

Today we did two sample problems:

19.20)

a) I need to solve for a total of 3 amperages. Thus, I should need three equations in three unknowns. Using the Loop rule for the first equation and the Node rule for the last equation, I have

$$1.5 + 1.5 - 15I_4 - 30I_2 - 20I_2 = 0$$

$$1.5 + 1.5 - 15I_4 - 20I_1 = 0$$

$$I_4 = I_1 + I_2$$

Solving, I get

$$I_1 = 0.073 \quad I_2 = 0.029 \quad I_4 = 0.102$$

b) Using $B = \frac{\mu_0 2I}{4\pi r}$ (assuming the wires are very long and very low resistance and that the compasses are equally spaced from the wires), I see that

$$3 \frac{\pi}{180} = \arctan\left(\frac{B}{B_{North}}\right) = \arctan\left(\frac{\frac{\mu_0 2I_2}{4\pi r}}{B_{North}}\right), \text{ and based on this I want to find}$$

$$\arctan\left(\frac{\frac{\mu_0 2I_4}{4\pi r}}{B_{North}}\right). \text{ To do so, I need to solve for } rB_{North}. \text{ Using my equation above, I have}$$

$$\tan\left(3 \frac{\pi}{180}\right) = \frac{\mu_0 2I_2}{4\pi rB_{North}} \quad rB_{North} = \frac{\mu_0 2I_2}{4\pi \tan\left(3 \frac{\pi}{180}\right)}. \text{ Finally, I have as my angle:}$$

$$\arctan\left(\frac{\mu_0 2I_4}{4\pi rB_{North}}\right) = \arctan\left(\tan\left(\frac{3\pi}{180}\right) \frac{I_4}{I_2}\right) = 0.182 \text{ radians} = 10.44^\circ. \text{ The compass}$$

needle 1 deflects clockwise, while the compass needle 2 deflects clockwise.

$$\text{c) } I = |q|nAuE \quad I_2 = \left(1.6 \cdot 10^{-19} \frac{C}{e^-}\right) \left(8 \cdot 10^{28} \frac{e^-}{m^3}\right) \left(6 \cdot 10^{-10} m^2\right) \left(3 \cdot 10^{-5} \frac{s}{N} \frac{m}{C}\right) E$$

$$E = 125 \frac{N}{C}$$

d) In resistor 1, amperage is greater and everything else is the same. Thus, electric field can only be greater.

e) If $\Delta V = EL$ in this case since electric field is constant, and $\Delta V = IR = 0.58$, then

$$L = \frac{\Delta V}{E} = \frac{0.58}{125} m = 5mm .$$

f) Since the voltmeter is connected backwards, the sign must be negative. Going backwards over the batteries, I pick up -3 volts. Going backwards over resistor 2, I get $V = IR = 0.58$ volts. Thus, I must have -2.48 volts.

g) The overall voltage drop between the beginning and end of the same battery is 1.5 volts. Now using an electron charge $U = q\Delta V = (1.6 \cdot 10^{-19} C) \left(1.5 \frac{J}{C}\right) = 2.4 \cdot 10^{-19} J$.

h) Power output is $P = I\Delta V = 0.102A \cdot 3V = .306W$.

19.22)

a) Capacitance is given by $C = \frac{Q}{V} = \frac{K\epsilon_0 A}{s} = \frac{(1)\epsilon_0(4m \cdot 3m)}{0.0025m} = 4800\epsilon_0 \frac{C}{V}$. At 9 volts, this

means that my charge per plate will be $9V \cdot 4800\epsilon_0 \frac{C}{V}$ C. Now I know that in the

parallel-plate capacitor, I have $E = \frac{Q}{A\epsilon_0} = \frac{9V \cdot 4800\epsilon_0 \frac{C}{V}}{(4m \cdot 3m)\epsilon_0} = 3600 \frac{N}{C}$, which is far less

than the $3 \cdot 10^6 \frac{N}{C}$ necessary to ionize the air.

b) From part a, the charge on the positive plate will be $9 \cdot 4800\epsilon_0 C = 3.82 \cdot 10^{-7} C$

c) Inserting the plastic sheet decreased the voltage somewhat. Let me use:

$$\int_0^{1.5mm} \left(3600 \frac{N}{C}\right) dl + \int_{1.5mm}^{2.5mm} \left(3600 \frac{N}{C} \cdot \frac{\epsilon_0}{\epsilon}\right) dl = 3600 \frac{N}{C} (1.5mm) + 3600 \frac{N}{C} \cdot \frac{1}{5} (1mm) = 6.12V .$$

d) Discharging over a 100 megohm resistor, I see that with my capacitance

$$C = \frac{3.82 \cdot 10^{-7} C}{6.12V} = 6.24 \cdot 10^{-8} F \text{ and my resistance } 10^8 \Omega, \text{ my time constant is}$$

$$6.24 \text{ sec} = RC . \text{ Now writing } \Delta V = \frac{Q}{C} = 6.12 \left(1 - e^{-\frac{t}{6.24}}\right) \text{ Volts and substituting 3}$$

seconds, I have $\Delta V = 6.12 \left(1 - e^{-\frac{3}{6.24}}\right) = 2.34 \text{ Volts}$. Indeed, when one tries to measure

voltage over a capacitor, one often finds that the voltage drops substantially over the time of measurement! Thus our dilemma: for a more accurate voltage reading, we need more amperage. However, for a more non-invasive reading, we need more resistance. These are not consistent variables, so we find that what is necessary is simply more advanced and precise internal circuitry for the voltmeter.