Recitation Notes April 27
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Ampere's Law: This is practically the same as Gauss's law, except instead of a spherical surface we take a two-dimensional sheet, instead of a surface normal we take a path, and instead of charge inside we consider piercing current.

In a way, all we're doing is taking the *average* magnetic field about the loop and asking, "If I were to say that this average were due to a single current, what would that current be?"

For justification, propose that you added up all of your little magnetic fields in the direction of an arbitrarily-shaped loop, and then deformed it to a circle. You now take the average field about the circle. Behold, from a single long wire:

$$2\pi r \cdot B_{avg} = \mu_0 I_{central}$$

$$B_{avg} = \frac{\mu_0 I_{central}}{2\pi r} = \mu_0 \frac{2 I_{central}}{4\pi r}$$

In a way, then, this is a process of averaging, but that works exactly (actually, the "real" result comes from differential geometry).

(Griffiths 5.16) A large parallel-plate capacitor with uniform surface charge density σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v.

(a) Find the magnetic field everywhere.

Using Ampere's law, I form a rectangular sheet of length L and center it on either sheet, with normal parallel to the direction of motion. By symmetry, I expect that the magnetic field is perpendicular to the vertical edges and uniform and parallel along the top edges of the loop. The current passing through it is $\pm \sigma L v$. I have sides of length L on top and bottom, each getting uniform field B. Now, by Ampere's law:

$$2BL = \mu_0 (\sigma L v)$$

$$B = \frac{\mu_0 \sigma v}{2}$$

B will point in a direction appropriate to the right-hand rule. I see that magnetic field is not a function of distance from the sheet, then, and so above the top sheet I will have by superposition

$$B_{above} = \frac{\mu_0 \sigma v}{2} + \frac{\mu_0 (-\sigma) v}{2} = 0$$

$$B_{between} = \mu_0 \sigma v$$

$$B_{below} = \frac{\mu_0 \sigma v}{2} + \frac{\mu_0 (-\sigma) v}{2} = 0$$

the direction of the magnetic field will be appropriate to the right hand rule for either sheet, perpendicular to both the sheets and direction of motion, e.g. $\hat{B} = \hat{v} \times \hat{n}$, where the velocity and surface normals are assumed perpendicular.

(b) Find the magnetic force per unit area on the upper plate, including direction.

$$\frac{F}{A} = \sigma \left| \vec{v} \times \vec{B}_{ONE \quad plate} \right| = \frac{\sigma^2 v^2 \mu_0}{2}$$

Using the right hand rule, I see that the magnetic force pushes the plates apart.

(c) At what speed v would the magnetic force balance the electrical force?

Obviously, it must be c. To substantiate, I use the electric field from a large plate:

$$\frac{\sigma^2 v^2 \mu_0}{2} + \sigma E = 0$$

$$\frac{\sigma^2 v^2 \mu_0}{2} + \sigma \frac{(-\sigma)}{2\varepsilon_0} = 0 = v^2 \mu_0 - \frac{1}{\varepsilon_0}$$

$$\frac{1}{\mu_0 \varepsilon_0} = v^2$$

It turns out, $\frac{1}{\mu_0 \varepsilon_0} = c^2$. The speed of light is not a constant free to change independently of the rest of the universe!

(Chabay, Sherwood 21.12) Remember how I showed you that you can find electric field knowing only the charge distribution and some symmetry relations from Gauss's Law? Let's do it in this trivial example with Ampere's law.

By symmetry:

$$B(r) \cdot L(r) = \mu_0 I_{enc}(r)$$

$$B(r) = \mu_0 \frac{I_{enc}(r)}{L(r)} \quad L(r) = 2\pi r \quad I_{enc}(r) = \int_0^r \frac{I}{\pi R^2} 2\pi r dr = \frac{I}{\pi R^2} \pi r^2 = \frac{r^2}{R^2}$$

$$B(r) = \frac{\mu_0}{2\pi} \frac{r}{R^2}$$

inside the wire. Outside the wire, incidentally, the result is even easier:

$$B(r) \cdot L(r) = \mu_0 I_{enc}(r)$$

$$B(r) = \mu_0 \frac{I_{enc}(r)}{L(r)} \quad L(r) = 2\pi r \quad I_{enc}(r) = I$$

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

this is identical to the result for a long wire.