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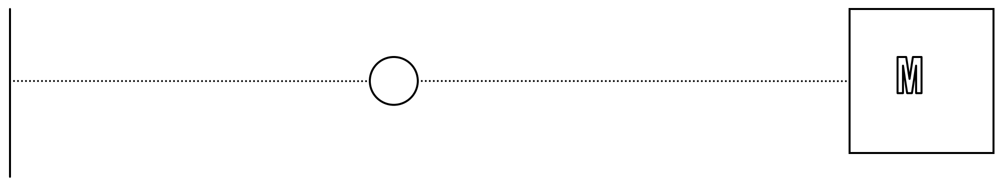
Physics 1 – Demonstration

The purpose of this problem is to show an extreme example of your procedure for solving free-body diagram problems.

Problem: Suppose two springs are attached end-to-end by a massless ring, and then a mass is attached to the far end and a wall is attached to the other, as shown below. What is the effective spring constant of the pair?



Define the System:

The system consists of two springs, a ring where they attach, and a box of mass M attached to the far end. Note that here I have to define two separate systems: one is the ring, one is the box.

Define what is inside the System:

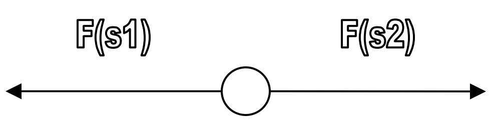
The ring and the box are inside their respective systems. The springs are to be treated as external forces.

Define what is outside the System:

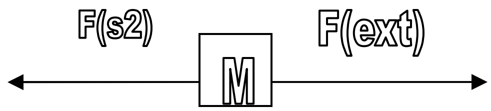
I consider an external force pulling on the box (someone hypothetically tugging on the box and feeing a spring force), and the two springs.

Draw a Free-Body Diagram:

Ring: (the ring was defined as massless—so I hope there is no net force on it!)



Box:



In both cases, both momentum and the change in momentum is zero. Thus, I consider this system in equilibrium.

Define a coordinate system:

Let x_1 be how far spring 1 has stretched, and x_2 be how far spring 2 has stretched.

Write $\Delta\vec{p}_{tot} = \vec{F}_{net,ext}\Delta t$ on your paper.

In component, form, for the ring, I have:

$$\Delta p_x = 0 = (k_1 x_1 - k_2 x_2)\Delta t$$
$$k_1 x_1 - k_2 x_2 = 0$$

Note that the spring was defined as massless, so I can only treat this in equilibrium.

For the box, I have:

$$\Delta p_x = 0 = (k_2 x_2 - F_{ext})\Delta t$$
$$k_2 x_2 - F_{ext} = 0$$

Now consider what I mean by the external force. Precisely, I want it to be a spring force, something like: $F_{ext} = k_{tot}(x_1 + x_2)$.

Solve each component equation:

$$k_1 x_1 - k_2 x_2 = 0$$
$$k_2 x_2 - k_{tot}(x_1 + x_2) = 0$$

Solving,

$$k_2 x_2 - k_{tot}\left(\frac{k_2}{k_1} x_2 + x_2\right) = 0$$
$$k_2 - k_{tot}\left(\frac{k_2}{k_1} + 1\right) = 0$$
$$k_{tot} = \frac{k_2}{\frac{k_2}{k_1} + 1} = \frac{1}{\frac{1}{k_2} + \frac{1}{k_1}}$$

So we have shown an "addition formula" for springs in series. If I had three springs in series, I could add the first two (considering the remaining part the "system"), then add the third one to that summed pair with the same formula!