the external magnetic flux. (Flux does not actually have a direction: it is a scalar quantity. Nonetheless, it is positive or negative with respect to a chosen normal, so by direction I mean flux times normal.)
  - Determine the final "direction" of the external magnetic flux.
  - Determine the change in flux as final minus initial.
  - The induced current would like to create a flux opposing this.
  - Use the right-hand rule to find the direction of the induced current (or the sense of the induced EMF).
- **Induced EMF:**
  - We may identify $EMF = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{A}$, where EMF is in a right-handed sense with respect to the chosen normal in $d\vec{A}$.
  - **Algorithm:**
    1. First, find magnetic field everywhere. This may be a function of time.
    2. Find $\oint \vec{B} \cdot d\vec{A}$ as a function of time. Note that the normal may be a function of time (as in a rotating loop) or the surface itself may be a function of time.
    3. Take negative the derivative of this result and interpret it as an electromotive force.
- **Induced Electric Field:**
  - We need not interpret the induced electric fields as an EMF as in the previous section. We may simply report the electric fields.
  - **Algorithm:**
    1. Use the symmetries from Gauss's Law for Electric Fields to determine the allowed directions of Electric Field.
    2. Replace the left-hand side of Faraday's Law as written above with a form appropriate to the symmetry from part (1).

3. Follow the algorithm from Induced EMF.
4. Solve for electric field on the left-hand side using the result from part (3) on the right-hand side of Faraday's Law.

- **Examples:**
  - Lenz's Law: Recitation 22 (29.17)
  - Induced EMF:
    - Changing Magnetic Field: Recitation 22 (29.8)
    - Changing Area: Recitation 22 (29.9)
  - Induced Electric Field:
    - Recitation 24 (Coaxial Cable)
    - Assignment 12 (29.29)

Motional EMF

- $EMF = \int_{start}^{finish} \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$
- Useful for finding induced EMF when a conductor is moving in a magnetic field. $\vec{v}$ is the velocity of the conductor.
- Most Frequently: $EMF = VBL$ when $\vec{v} \perp \vec{B}$ and $\vec{v} \times \vec{B} \parallel d\vec{\ell}$.
- Interpret the result as a battery with the positive terminal at "finish" and the negative terminal at "start".
- **Examples:**
  - Recitation 23 (Rotating Rod)
  - Assignment 12 (Special Problem)

Waves

- Any function that satisfies $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ for some characteristic velocity v.
- Any function that can be parameterized as $y(x,t) = y(x + vt)$.
  - **Challenge:** Can you show that this is a sufficient condition for satisfying the equation above? You can do it in just one line.
- **For a wave function** $y(x,t) = A\cos(kx - \omega t)$
  - A = amplitude
  - k = wavenumber
  - ω = angular frequency
  - $\frac{2\pi}{k}$ = wavelength
  - $\frac{2\pi}{\omega}$ = period = $\frac{1}{f}$ = inverse of frequency
  - $\frac{\omega}{k} = \lambda f$ = velocity of propogation
  - The fact that you have a function of $kx - \omega t$ indicates that the wave is propogating to the right. $kx + \omega t$ would have indicated a left-propogating wave.
  - $\frac{\partial y}{\partial t}$ gives the transverse velocity.

- For a mechanical wave on a wire or rope, $v = \sqrt{\frac{S}{\mu}}$ for tension S and linear mass density μ.
- Average power transmitted by a mechanical wave: $P = \frac{1}{2}\mu v \omega^2 A^2$
- **Examples:**
  - Recitation 27 (First problem)

Final Thoughts:

- Do the practice final exam provided. If you can't do a problem on the practice final, you can be assured that a problem like it will be on the upcoming final and that you need to study that material.
- The formulae provided on this sheet are probably sufficient, but if you don't know how to use them they will be useless to you. If there are a few areas you are shaky on, you may be better off bringing some worked examples or more detailed notes on how to use the formulae here on your formula sheet.
- On your final exam, be sure not to write answers where you set a vector quantity equal to a scalar quantity. Always include units on your answer,

Good Luck!
~Ben Sauerwine