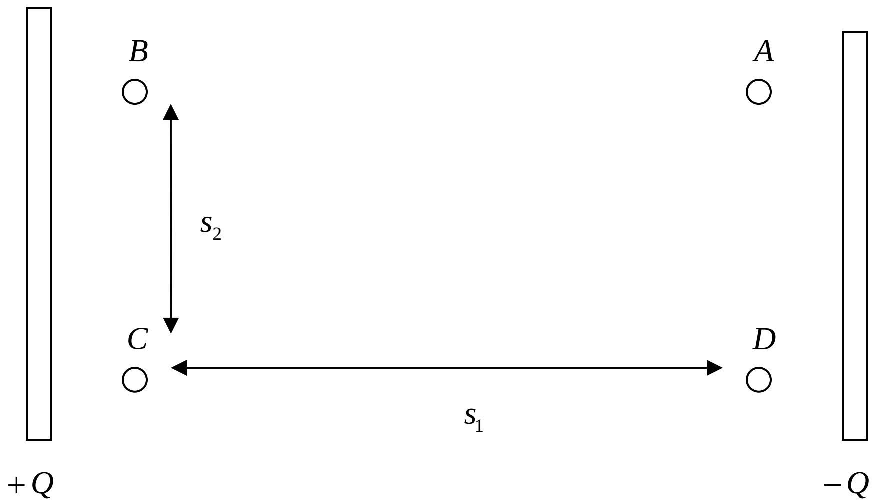
Ben Sauerwine

Chabay and Sherwood
Problem 16.P.37

A capacitor consists of two charged disks of radius $R >> s$. The magnitude of the charge on each disk is Q.



(a) Show that $\Delta V = V_c - V_A$ is the same for these paths by calculating ΔV along each path.

Path 1: $A \to B \to C$
Path 2: $A \to C$
Path 3: $A \to D \to B \to C$

Inside this capacitor, I expect a constant electric field $\vec{E}_{cap} = \frac{Q}{A\varepsilon_0}\hat{i}$, where I define the positive x-direction to be pointing from the positive capacitor to the negative capacitor and the positive y-direction to be pointing up the paper.

Now, I write: $\Delta V = -\int_{start}^{finish} \vec{E}_{cap} \cdot d\vec{l} = -\vec{E}_{cap} \cdot \int_{start}^{finish} d\vec{l} = -\vec{E}_{cap} \cdot \vec{l}_{start \to finish}$

I was able to factor out the capacitor field since it is constant in this problem.

Next, I'll tabulate a bunch of results using the formula I just derived and string them together:

$$A \to B \qquad = -\frac{Q}{A\varepsilon_0}\hat{i}\cdot\left(-s_1\hat{i}\right) = \frac{Q}{A\varepsilon_0}s_1$$
$$A \to C \quad = -\frac{Q}{A\varepsilon_0}\hat{i}\cdot\left(-s_1\hat{i} - s_2\hat{j}\right) = \frac{Q}{A\varepsilon_0}s_1$$
$$A \to D \qquad = -\frac{Q}{A\varepsilon_0}\hat{i}\cdot\left(-s_2\hat{j}\right) = 0$$
$$B \to C \qquad = -\frac{Q}{A\varepsilon_0}\hat{i}\cdot\left(-s_2\hat{j}\right) = 0$$
$$B \to D \quad = -\frac{Q}{A\varepsilon_0}\hat{i}\cdot\left(s_1\hat{i} - s_2\hat{j}\right) = -\frac{Q}{A\varepsilon_0}s_1$$
$$C \to D \qquad = -\frac{Q}{A\varepsilon_0}\hat{i}\cdot\left(s_1\hat{i}\right) = -\frac{Q}{A\varepsilon_0}s_1$$

Now adding up the results from each sub-trip, I have

$$A \to B \to C = A \to B + B \to C = \frac{Qs_1}{A\varepsilon_0} + 0$$

$$A \to C = \frac{Qs_1}{A\varepsilon_0}$$

$$A \to D \to B \to C = A \to D + D \to B + B \to C = 0 + \frac{Qs_1}{A\varepsilon_0} + 0$$

(b) If $Q = 43\mu C$, $R = 4.0m$, $s_1 = 1.5mm$, and $s_2 = 0.7mm$, what is the value of $\Delta V = V_c - V_A$?

$$\frac{Qs_1}{A\varepsilon_0} = \frac{43\cdot 10^{-6}\cdot(0.0015)}{\pi(4.0)^2\varepsilon_0}\frac{J}{C} = \frac{1}{4\pi\varepsilon_0}\frac{43\cdot 10^{-6}\cdot(0.0015)}{4.0}\frac{J}{C} = 150\frac{J}{C}$$

to the proper number of significant digits.

(c) Choose two different paths from point A back to point A again and show that $\Delta V = 0$ for a round trip along both of these.

$$A \to B \to C \to A = A \to B + B \to C + C \to A = \frac{Qs_1}{A\varepsilon_0} + 0 - \frac{Qs_1}{A\varepsilon_0} = 0$$

$$A \to D \to A = A \to D + D \to A = 0 + 0 = 0$$