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Recitation Notes April 6

Your Requests:

19.16) a) Find the capacitance of two spherical shells with the same center of inner radius indexed 1 and outer radius indexed 2. The inner carries positive charge Q and the outer negative charge Q.

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\Delta V = -\int_{r_2}^{r_1} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot\hat{r}dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_{r_2}^{r_1} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{1}{r_1}-\frac{1}{r_2}\right)^{-1}$$

b) Show that when the radii are very near one another and large that the capacitance is much like that of the parallel plate capacitor with the same surface area and separation.

$$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{1}{r_1}-\frac{1}{r_2}\right)^{-1} = 4\pi\varepsilon_0\left(\frac{r_2-r_1}{r_1 r_2}\right)^{-1} =$$

$$A = 4\pi r^2 \quad r_1 \approx r_2 \quad s = r_2 - r_1$$

$$C \approx 4\pi\varepsilon_0 \frac{\left[r_1 r_2 \to r^2\right]}{\left[r_2 - r_1 \to s\right]} = \frac{A\varepsilon_0}{s}$$

20.12) A zero-resistance metal rod is sliding down between two zero-resistance metal armatures over a base with resistance R. The rod has mass M and length L and travels with constant velocity. Perpendicular to this equipment is a uniform magnetic field B pointing out of the page.

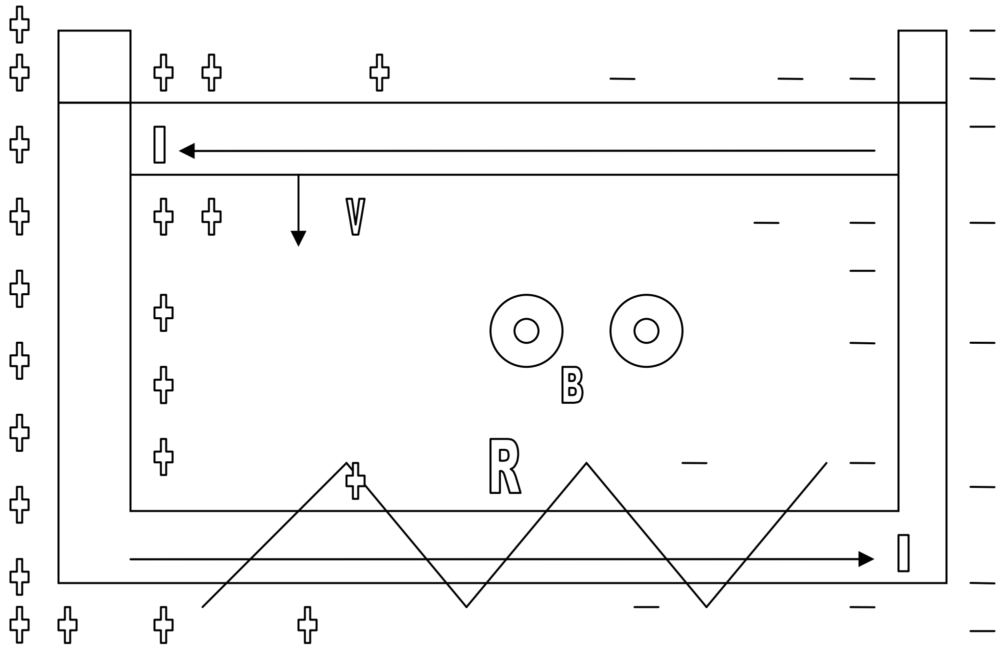
a) To find the current through the resistor, I consider the equation

$$d\vec{F}_{Mag} = Id\vec{l}\times\vec{B} \quad \vec{F}_{Mag} = \int d\vec{F}_{Mag} \quad \left|\vec{F}_{Mag}\right| = ILB$$

$$\left|\vec{F}_{Gravity}\right| = MG = \left|\vec{F}_{Mag}\right| = ILB$$

$$I = \frac{MG}{LB}$$

b) Draw a diagram showing the conventional current direction and surface charge distribution.



IMPORTANT POINT:

Wait a minute! We've been duped! Someone told us that magnetic field was non-potential, but here we are with real potential and real energy from a magnetic field! What's gone horribly wrong? Where is this energy coming from?

In fact, magnetic forces do no work. In this case, the magnetic force is perpendicular to the direction of motion. The work here was done by gravity, not the magnetic force! The magnetic force simply redirected the work done by gravity.

c) What must the constant velocity be?

$$P = IV = I^2 R \quad V = IR$$

$$Work = Fdx$$

$$I^2 R = Power = Work / \sec = F \frac{dx}{dt} = MGv$$

$$\frac{I^2 R}{MG} = v$$

d) Show that the rate of change of the gravitational energy of the universe is equal to the rate of energy dissipation in the resistor.

$$MGH = G.P.E$$

$$MG \frac{\partial H}{\partial t} = \frac{\partial G.P.E}{\partial t} = MG \frac{I^2 R}{MG} = I^2 R = P$$

20.20) Find the magnetic field in a mass spectrometer given the accelerating potential, diameter of arc, mass of particle and that it is singly-positive charged.

$$U = \frac{1}{2}mv^2 = q_p \Delta V$$

$$F = q\vec{v} \times \vec{B} = q_p vB = ma = m\frac{v^2}{r} = m\frac{v^2}{\frac{d}{2}}$$

Substituting,

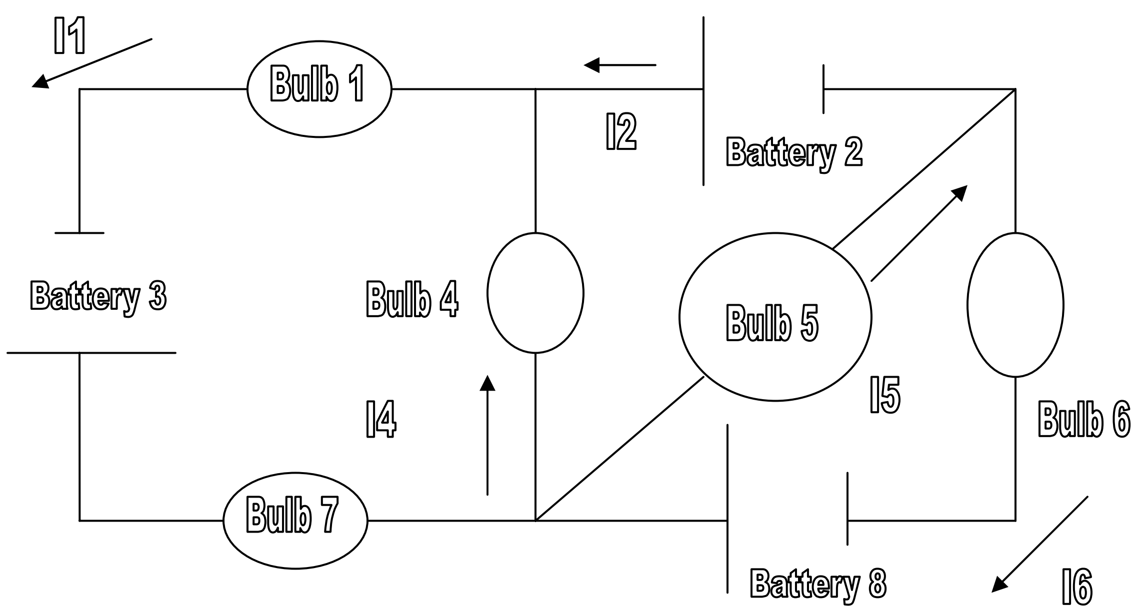
$$v = \sqrt{\frac{2q_p \Delta V}{m}} \quad B = m\frac{2v}{q_p d} \quad B = m\frac{2}{q_p d}\sqrt{\frac{2q_p \Delta V}{m}} = \frac{2}{d}\sqrt{\frac{2m\Delta V}{q_p}}$$

Quiz) Given a circuit diagram, the problem was to determine a set of equations that could be solved to give the currents through each portion. The key points to understand were:

1) **You need the same number of equations unknowns.**
2) **You need several loop rule equations and several node rule equations. You must be certain to include no redundant information, e.g., loops that are linear combinations of one another, and to include each variable at least once.**
3) **You should understand the type of behavior over each type of component**

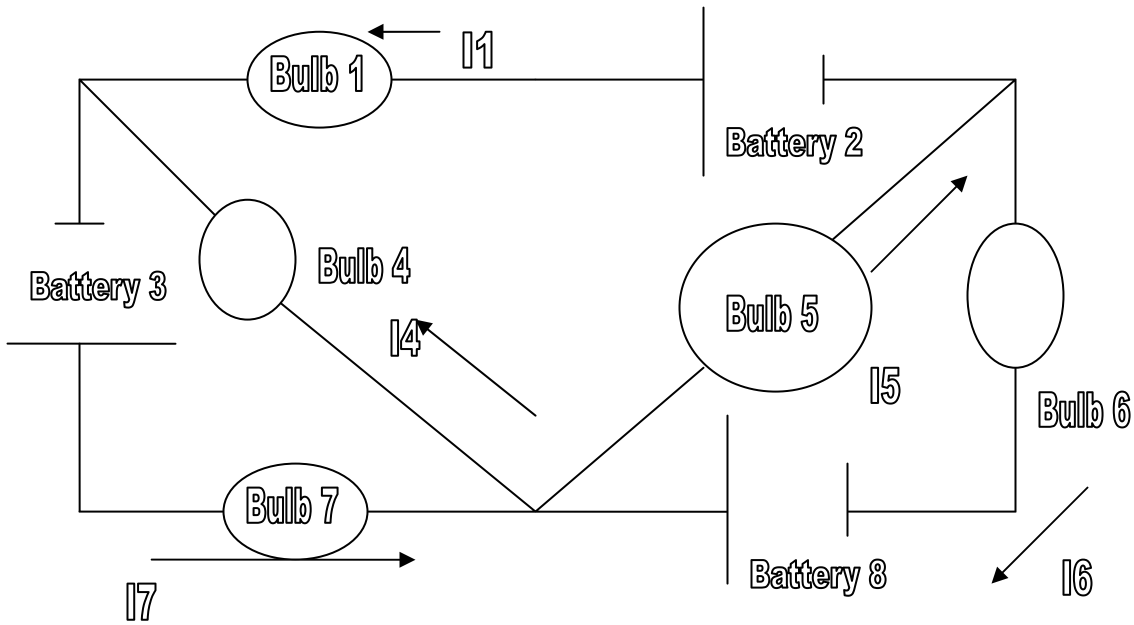
Many different solutions are possible. Here are two.

Section B:



$$-I_1R_1 + V_3 - I_1R_7 - I_4R_4 = 0$$
$$V_2 + I_4R_4 - I_5R_5 = 0$$
$$V_8 - I_5R_5 - I_6R_6 = 0$$
$$I_1 = I_2 + I_4$$
$$I_5 = I_2 + I_6$$

Section C:



$$-I_1R_1 + V_2 + I_4R_4 - I_5R_5 = 0$$
$$V_3 - I_7R_7 - I_4R_4 = 0$$
$$V_8 - I_5R_5 - I_6R_6 = 0$$
$$I_7 = I_4 + I_1$$
$$I_5 = I_1 + I_6$$